A THEORY OF THE EARTH'S PRECESSION RELATIVE TO THE INVARIABLE PLANE OF THE SOLAR SYSTEM

By

WILLIAM MANN OWEN, JR.

A DISSERTATION PRESENTED TO THE GRADUATE SCHOOL
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DEDICATION

In the wee hours of March 13, 1960, a five-year-old boy was awakened by his grandfather in order to see a total eclipse of the moon. Thirty years have passed; the boy has become a man, and his interest in astronomy, planted that night, has taken root and blossomed. It is therefore to the memory of

R. Waldo Hambleton (1901–1981)

that this dissertation is lovingly dedicated.

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No enterprise of this nature can be accomplished without the support of many others. My chairman, Dr. Heinrich K. Eichhorn, and my colleague at the Jet Propulsion Laboratory, Dr. Jay H. Lieske, reviewed early versions of the manuscript; the quality of this dissertation has been helped immeasurably by their critiques. The author benefited from many conversations with them and with Drs. E. Myles Standish and X X Newhall of JPL and with Dr. Jacques Laskar of the Bureau des Longitudes. Laskar's cooperation in providing his tabulation of the Earth's orbital parameters in advance of publication is greatly appreciated. The unwavering encouragement of my supervisors at JPL, Drs. Stephen P. Synnott and James P. McDanell, was most helpful as well. Finally, the author is indebted to Eva Eichhorn for translating the 1967 paper by Sharaf and Budnikova.

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The $\TeX$ program (Knuth 1984) typeset the dissertation. The PGPLOT subroutine library (Pearson 1989) was used to create the figures. The high quality of the appearance of this dissertation is due to these two programs; nevertheless, any defects in layout or errors in substance are solely the responsibility of the author.

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William Mann Owen, Jr.

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The most commonly used formulas for the rigorous application of general precession employ three successive small rotations by which the equatorial coordinate system at epoch is aligned with the equatorial system of date. This dissertation presents an alternate method for applying precession in which the rotation angles are intimately tied to the invariable plane of the Solar System.

The work involved in constructing this theory divides neatly into three parts. First, the newest planetary ephemeris from the Jet Propulsion Laboratory, produced after the Voyager 2 encounter with Neptune, allows one to infer the orientation of the invariable plane with a standard error on the order of $0''.04$. Second, the coefficients in the approximation polynomials for the new angles are found in terms of their counterparts in the currently-accepted IAU theory; this "short-term theory," like the IAU one, is valid for a few centuries near the present time. Third, the motion of the Earth's north pole is integrated numerically over a million-year time span; values for both the standard precession angles and the new ones are inferred at discrete times from the integration, and Chebyshev polynomials are fit to these values, giving the "long-term theory." Both the long-term and short-term theories are simpler to use than the current ones.

A comparison of the long-term and short-term theories reveals two possible improvements to the IAU system of constants. The higher-order terms in Kinoshita's model for the rigid Earth, used in the long-term theory, change the time derivative of Newcomb's Precessional Constant from $-0\overset{''}{.}00369$ per Julian century to $-0\overset{''}{.}00393$/century at the standard epoch J2000.0. Laskar's investigation into the motion of the ecliptic, also used in the long-term theory, yields $-46\overset{''}{.}8065$/century for the rate of change of the obliquity versus the currently-adopted $-46\overset{''}{.}8150$/century.

CHAPTER 1
INTRODUCTION

1.1. Background

The slow motion of the celestial pole (however one defines it) among the stars and the concomitant slow westward drift of the location of the vernal equinox are the result of the physical process known as astronomical precession. The former effect results from the gravitational interaction of the Sun and Moon with the oblate Earth; this produces a torque which changes the direction of the Earth's rotational angular momentum vector. The short-period portion of this phenomenon is known as "nutation" and will not be treated in depth here; the long-period part is "luni-solar precession." (As the longest nutation period is about 18.6 years while the shortest period of luni-solar precession is about 25,000 years, there is a clean division between the two manifestations of the same physical process.) In addition, the gravity of the other planets in the Solar System perturbs the Earth's orbit, changing the orientation of its mean orbital plane (the ecliptic). The influence of this orientation change on the direction of the mean vernal equinox is called "planetary precession." Since the vernal equinox lies at the intersection of the Earth's mean equator and the ecliptic, its location is affected by both motions. The resultant of luni-solar and planetary precession is accordingly called "general precession."

Luni-solar precession was discovered in the second century B.C. by Hipparchus, who noted systematic differences between ecliptic longitudes which he observed and those observed by Timocharis over a century earlier (Lieske 1985). The speed of planetary precession is slower by a factor of about 500 than that of luni-solar precession; consequently the

motion of the ecliptic was not suspected until Newton applied his theory of gravitation to the motions of the planets.

The theory of general precession is both dynamic and kinematic in nature. The dynamics enters into the differential equations of motion for the ecliptic (or equivalently for the pole of the ecliptic) and for the equator (or the celestial pole). Once these equations are integrated, the motion of the vernal equinox and of the equatorial and ecliptic coordinate axes becomes a problem in rotational kinematics. Precession theory accordingly is developed in terms of angles by which one can rotate coordinate frames from their orientation at one epoch to that at another.

The current theory of precession was developed by Lieske *et al.* (1977). Its time origin is the beginning of the astronomical Julian year 2000, or noon dynamical time on 2000 January 1 (Julian Ephemeris Date 2451545.0). This zero epoch is denoted "J2000.0," and time T in this theory is measured in units of Julian centuries of 36525 days of dynamical time. (The current distinction between "terrestrial dynamical time" and "barycentric dynamical time," a periodic difference of no more than two milliseconds, is of no consequence in a theory whose rates do not exceed two degrees per century.) The paper of Lieske *et al.* is based on Fricke's (1971) determination of the speed of luni-solar precession and on Newcomb's (1894) work on the motion of the ecliptic, the latter updated with modern values of the masses of the planets.

While the rotation matrix approach to precession is fully rigorous, giving exact results for the effects of precession on equatorial coordinates, the formulas for evaluating the precession angles themselves are only approximate: Lieske *et al.* provide third-degree polynomials in T for each angle. These polynomials are accurate to within a milliarcsecond for about two centuries on either side of J2000.0, so they are easily sufficient for the

reduction of modern precise astrometric observations. However, a blind application of the polynomials over millenia will produce grotesquely wrong results.

Several authors have examined precession theory over much longer time intervals. Here one generally expresses the orientation of the ecliptic as a sum of trigonometric terms derived from long-period planetary perturbations; expressions for the accumulated general precession in longitude follow. This method was developed by Brouwer and van Woerkom (1950), corrected by Sharaf and Budnikova (1967), and extended to five million years by Berger (1976). Berger's work was based on Bretagnon's (1974) theory, which was complete to second order in the planetary masses and to third degree in eccentricity and inclination.

The goal of this dissertation is to develop a theory of precession in which the invariable plane of the Solar System is used as a fundamental reference plane. Precession angles referred to the invariable plane can be used in both short-term and long-term applications. With a suitable change in the origin of right ascensions, the new precession theory becomes simpler computationally than the current theory.

The work described here divides neatly into three parts. In Chapter 2 the orientation of the invariable plane is found from the most recent planetary ephemeris produced at the Jet Propulsion Laboratory. Chapter 3 presents formulas by which the polynomial coefficients of the new precession angles may be calculated from those of the current theory; the results when the values in the Lieske *et al.* (1977) paper are adopted may be called the "short-term theory." Chapter 4 is devoted to the "long-term theory," developed by numerical integration of the position of the celestial pole; the equations of motion are those of Kinoshita (1977), while the "Numerical General Theory" of Laskar (1985) provided the orientation of the ecliptic. A final chapter compares the two theories.

The remainder of this chapter presents the notation and definitions that are used throughout the dissertation.

1.2. Notation

Table 1–1, found at the end of this chapter, presents a list of all the symbols used throughout this dissertation with their definitions and the number of the section in which the symbol first appears. The table contains Latin symbols first, then Greek, then special astronomical symbols, with the first two parts in alphabetical order.

Scalar quantities (including components of vectors and elements of matrices) are set in italic type throughout this work. Vectors are given in boldface roman type; individual rows or columns of matrices are considered to be vectors for this purpose. Matrices appear in a boldface sans-serif font; the same font is used for both rotation matrices and covariance matrices. This scheme follows standard typographical practice for scalars and vectors; the typography for matrices is that used by Goldstein (1980).

This paper follows commonly accepted notation for the precession angles. The system of notation for the coefficients of these angles pioneered by Lieske *et al.* (1977) is complete and unambiguous; at the same time the diacritical marks can be awkward. Deviations from their system are carefully noted both in Table 1–1 and at their first use in the text.

Equations are numbered by chapter in order of their first appearance. When an equation appears more than once, all occurrences are labeled with the original equation number.

1.3. Definitions

Insofar as possible, all the terminology used in this dissertation is currently in use by the astronomical community. An excellent glossary appears in section M of the annual volumes of *The Astronomical Almanac*, published jointly by the United States Naval Observatory and the Royal Greenwich Observatory. The reader is referred there for terms not defined below.

A "coordinate system" is a set of three mutually perpendicular Cartesian coordinate axes, with equal units of length along each axis. All coordinate systems used in this

dissertation are right-handed: if the three axes be denoted (in order) by x, y, and z, and if the x- and y-axes are placed on a piece of paper such that x increases to the right and y increases upward, then the z-axis points out of the paper toward the observer.

The rectangular components of a vector are taken to form a column vector (a matrix with only one column). Vectors are also specified in terms of their spherical coordinates: length (or magnitude) r, longitude angle λ, and latitude angle β. The transformation from $\{r, \lambda, \beta\}$ into rectangular components $(x, y, z)^{\mathrm{T}}$ is

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} r\cos\lambda\cos\beta \\ r\sin\lambda\cos\beta \\ r\sin\beta \end{pmatrix}; \tag{1–1}$$

this is a vector equation. The inverse transformation consists of the three scalar equations

$$r = \sqrt{x^2 + y^2 + z^2}; \tag{1–2}$$

$$\lambda = \mathrm{plg}(y, x); \tag{1–3}$$

$$\beta = \tan^{-1}(z/\sqrt{x^2 + y^2}). \tag{1–4}$$

The notation "$\mathrm{plg}(y, x)$" in equation (1–3) was introduced by Eichhorn (1987/88) as an alternate to the more usual $\tan^{-1}(y/x)$. The range of the plg function is $0 \leq \mathrm{plg}(y, x) < 360°$; the quadrant depends on the signs of x and y individually. In the Fortran programming language, λ would be evaluated as ATAN2 (y, x). No such confusion exists for equations (1–2) and (1–4); r is nonnegative, and β has the same range as the principal value of the arctangent. Equation (1–4) is given in terms of the arctangent rather than the more usual $\beta = \sin^{-1}(z/r)$ for numerical reasons: the arcsine loses its precision when its argument approaches ± 1. Finally, if $r = 0$, the equations for λ and β are technically indeterminate; the angles by convention assume their limiting values as $r \to 0$.

Rotations are given in matrix form. A rotation matrix is thought of as an operator rotating the coordinate axes rather than as a physical rotation of the vector to which

the matrix is applied. There are three elementary rotation matrices, denoted $\mathbf{R}_i(\theta)$, each producing a rotation by some angle θ about one of the three coordinate axes. The standard sign convention applies to the angles; for instance, a rotation about the z-axis by a positive angle carries the x-axis toward the old y-axis. Written out, the three elementary rotation matrices are

$$\mathbf{R}_1(\theta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{pmatrix}; \tag{1–5}$$

$$\mathbf{R}_2(\theta) = \begin{pmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{pmatrix}; \tag{1–6}$$

$$\mathbf{R}_3(\theta) = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{1–7}$$

Precession theory is intimately concerned with the orientation of the "mean equatorial coordinate system." The z-axis (denoted $\mathbf{Q}$) of this system is directed to the mean Celestial Ephemeris Pole; the x-axis is directed toward the mean vernal equinox and is symbolized by the "ram's horns" of Aries (♈); the y-axis ($\mathbf{y}_Q$), directed toward right ascension 6^{h} and declination 0°, completes the right-handed triad. (The word "mean" in this context indicates that the effects of nutation are disregarded.) This system is also called the "mean equator and equinox," either "of epoch" or "of date." The "epoch" is the initial time for precession, usually J2000.0, and the "date" refers to an arbitrary final time for precession. Figure 1–1 shows the coordinate axes of this system, with circles marking the planes of the equator and the ecliptic.

The other coordinate system used in this work is the "ecliptic coordinate system," also called the "ecliptic and equinox of epoch" or "of date." Here the z-axis is directed toward the north ecliptic pole; the x-axis is identical to that of the mean equatorial coordinate

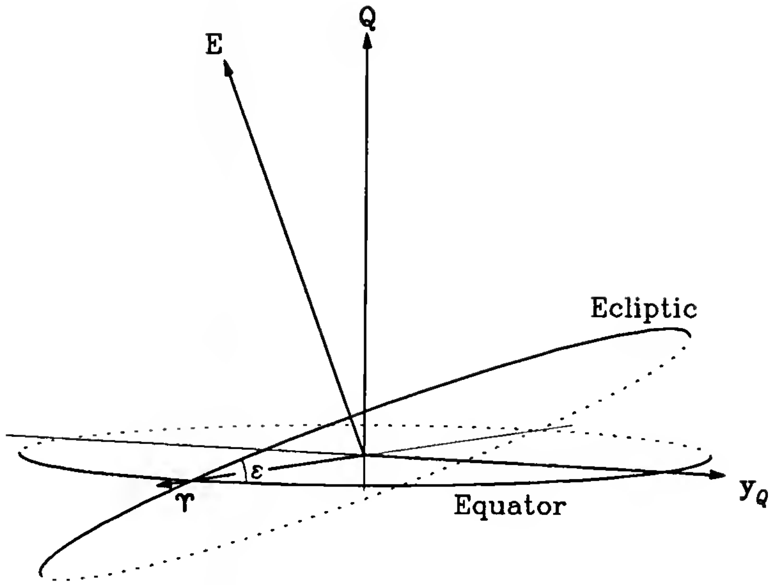

Figure 1–1. The Equatorial Coordinate System

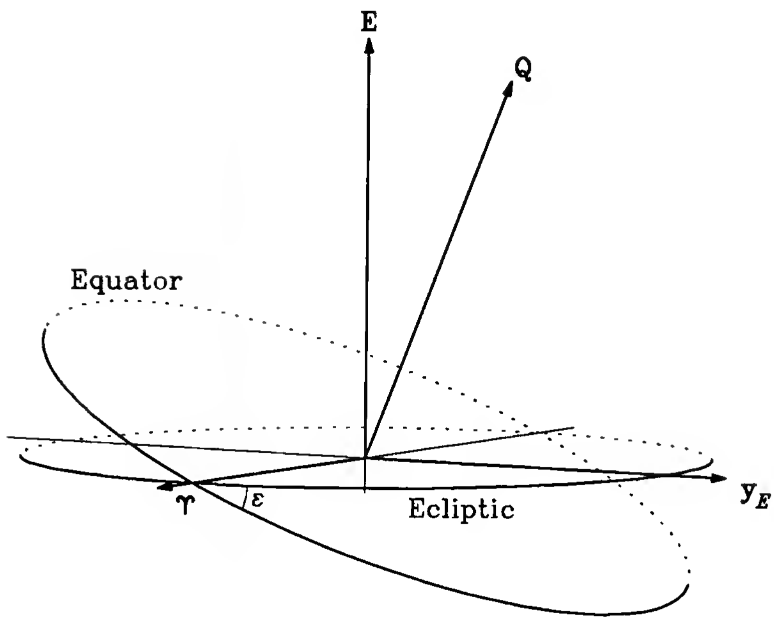

Figure 1–2. The Ecliptic Coordinate System

system; and again the y-axis completes the right-handed triad. The basis unit vectors are denoted Υ for the x-axis, $\mathbf{y}_E$ for y, and $\mathbf{E}$ for z. Figure 1–2 depicts the ecliptic coordinate system. The vectors $\mathbf{E}$ and $\mathbf{Q}$ appear in both figures; the angle between them is ε, the obliquity of the ecliptic.

For the sake of brevity, Eichhorn's (1987/88) single-letter notation for coordinate systems will often be used. The mean equatorial system of date is called the "Q system," and the ecliptic coordinate system of date is likewise called the "E system." Their counterparts at epoch carry a subscript zero: Q_0 and E_0, respectively. The orientation of the Q_0 system is defined implicitly by the right ascensions and declinations at J2000.0 of the 1535 fundamental stars in the FK5 star catalog (Fricke *et al.* 1988).

Two other equatorial systems are still in frequent use. The "B1950 (FK4) system" is defined by the positions and proper motions of the FK4 star catalog (Fricke and Kopff 1963) at epoch B1950.0 (the beginning of Besselian year 1950, or Julian Ephemeris Date 2433282.42345905). Observations subsequent to those used in the FK4 have revealed a small but significant systematic error in the FK4 proper motions in right ascension. Consequently this system, although intended to be inertial, is in slow rotation about its z-axis. (No doubt future observations will reveal a similar flaw, albeit of smaller magnitude, in the FK5.)

By contrast, the planetary ephemerides developed at the Jet Propulsion Laboratory (JPL) are integrated numerically in a coordinate system that is inertial by construction. The latest available ephemeris, produced for the Voyager project after the Neptune encounter, has its coordinate axes aligned with the B1950 (FK4) system above at epoch B1950.0. This non-rotating system will be denoted "EME50" in accordance with JPL parlance in order to distinguish it from the rotating B1950 (FK4) system.

Table 1–1. Definitions of Symbols

Symbol	Section	Definition
$\mathbf{A}$	4.5	The rotation matrix which transforms from the mean ecliptic and equinox of J2000.0 into the mean equator and equinox of date.
A	4.2.3 3.1	1) The smallest principal moment of inertia of the Earth. 2) Subscript denoting the accumulated precession angle rather than its rate.
a	2.5	The semimajor axis of an orbit, usually with a subscript to indicate the orbiting body.
a_k	2.2	Coefficients of Chebyshev polynomials obtained from JPL planetary ephemerides.
B	1.3	Prefix denoting a Besselian year.
B	4.2.3 2.4	1) The intermediate principal moment of inertia of the Earth. 2) Subscript denoting the Earth-Moon barycenter.
B1950	1.3	The mean equatorial coordinate system at epoch B1950.0; used with a following (FK4) to denote the rotating system of the FK4 star catalog.
B1950.0	1.3	The instant of time corresponding to the start of the Besselian year 1950; Julian Ephemeris Date 2433282.42345905.
C	4.2.3	The largest (polar) principal moment of inertia of the Earth.
c	2.5 4.2	1) The speed of light *in vacuo*. 2) The product $\sin \pi_A \cos \Pi_A$.
c_k	3.3.1	The coefficient of T^k in the expression for the cosine of the angle I.
$\mathbf{D}$	2.5	The Jacobian of the transformation from rectangular coordinates into right ascension and declination.
d	3.3	The polynomial forming the denominator in the quotient n/d.
d_k	3.3	The coefficient of T^k in the denominator polynomial d.
$\mathbf{E}$	4.5	The rotation matrix which transforms from E_0 to E coordinates.
$\mathbf{E}$	1.3	The north ecliptic pole vector.

Table 1–1, continued.

Symbol	Section	Definition
E	1.3	The coordinate system defined by the ecliptic and mean equinox of date.
E_0	1.3	The coordinate system defined by the ecliptic and mean equinox of epoch.
e	2.5	The orbital eccentricity, often used with a subscript to denote the orbiting body.
EME50	1.3	The nonrotating coordinate system aligned with the B1950 (FK4) system at epoch B1950.0.
G	2.1	The universal constant of gravitation.
$\mathbf{H}$	2.5	The Jacobian of the transformation from Set III coordinates into rectangular coordinates in the orbital system.
H	4.3.3	The ratio of the moments of inertia of the Earth.
$\mathbf{h}$	2.4	An angular momentum vector.
$\hat{\mathbf{h}}$	2.5	The unit vector in the direction of the orbital angular momentum of a planet.
h	2.6	1) The magnitude of angular momentum.
	4.2	2) One of the two rectangular components giving the eccentricity and longitude of perihelion of the Earth's orbit.
	4.7	3) The stepsize in a numerical integration.
I	2.6	1) The rotational moment of inertia of a body.
	3.1	2) The inclination of the invariable plane to the mean equator of date.
I_0	3.1	The inclination of the invariable plane to the mean equator of epoch.
I_k	3.3.1	The coefficients of T^k in the polynomial expansion of I.
i	2.4	1) Running index over all bodies on a JPL planetary ephemeris file.
	2.5	2) The inclination of a planetary orbit to the EME50 x-y plane.
	4.6	3) Ordinal number of the set of Chebyshev coefficients to be used for finding precession angles at a given date in the long-term theory.

Table 1–1, continued.

Symbol	Section	Definition
J	1.3	Prefix denoting a Julian year.
J2000	1.3	The mean equatorial coordinate system at epoch J2000.0, as realized by the FK5 star catalog.
J2000.0	1.3	The standard epoch for precession theory: Julian Ephemeris Date 2451545.0, or 2000 January 1 12:00 dynamical time.
k	2.1 3.2 4.2	1) The Gaussian gravitational constant. 2) A running index, or the power of time T. 3) One of the two rectangular components giving the eccentricity and longitude of perihelion of the Earth's orbit.
$k_\odot$	4.3.3	Fundamental rate of luni-solar precession attributable to the Sun.
$k_☾$	4.3.3	Fundamental rate of luni-solar precession attributable to the Moon.
L	3.1	The angle, measured along the equator of date, from the mean vernal equinox of date to the intersection of the invariable plane and the equator of date; the right ascension of this point.
L_0	3.1	Same as above, but using the equator and mean vernal equinox of epoch.
L_k	3.3.2	The coefficients of T^k in the polynomial expansion of L.
$\mathbf{M}$	2.5	The rotation matrix which transforms from the EME50 system into the orbital system (p, q, h) of a planet.
M	2.5	The mean anomaly of a planet.
M_k	4.3.3	The lunar coefficients in Kinoshita's expression for luni-solar precession.
m	2.1	Mass.
$\overline{m}$	2.5	The reciprocal mass of a planet $(m_\odot/m)$.
$\mathbf{N}$	5.2	The rotation matrix which transforms from mean equatorial coordinates to true equatorial coordinates; the "nutation matrix."
$\mathbf{N}'$	5.2	The analog to $\mathbf{N}$ above with the intersection of the invariable plane and equator replacing the vernal equinox.

Table 1–1, continued.

Symbol	Section	Definition
$\mathbf{N}$	4.5	The vector directed toward the ascending node of the ecliptic of J2000 on the ecliptic of date (for $T > 0$) or toward the descending node if $T < 0$.
$\mathbf{n}$	2.7	The vector directed toward the ascending node of the invariable plane on the mean equator of J2000.
n	2.2	1) The degree of the highest Chebyshev polynomial whose coefficient is included in the data records of a JPL planetary ephemeris file.
	3.3	2) The polynomial forming the numerator in the quotient n/d.
	4.3.3	3) The mean motion of a planet.
n_k	3.3	The coefficient of T^k in the numerator polynomial n.
n_{Ω}	4.3.3	The rate of regression of the nodes of the Moon's orbit on the ecliptic (a negative quantity).
O	2.1	Order of terms which are omitted from an equation, as in $O(T^5)$.
P	3.1	The rotation matrix which transforms from the mean equator and equinox of epoch to the mean equator and equinox of date; the "precession matrix."
P'	3.6	The analog to P above with the intersection of the invariable plane and equator replacing the vernal equinox.
P	4.3.1	Newcomb's "Precessional Constant." This is not the same as the so-called "constant of precession" p below.
P_k	4.3.1	The coefficient of T^k in the polynomial for Newcomb's "Precessional Constant."
$\hat{\mathbf{p}}$	2.5	The unit vector directed toward perihelion of a planet.
p	4.2	1) One of the two rectangular components giving the inclination and node of the ecliptic of date on the ecliptic of J2000.
	4.3.1	2) The speed of general precession in longitude, also known as the "constant of precession."
p_A	4.5	The accumulated general precession in longitude from J2000 to date; denoted $\tilde{p}_A$ by Lieske *et al.*(1977).
p_g	4.3.1	The rate of geodesic precession.

Table 1–1, continued.

Symbol	Section	Definition
p_1	4.4	The speed of general precession in longitude at J2000.
$\mathbf{Q}$	4.5	The rotation matrix which transforms from the mean equator and invariable plane of J2000 to the mean equator and vernal equinox of date.
$\mathbf{Q}$	1.3	The vector directed toward the Celestial Ephemeris Pole.
Q	1.3	The coordinate system defined by the mean equator and vernal equinox of date.
Q_0	1.3	The coordinate system defined by the mean equator and vernal equinox of epoch.
$\hat{\mathbf{q}}$	2.5	The unit vector directed toward 90° true anomaly in the orbital plane of a planet.
q	3.3 4.2	1) The quotient of two polynomials, $q = n/d$. 2) One of the two rectangular components giving the inclination and node of the ecliptic of date on the ecliptic of J2000.
q_k	3.3	The coefficient of T^k in the quotient of two polynomials.
$\mathbf{R}_i(\theta)$	1.3	The elementary rotation matrix which rotates the coordinate system by angle θ about axis i, where $i = 1$, 2, or 3 for the x-, y-, or z-axis.
R	2.6 4.3.3	1) The radius of a rotating body. 2) Kinoshita's rate of luni-solar precession.
$\mathbf{r}$	2.4	The position of a body relative to the barycenter of the Solar System.
r	1.3 2.4	1) The magnitude of an arbitrary vector. 2) The magnitude of the difference of the barycentric position vectors of two bodies.
$\mathbf{S}_{\mathrm{III}}$	2.5	The covariance matrix of the Set III elements and mass of a planet.
$\mathbf{S}_{pqh}$	2.5	The covariance matrix of the angular momentum of a planet referred to its orbital plane.

Table 1–1, continued.

Symbol	Section	Definition
$\mathbf{S}_{xyz}$	2.5	The covariance matrix of the angular momentum of a planet referred to B1950.0 equatorial coordinates.
$\mathbf{S}_{\alpha\delta}$	2.5	The covariance matrix of the orbital angular momentum of the Solar System in terms of the right ascension and declination of the angular momentum vector.
S_k	4.3.3	The solar coefficients in Kinoshita's expression for luni-solar precession.
s	2.6	1) In cylindrical coordinates, the distance from a point to the z-axis.
	3.2	2) The quantity $\sin \pi_A \sin \Pi_A$.
$\mathbf{T}$	2.7	The rotation matrix which transforms from the EME50 system (as realized by the DE130 planetary ephemeris) into the J2000 system (as realized by the DE202 planetary ephemeris).
$^{\mathrm{T}}$	1.3	The transpose of a matrix or column vector.
T	1.1	Time, measured in Julian centuries past J2000.
T_c	4.6	The central time in a time interval of the table of long-term Chebyshev coefficients.
T_1	2.2	1) The earliest time covered by one record of a JPL planetary ephemeris file.
	3.5	2) The initial time when one desires to apply precession between two arbitrary times.
T_2	2.2	1) The latest time covered by one record of a JPL planetary ephemeris file.
	3.5	2) The final time when one desires to apply precession between two arbitrary times.
$T_k(x)$	2.2	The Chebyshev polynomial of the first kind of degree k.
t	3.5	The time interval $T_2 - T_1$ when one desires to apply precession between two arbitrary times.
$\mathbf{v}$	2.4	The velocity of a body relative to the barycenter of the Solar System.
v	2.4	The speed of a body relative to the barycenter of the Solar System.

Table 1–1, continued.

Symbol	Section	Definition
x	1.3	The first Cartesian coordinate axis, or the projection of a vector in that direction.
$\mathbf{y}_E$	1.3	The unit vector in the y direction of the E system.
$\mathbf{y}_Q$	1.3	The unit vector in the y direction of the Q system.
y	1.3	The second Cartesian coordinate axis, or the projection of a vector in that direction.
z	1.3	1) The third Cartesian coordinate axis, or the projection of a vector in that direction.
	3.1	2) The accumulated angle, measured along the equator of date, from the intersection of the equator of date and equator of epoch to the y-axis of the mean equatorial system of date; this is denoted as $\tilde{z}_A$ by Lieske *et al.* (1977).
z_A	4.6	Another notation for $\tilde{z}_A$ above.
z_k	3.3	The coefficients of T^k in the polynomial expansion for $\tilde{z}_A$; the coefficients z_2 and z_3 are denoted z_1' and z_1'' respectively by Lieske *et al.* (1977).
α	2.4	1) Right ascension, in particular of the angular momentum vector of the Solar System.
	4.6	2) Any one of the precession angles.
α_0	2.6	The right ascension of the angular momentum vector of the Solar System in the Q_0 system.
$\alpha_k^{(p)}$	5.1	The coefficient of T^k (a polynomial coefficient) for any one of the precession angles.
$\alpha_k^{(C)}$	5.1	The coefficient of $T_k(\tau)$ (a Chebyshev coefficient) for any one of the precession angles.
β	1.3	The generalized latitude angle of a vector.
γ	2.6	The coefficient of moment of inertia of a body.
Δ	3.1	The angle, measured along the invariable plane, from the intersection of the invariable plane and the mean equator of J2000 to the intersection of the invariable plane and the mean equator of date.

Table 1–1, continued.

Symbol	Section	Definition
Δ_k	3.3.3	The coefficient of T^k in the polynomial expansion for Δ.
ΔI	5.2	The difference between the inclination of the invariable plane to the true equator of date and the inclination of the invariable plane to the mean equator of date.
Δp	2.5	The Set III parameter specifying a rotation of an orbital plane about its line of apsides.
Δq	2.5	The Set III parameter specifying a rotation of an orbital plane about the latus rectum.
$\Delta\varepsilon$	5.2	Nutation in obliquity.
$\Delta\psi$	5.2	Nutation in longitude.
δ	2.4	Declination, in particular of the angular momentum vector of the Solar System.
δ_0	2.5	The declination of the angular momentum vector of the Solar System in the Q_0 system.
$\delta\Delta$	3.4	1) The difference between the angle Δ (*q.v.*) as computed from the rigorous equation and from the polynomial approximation.
	5.2	2) The difference between the angle Δ measured to the true equator of date and the analogous angle measured to the mean equator of date.
δI	3.4	The difference between the angle I (*q.v.*) as computed from the rigorous equation and from the polynomial approximation.
δL	3.4	The difference between the angle L (*q.v.*) as computed from the rigorous equation and from the polynomial approximation.
ε	1.3	The obliquity of the ecliptic; the inclination of the ecliptic of date to the equator of date.
ε_0	4.3.1	The obliquity of the ecliptic at J2000.
ζ	3.1	The accumulated angle, measured along the equator of J2000, from the y-axis of the Q_0 system to the intersection of the equator of J2000 and the equator of date; denoted $\tilde{\zeta}_A$ by Lieske *et al.* (1977).

Table 1–1, continued.

Symbol	Section	Definition
ζ_A	4.6	Another notation for $\tilde{\zeta}_A$ above.
ζ_k	3.3	The coefficients of T^k in the polynomial expansion for $\tilde{\zeta}_A$; the coefficients ζ_2 and ζ_3 are denoted ζ_1' and ζ_1'' respectively by Lieske *et al.* (1977).
θ	2.6 3.1	1) The longitude angle in cylindrical coordinates. 2) The dihedral angle between the equator of J2000 and the equator of date; the angle between the Celestial Ephemeris Poles of J2000 and of date; denoted $\tilde{\theta}_A$ by Lieske *et al.* (1977).
θ_A	4.6	Another notation for $\tilde{\theta}_A$ above.
θ_k	3.3	The coefficients of T^k in the polynomial expansion for $\tilde{\theta}_A$; the coefficients θ_2 and θ_3 are denoted θ_1' and θ_1'' respectively by Lieske *et al.* (1977).
Λ	4.6	The angle, measured in the ecliptic of date, from the mean vernal equinox of date to the intersecion of the ecliptic of date and the ecliptic of J2000; denoted $\tilde{\Lambda}_A$ by Lieske *et al.* (1977).
λ	1.3	The generalized longitude angle of a vector.
μ	2.1	The rest mass of a body multiplied by G.
μ^*	2.4	The relativistic mass of a body multiplied by G.
Π_A	4.2	The angle, measured in the ecliptic of J2000, from the mean vernal equinox of J2000.0 to the intersection of the ecliptic of date and the ecliptic of J2000.0; the J2000 ecliptic longitude of the ascending node of the ecliptic of date; denoted $\tilde{\Pi}_A$ by Lieske *et al.* (1977).
π_A	4.2	The angle between the ecliptic of date and the ecliptic of J2000, taken to be negative for $T < 0$; denoted $\tilde{\pi}_A$ by Lieske *et al.* (1977).
ϖ	4.2	The longitude of the Earth's perihelion in the E_0 system.
ρ	2.6	The density of a rotating object.
σ_x	2.5	The standard deviation (or dispersion) of the random quantity x.
τ	2.2	The dimensionless time argument to Chebyshev polynomials.

Table 1–1, continued.

Symbol	Section	Definition
ϕ	4.3.2	The colatitude angle of the Celestial Ephemeris Pole referred to the ecliptic and mean equinox of J2000.0.
ψ	4.3.1	The instantaneous speed of luni-solar precession; the rate at which the mean vernal equinox of date moves along the moving ecliptic of date.
ψ_1	4.3.1	The speed of luni-solar precession at J2000.
ψ_A	4.6	The angle, measured along the ecliptic of J2000, from the mean vernal equinox of J2000 to the intersection of the ecliptic of J2000 and the equator of date; the accumulated luni-solar precession; denoted $\tilde{\psi}_A$ by Lieske *et al.* (1977).
χ	4.3.1	The instantaneous speed of planetary precession; the rate at which the mean vernal equinox of date moves along the moving mean equator of date.
χ_1	4.3.1	The speed of planetary precession at J2000.
χ_A	4.6	The angle, measured along the equator of date, from the vernal equinox of date to the intersection of the mean equator of date and the ecliptic of J2000; the accumulated planetary precession; denoted $\tilde{\chi}_A$ by Lieske *et al.* (1977).
Ω	2.6 4.3.1	1) The rotation rate of a body. 2) The rotation rate of the Earth.
ω	2.5	The argument of perihelion of a planetary orbit, referred to the EME50 coordinate system.
ω_A	4.6	The inclination of the ecliptic of J2000 to the mean equator of date; denoted $\tilde{\omega}_A$ by Lieske *et al.* (1977).
$\odot$	2.4	The Sun.
☿	2.4	The planet Mercury.
♀	2.4	The planet Venus.
$\oplus$	2.4	The Earth.
♂	2.4	The planet Mars.

Table 1–1, continued.

Symbol	Section	Definition
♃	2.4	The planet Jupiter.
♄	2.4	The planet Saturn.
⛢	2.4	The planet Uranus.
Ψ	2.4	The planet Neptune.
♇	2.4	The planet Pluto.
☾	2.4	The Earth's Moon.
♈	1.3	The unit vector directed toward the mean vernal equinox of date.
☊	2.5	The longitude of the ascending node.

CHAPTER 2
THE DETERMINATION OF THE INVARIABLE PLANE

2.1. Introduction

The invariable plane of the Solar System is rigorously defined as that plane which contains the center of mass of the Solar System and is perpendicular to the total angular momentum of the Solar System. This definition holds for both classical and relativistic physics.

The "total angular momentum" must rigorously include rotational angular momentum as well as orbital angular momentum, and it must account for all the mass in the Solar System: satellites, asteroids, and comets in addition to the Sun and planets. Nevertheless, the major contributor to the total angular momentum of the Solar System is the orbital angular momentum of the planets, in particular of Jupiter.

The goal of this chapter is to determine the orientation of the invariable plane, or equivalently to estimate the direction of the total angular momentum. This task is made much easier thanks to the availability of computer-readable planetary ephemeris files, and it can provide reliable results now that the masses of the outer planets have been determined from spacecraft encounters.

The angular momenta of the largest asteroids must be included. The largest asteroid (1 Ceres) has about 6×10^{-10} solar mass (Schubart 1974; Newhall *et al.* 1983) or 6×10^{-7} Jupiter mass. With a semimajor axis of 2.767 AU (Inst. Teor. Astron. 1989), the magnitude of its angular momentum is 3×10^{-7} that of Jupiter. Due to the fairly low inclination of Ceres' orbit, the effect of Ceres on the direction of the invariable plane will be a factor

of about five less than this: about 6×10^{-8} radian, or $0''.01$. This is comparable to the uncertainty found in Section 2.5 and therefore must be included.

Smaller asteroids individually will have an effect correspondingly less. Taken as an ensemble, their angular momentum should lie very close to the normal to the invariable plane; thus while the magnitude of the total angular momentum of the Solar System would be increased, the direction of the total angular momentum would be virtually unchanged. So small asteroids, and by implication comets and smaller bodies, can be safely ignored at this time.

Rotational angular momentum will also be ignored, but for an entirely different reason: the large uncertainty in the Sun's rotational angular momentum. This topic is explored in more depth in Section 2.6.

The approach adopted here will be to compute the orbital angular momentum of each planetary system (comprised of the planet and its satellites) under the assumption that each planetary system is a point mass. Accordingly, this chapter will first describe the most recent planetary ephemeris file produced at the Jet Propulsion Laboratory. The relativistic equations giving the orbital angular momentum of the Sun and planets are presented next. Then the total angular momentum will be found and its uncertainty estimated.

For most planets, the product of the universal gravitational constant G and the body's mass is known much more precisely than the mass itself. This arises because only their product (typically denoted by μ) enters into the equations of motion. The value of G must be determined in the laboratory by measuring the gravitational force between two bodies of known mass; modern measurements (*e.g.*, Luther and Towler 1982) give its value to only five places. Since a vector maintains its orientation when multiplied by a non-zero constant scalar, the direction of the angular momentum of the Solar System does not change if all masses m_i are everywhere replaced by the corresponding μ_i.

Throughout this remainder of this chapter, the word "mass" will therefore be taken to mean $\mu_i \equiv Gm_i$ rather than m_i itself. "Mass" accordingly is measured in units of length^3/time^2. (In passing, the square of the Gaussian constant of gravitation k gives the solar mass, exactly 1 AU^3/day^2; if $\mu_\odot$ is specified in km^3/sec^2, the length of the AU follows.)

2.2. JPL Planetary Ephemerides

The planetary ephemeris files produced by the Jet Propulsion Laboratory provide the foundation for all NASA's deep-space missions. In addition to providing the position and velocity of the Sun, Moon, and nine planets, these files also include the values of the physical constants that were used in their generation. These constants include the speed of light, the length of the astronomical unit (expressed in kilometers), and the masses of the planets. The constants are read by the programs that integrate the equations of motion of natural satellites or of a spacecraft; this insures compatibility between files.

Planetary ephemeris files are produced by the "Solar System Data Processing System" (SSDPS), which was originally designed by Charles L. Lawson (1981) and is now maintained by E. Myles Standish, Jr., and X X Newhall. The SSDPS has three main functions: it reduces astrometric observations (both ground-based and from spacecraft) to determine a set of initial conditions for the Solar System bodies; it integrates the equations of motion; and it transforms the integrator output into an easily interpolated ephemeris file. The reader is referred to Newhall *et al.* (1983), Standish (1990*a*), or Newhall (1989) for a more complete description of the SSDPS; an overview of the system for our purposes here is presented below.

The coordinate system used in the SSDPS is relativistic: the isotropic "Parameterized Post-Newtonian" (PPN) metric (Will 1974; Newhall *et al.* 1983). The origin of the system is the Solar System barycenter. The z-axis is identical to the Earth mean celestial north

pole at a standard epoch, either B1950.0 (as realized by the FK4 catalog) or J2000.0 (FK5). The x-axis is directed to the corresponding (FK4 or FK5) "catalog equinox" (the zero point for measuring right ascensions) at that epoch. Since the equations of motion integrated by the SSDPS are expressed in an inertial coordinate system (*i.e.*, there are no Coriolis accelerations in the differential equations), the resulting ephemeris realizes, by construction, an inertial coordinate system. When the B1950.0 epoch is used, the resulting inertial coordinate system is termed EME50 (an abbreviation for "Earth mean equator of 1950") in order to distinguish it from the slowly rotating coordinate system of the FK4 catalog.

The first step in the production of a new planetary ephemeris is the estimation of the masses and initial positions and velocities of the principal bodies in the Solar System. The *a priori* model is an earlier ephemeris. The data (Standish 1990*a*) include ground-based optical observations, radar ranging data, and spacecraft ranging data. The latter two classes provide accurate synodic periods independent of transit circle observations; this in turn gives the planets' inertial mean motions. Since transit observations are made relative to fundamental stars, the offset and drift of the fundamental catalog (FK4 or FK5) vernal equinox are therefore visible in the residuals, and these quantities can be estimated.

The second step in the preparation of an ephemeris is the numerical integration. The equations of motion (Newhall *et al.* 1983) are fully relativistic in their treatment of the gravitational forces. Each planet other than the Earth is taken to be a point whose mass is the sum of the masses that comprise that planetary system. Each point mass therefore defines the barycenter of its planetary system. (This scheme is consistent with the other JPL ephemeris files; for instance, a satellite ephemeris file associated with a particular planet will yield positions of the satellites and of that planet relative to the same barycenter.)

Several asteroids are included as perturbing bodies; their orbits are given analytically, but they themselves are not integrated.

The Earth and Moon, however, are included as separate bodies. Here the low-order gravity harmonics of both Earth and Moon are modeled, as are the Earth tides and lunar librations. These accelerations are vital for the production of an accurate geocentric ephemeris for the Moon. Therefore the SSDPS integration is effectively of an eleven-body system: the nine planets, the Sun, and the Moon. The integrated positions and velocities are written to a file for further processing.

The third step produces the planetary ephemeris file itself. To save storage space, and to make interpolating easier, a planetary ephemeris file contains a Chebyshev representation of the motions of the planets (Newhall 1989). The final ephemeris file contains many records, with each record covering a fixed time interval. Each record in turn contains coefficients of Chebyshev polynomials used in the interpolation of position and velocity. If $T_1 \leq T \leq T_2$, where T is the desired interpolation time and T_1 and T_2 are the start and end times of the record bracketing T, then the x component of position is found by

$$x = \sum_{k=0}^{n} a_k T_k(\tau), \tag{2–1}$$

where the T_k are the Chebyshev polynomials of the first kind (see, for instance, Rivlin 1974); τ, in the range $|\tau| \leq 1$, is a dimensionless measure for time defined by

$$\tau = \frac{T - \frac{1}{2}(T_1 + T_2)}{\frac{1}{2}(T_2 - T_1)}; \tag{2–2}$$

the a_k are the coefficients read from the file; and n is the degree of the highest polynomial retained, typically 15. Similarly, the x component of velocity is found by differentiating equation (2–1):

$$\frac{dx}{dT} = \sum_{k=1}^{n} a_k T_k'(\tau) \frac{d\tau}{dT}. \tag{2-3}$$

The y and z components are found in the same manner. In practice, the values of $T_k(\tau)$ and $T_k'(\tau)$ are found recursively in a low-level subroutine. Users need only call a higher-level subroutine, passing the time and desired planets, and retrieving the position and velocity vectors.

The Chebyshev coefficients are fit to positions and velocities output from the numerical integrator; the fitting process, which is constrained to match both position and velocity across a record boundary, is described by Newhall (1989). The Chebyshev representation has been found to match the integrated trajectory to within a tolerance of 0.5 mm in position; this is many orders of magnitude below the accuracy to which the planets' positions are currently known.

2.3. The M04786 Planetary Ephemeris

The particular ephemeris file used in this work, known internally as M04786 after the identification number of the run which created it, has a slightly different history from the scheme outlined above. It was produced by Jacobson *et al.* (1990) using JPL's Orbit Determination Program (ODP; Moyer 1971) in the final steps of postflight analysis of the 1989 Voyager encounter with Neptune (Stone and Miner 1989). Its precursor, known within JPL as DE130, was created by Standish (1987) to serve as an *a priori* ephemeris for the Neptune encounter. In accordance with Voyager project requirements, DE130 (and therefore M04786) is on the EME50 system; a J2000 equivalent is known as DE202. These ephemerides supersede DE200, which is still used in the production of the annual volumes of the *Astronomical Almanac* (*e.g.*, USNO and RGO 1990). The DE130 ephemeris included five asteroids as perturbing bodies: 1 Ceres, 2 Pallas, 4 Vesta, 7 Iris, and 324 Bamberga.

These five asteroids were found to have the largest effect on the Viking lander ranging observations.

As Voyager encountered Neptune, the data it collected (both radiometric and optical) helped to render Neptune's ephemeris more accurate. The planetary ephemeris file used during encounter operations was accordingly updated from time to time. Due to time constraints, the full SSDPS was not executed. Rather, Neptune's orbit and mass were merely linearly corrected based on parameters estimated by the ODP.

The masses and other physical constants used by the M04786 ephemeris are presented in Table 2–1. As mentioned above, the "masses" all include a factor of G, and the satellites of the superior planets are included with their primaries.

Table 2–1. Physical Constants from the M04786 Ephemeris

Mass of Mercury, $\mu_{☿}$	22032.08045038011 km^3/sec^2
Mass of Venus, $\mu_{♀}$	324857.4782760278 km^3/sec^2
Mass of Earth, $\mu_{\oplus}$	398600.4420277941 km^3/sec^2
Mass of Mars system, $\mu_{♂}$	42828.28654533971 km^3/sec^2
Mass of Jupiter system, $\mu_{♃}$	126712700.9827376 km^3/sec^2
Mass of Saturn system, $\mu_{♄}$	37940448.53389772 km^3/sec^2
Mass of Uranus system, $\mu_{♅}$	5794559.117031542 km^3/sec^2
Mass of Neptune system, μ_{Ψ}	6836534.716231512 km^3/sec^2
Mass of Pluto plus Charon, $\mu_{♇}$	1020.864919671097 km^3/sec^2
Mass of the Sun, $\mu_{\odot}$	132712439800.9096 km^3/sec^2
Mass of the Moon, $\mu_{☾}$	4902.799066232991 km^3/sec^2
Mass of Earth plus Moon, μ_B	403503.2410940270 km^3/sec^2
Mass of 1 Ceres	$1.54674588 \times 10^{-13}$ AU^3/day^2
Mass of 2 Pallas	$3.8448884762 \times 10^{-14}$ AU^3/day^2
Mass of 4 Vesta	$7.2858525 \times 10^{-14}$ AU^3/day^2
Mass of 7 Iris	1.6×10^{-15} AU^3/day^2
Mass of 324 Bamberga	2.6×10^{-15} AU^3/day^2
Length of astronomical unit	149597870.6094344 km
Speed of light *in vacuo*, c	299792.458 km/sec
Earth/Moon mass ratio, $\mu_{\oplus}/\mu_{☾}$	81.300587

2.4. The Total Orbital Angular Momentum of the Solar System

Since a planetary ephemeris is produced according to the precepts of general relativity, finding the angular momentum of a body about the Solar System barycenter is not a trivial

task. The location of the Solar System barycenter is defined implicitly by equation (2) of Standish *et al.* (1976):

$$\sum_i \mu_i^* \mathbf{r}_i = \mathbf{0} \tag{2–4}$$

where $\mathbf{r}_i$ is the position of body i relative to the Solar System barycenter and μ_i^* is its relativistic mass (corrected for both special and general relativity effects). One calculates μ_i^* using equation (3) of the same report:

$$\mu_i^* = \mu_i \left[1 + \frac{v_i^2}{2c^2} - \frac{1}{2c^2} \sum_{j \neq i} \frac{\mu_j}{r_{ij}} \right] . \tag{2–5}$$

In this equation, μ_i is the rest mass of body i; $v_i = |\mathbf{v}_i| = |\dot{\mathbf{r}}_i|$ is the speed of body i relative to the Solar System barycenter; c is the speed of light; and $r_{ij} = |\mathbf{r}_i - \mathbf{r}_j|$ is strictly speaking the magnitude of the difference of the barycentric position vectors of body i and another body j. Newhall *et al.* (1983) point out that equations (2–4) and (2–5) are interdependent; in practice, the Sun's position is not integrated at all but is inferred from equation (2–4).

The orbital angular momentum of body i is then found by (Burkhardt 1982):

$$\mathbf{h}_i = \mu_i^* \mathbf{r}_i \times \mathbf{v}_i \tag{2–6}$$

and the total orbital angular momentum is

$$\mathbf{h} = \sum_{i=1}^{16} \mu_i^* \mathbf{r}_i \times \mathbf{v}_i . \tag{2–7}$$

The sixteen bodies are the nine planets, the Sun, and the Moon, all interpolated from the planetary ephemeris file; and the five asteroids, whose positions and velocities are found from the orbital elements input to the DE130 integration.

The M04786 ephemeris was interpolated at 1989 August 25 04:00 TDB (Julian Ephemeris Date 2447763.6667), the nominal epoch of the closest approach of Voyager 2 to Neptune;

this will presumably give the most accurate position and velocity for Neptune. The resulting positions, velocities, and angular momenta from equation (2–6) of the eleven bodies on the file are presented in Table 2–2; those of the five asteroids appear in Table 2–3; and the relativistic masses, from equation (2–5), are presented in Table 2–4. In Table 2–2, the solar position and velocity are found from equation (2–4). The total of these angular momenta is:

$$\mathbf{h} = \begin{pmatrix} 5.5138791989915901 \times 10^{16} \\ -8.1599811430560712 \times 10^{17} \\ 1.9241747810631041 \times 10^{18} \end{pmatrix} \ \mathrm{km}^5/\mathrm{sec}^3. \tag{2–8}$$

The spherical coordinates of $\mathbf{h}$ are

$$|\mathbf{h}| = 2.0907754056836565 \times 10^{18}\ \mathrm{km}^5/\mathrm{sec}^3; \tag{2–9}$$

$$\alpha = 273\overset{\circ}{.}865725677 = 18^{\mathrm{h}}15^{\mathrm{m}}27\overset{\mathrm{s}}{.}774162; \tag{2–10}$$

$$\delta = 66\overset{\circ}{.}972373413 = 66^{\circ}58'20\overset{''}{.}54429. \tag{2–11}$$

The right ascension and declination here are still expressed in the EME50 coordinate system of the ephemeris; the uncertainty in these numbers is discussed in the next section.

A straightforward application of equation (2–7) will not yield an unvarying result. The nodes on the ecliptic of the Moon's geocentric orbit regress with a well-known period of 18.6 years; this regression is the result of the gravitational influence of both the Earth's oblateness and the Sun. Since the Moon's geocentric orbital plane thus changes, the contribution of the Moon to $\mathbf{h}$ also varies. (The direction of the Earth's rotational angular momentum varies due to nutation with the same period—an equal and opposite effect—thus ensuring that the total angular momentum is conserved.) Therefore it is preferable to consider the Earth-Moon barycenter itself to be a single body, as is already done implicitly by the SSDPS for the superior planets. The Earth-Moon barycenter (denoted by subscript B) has position

Table 2–2. Vectors from the M04786 Ephemeris at 1989 August 25 04:00 ET

Body		**r**	**v**	**h**
Mercury	x	−16158199.07503421	37.68574922489486	$5.383098408817692 \times 10^{12}$
	y	−60540281.38189575	−6.219471575576255	$-2.812992786916346 \times 10^{13}$
	z	−30792661.60242493	−7.199241280123081	$5.248044464575123 \times 10^{13}$
Venus	x	−48044757.13184071	31.19917668472052	$7.071007368738365 \times 10^{13}$
	y	−89820030.22659636	−13.50936340740929	$-5.054773396094979 \times 10^{14}$
	z	−37464034.81962667	−8.058115422625290	$1.121201755436503 \times 10^{15}$
Earth	x	132534047.2682675	13.80094834015177	$3.138869950535301 \times 10^{10}$
	y	−66391831.78962909	23.89128363392751	$-7.056237004025107 \times 10^{14}$
	z	−28790114.33862247	10.35901588646045	$1.627357555590895 \times 10^{15}$
Mars	x	−244898357.5855488	−3.157305918724009	$5.705573328457442 \times 10^{12}$
	y	35884297.47698200	−19.87024113963734	$-9.788994312814919 \times 10^{13}$
	z	23023213.81212496	−9.036182675380297	$2.132629013701246 \times 10^{14}$
Jupiter	x	71436178.22611272	−13.17134485303879	$2.887703775435171 \times 10^{16}$
	y	699498249.8393588	1.568629228634501	$-5.068779699170997 \times 10^{17}$
	z	298310659.1621094	0.994759615211981	$1.181645297602056 \times 10^{18}$
Saturn	x	303660932.6890458	8.938087519218495	$2.082055996257906 \times 10^{16}$
	y	−1353840709.129601	1.928822496211554	$-1.990622886004587 \times 10^{17}$
	z	−573039586.0391354	0.411069166116023	$4.813297847774068 \times 10^{17}$
Uranus	x	176076406.2304637	6.742392857724443	$1.460888460184428 \times 10^{15}$
	y	−2646457517.863089	0.124628946369494	$-4.535922436215454 \times 10^{16}$
	z	−1162055888.894550	−0.040540240190316	$1.035221192520042 \times 10^{17}$
Neptune	x	843360977.1913863	5.305123897076706	$3.892369040033341 \times 10^{15}$
	y	−4101540837.662588	1.016154761387040	$-6.333659888390561 \times 10^{16}$
	z	−1701376011.969666	0.282701827614558	$1.546162148446206 \times 10^{17}$
Pluto	x	−3070732451.816337	4.079629959404747	$7.691217076519461 \times 10^{12}$
	y	−3201106574.346603	−3.842341036034878	$-8.016739555119871 \times 10^{12}$
	z	−80576284.87092758	−2.450284528360149	$2.537679451419032 \times 10^{13}$
Sun	x	−206331.5970958987	0.009329389609560	$-1.720703970228159 \times 10^{12}$
	y	46449.36015359861	−0.002139033914718	$-8.045154208580101 \times 10^{12}$
	z	17559.05332732530	−0.001087745178433	$1.062509844429565 \times 10^{12}$
Moon	x	132604706.2412162	12.79303203667523	$8.598343384697650 \times 10^{9}$
	y	−66064972.74889723	24.12556874324915	$-8.570777021009418 \times 10^{12}$
	z	−28613688.54883796	10.42258430286988	$1.982856317738149 \times 10^{13}$
Earth-Moon baryctr.	x	132534905.8158858	13.78870157083678	$4.008660038070628 \times 10^{10}$
	y	−66387860.26217532	23.89413033427148	$-7.141935944349106 \times 10^{14}$
	z	−28787970.66266322	10.35978827969271	$1.647184443007372 \times 10^{15}$

Table 2–3. Asteroid Vectors at 1989 August 25 04:00 ET

Body		**r**	**v**	**h**
Ceres	x	186281992.1788968	−16.35952815373246	92992435509.65316
	y	343470578.6841611	5.116105802899675	−214825510453.6537
	z	123850658.9230570	5.747722186944968	455899280135.5158
Pallas	x	436755840.8648572	−5.507766068499227	8893557968.250119
	y	38833040.99610688	15.64886644512920	24044304801.65627
	z	−39639922.64470550	−2.692689866417276	121544809850.9764
Vesta	x	125506706.2166484	19.46247210077522	26912693424.32996
	y	−275465663.8456852	7.392776730344995	−82010229009.18285
	z	−126367091.5783639	0.401434801390846	205502281954.1065
Iris	x	−409332580.4006283	−1.124700364629995	−453228983.1614052
	y	−24051697.50313012	−15.12047499555895	−1827087511.594522
	z	−51893151.65421642	−6.362926495829524	4421892639.311001
Bamberga	x	−524855815.2412224	2.253161649691973	−818264398.8558691
	y	−53995531.06175173	−10.70612252024783	−4341672429.418912
	z	−99176075.74679006	−6.668306875225594	6694172275.600386

Table 2–4. Relativistic Masses from the M04786 Ephemeris at 1989 August 25 04:00 ET

Body	μ_i^* (km^3/sec^2)
Mercury	22032.08040253906
Venus	324857.4782712153
Earth	398600.4418123855
Mars system	42828.28653388460
Jupiter system	126712700.9849606
Saturn system	37940448.53290854
Uranus system	5794559.117018051
Neptune system	6836534.716225980
Pluto plus Charon	1020.864919713455
Sun	132712439800.7594
Moon	4902.799065514813
Earth plus Moon	403503.2408779003
Ceres	69.36936487995916
Pallas	17.24378096264524
Vesta	32.67601789945270
Iris	0.717577368136944
Bamberga	1.166063223013404

$$\mathbf{r}_B = \frac{\mu_\oplus \mathbf{r}_\oplus + \mu_☾ \mathbf{r}_☾}{\mu_\oplus + \mu_☾} \tag{2–12}$$

and is assigned a mass

$$\mu_B = \mu_\oplus + \mu_☾. \tag{2–13}$$

When the Earth-Moon barycenter is treated as a single fictitious body, the total orbital angular momentum is instead

$$\mathrm{h} = \begin{pmatrix} 5.5138792089473392 \times 10^{16} \\ -8.1599811342261851 \times 10^{17} \\ 1.9241747793873432 \times 10^{18} \end{pmatrix} \ \mathrm{km}^5/\mathrm{sec}^3, \tag{2–14}$$

with corresponding spherical coordinates

$$|\mathrm{h}| = 2.0907754037994348 \times 10^{18}\ \mathrm{km}^5/\mathrm{sec}^3; \tag{2–15}$$

$$\alpha = 273^{\circ}\!.865725688 = 18^{\mathrm{h}}15^{\mathrm{m}}27^{\mathrm{s}}\!.774165; \tag{2–16}$$

$$\delta = \ 66^{\circ}\!.972373417 = 66^{\circ}58'20''\!.54430. \tag{2–17}$$

Figure 2–1 shows the difference between the vector **h** computed by equation (2–7) and that computed using the Earth-Moon barycenter instead of the Earth and Moon individually. This plot was actually generated from the longer DE130 ephemeris in order to show the effect more clearly; the results from the M04786 ephemeris are nearly identical. The 18.6-year oscillation, with an amplitude of 17 μarcsec, dominates. An annual perturbation in the lunar orbit is also apparent.

Neither equation (2–8) nor equation (2–14) gives the desired result. If the M04786 ephemeris is interpolated at various other times, **h** is seen to be a function of time, with a twelve-year periodicity indicative of an unbalanced Jupiter-Neptune couple. The explanation for this periodicity is rather simple: the ephemerides of both Jupiter and Neptune

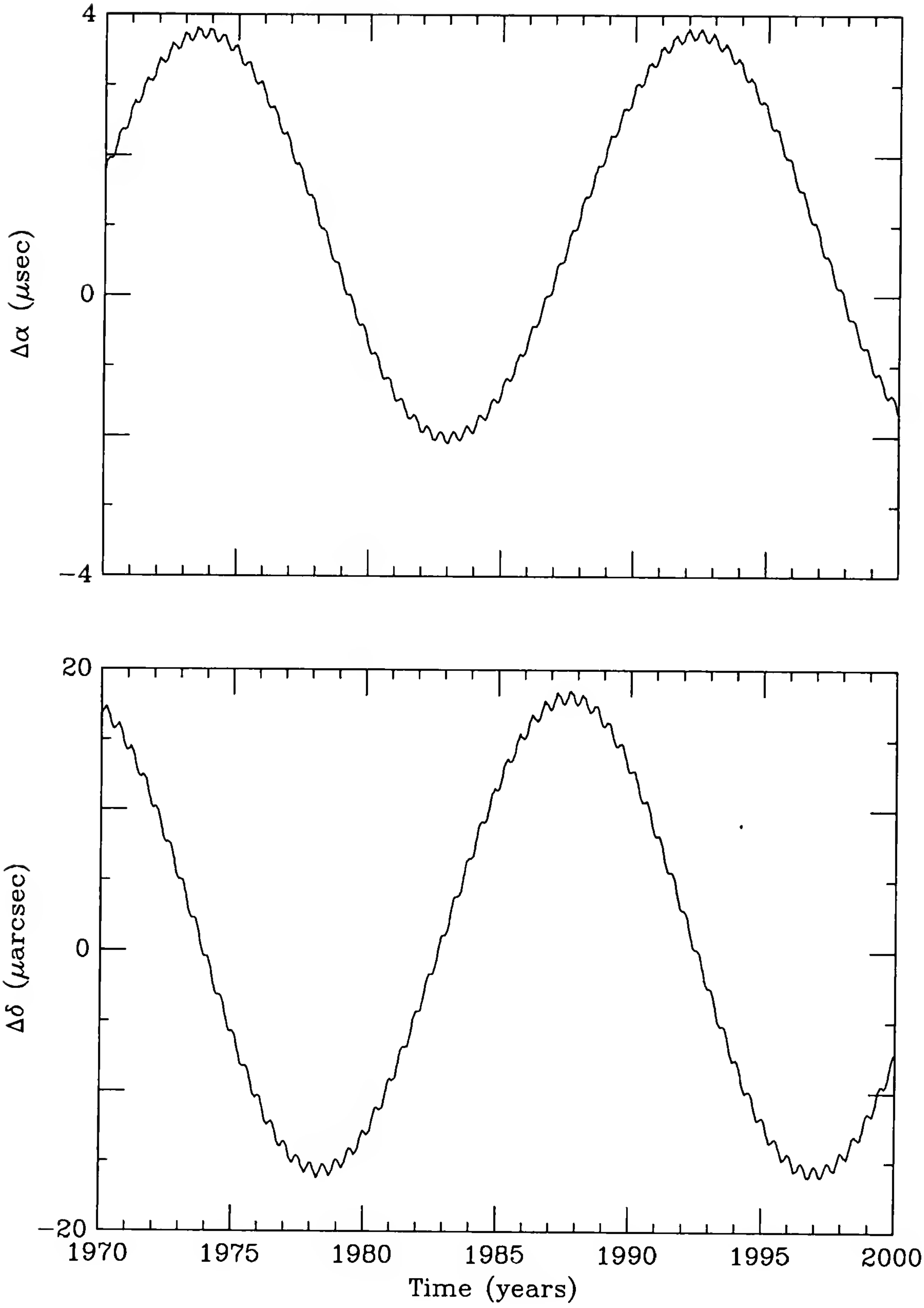

Figure 2–1. The Effect of the Geocentric Lunar Orbit on the Invariable Plane

were integrated using the DE130 value of Neptune's mass, 6828879.09654 km^3/sec^2 (Standish 1987). The Voyager data cause the estimate of Neptune's mass to increase by 0.112%, to the value shown in Table 2–1. When the estimate of Neptune's mass increased, the gravitational attraction by Neptune on Jupiter ought also to have increased. However, the ODP corrected only Neptune's ephemeris; the operational software cannot change the orbits of the other planets when the mass of one planet changes. Therefore the M04786 ephemeris contains, in effect, a pair of forces (Jupiter on Neptune and *vice versa*) that are not equal and opposite. The total angular momentum of the system is therefore not strictly conserved. The magnitude of the imbalance varies with the twelve-year synodic period of Jupiter and Neptune.

Figure 2–2 displays the right ascension and declination of **h**, as a function of time, for the entire sixteen-year span of the M04786 ephemeris. The twelve-year periodicity is quite obvious in the figure. (Corresponding plots using the DE130 ephemeris, shown to the same scale in Figure 2–3, are quite stable by contrast.) The average value of these coordinates will be taken as the direction of the total orbital angular momentum of the Solar System; the magnitude is given by equation (2–15). Consequently we have

$$\mathbf{h} = \begin{pmatrix} 5.5138790907107613 \times 10^{16} \\ -8.1599810902131458 \times 10^{17} \\ 1.9241747812877163 \times 10^{18} \end{pmatrix} \text{ km}^5/\text{sec}^3; \tag{2–18}$$

$$|\mathbf{h}| = 2.0907754037994348 \times 10^{18} \text{ km}^5/\text{sec}^3; \tag{2–15}$$

$$\alpha = 273\overset{\circ}{.}865725626 = 18^{h}15^{m}27\overset{s}{.}774150; \tag{2–19}$$

$$\delta = \ 66\overset{\circ}{.}972373550 = 66^\circ 58' 20\overset{''}{.}54478. \tag{2–20}$$

This result, which is still expressed in EME50 coordinates, will be adopted for the total orbital angular momentum of the Solar System.

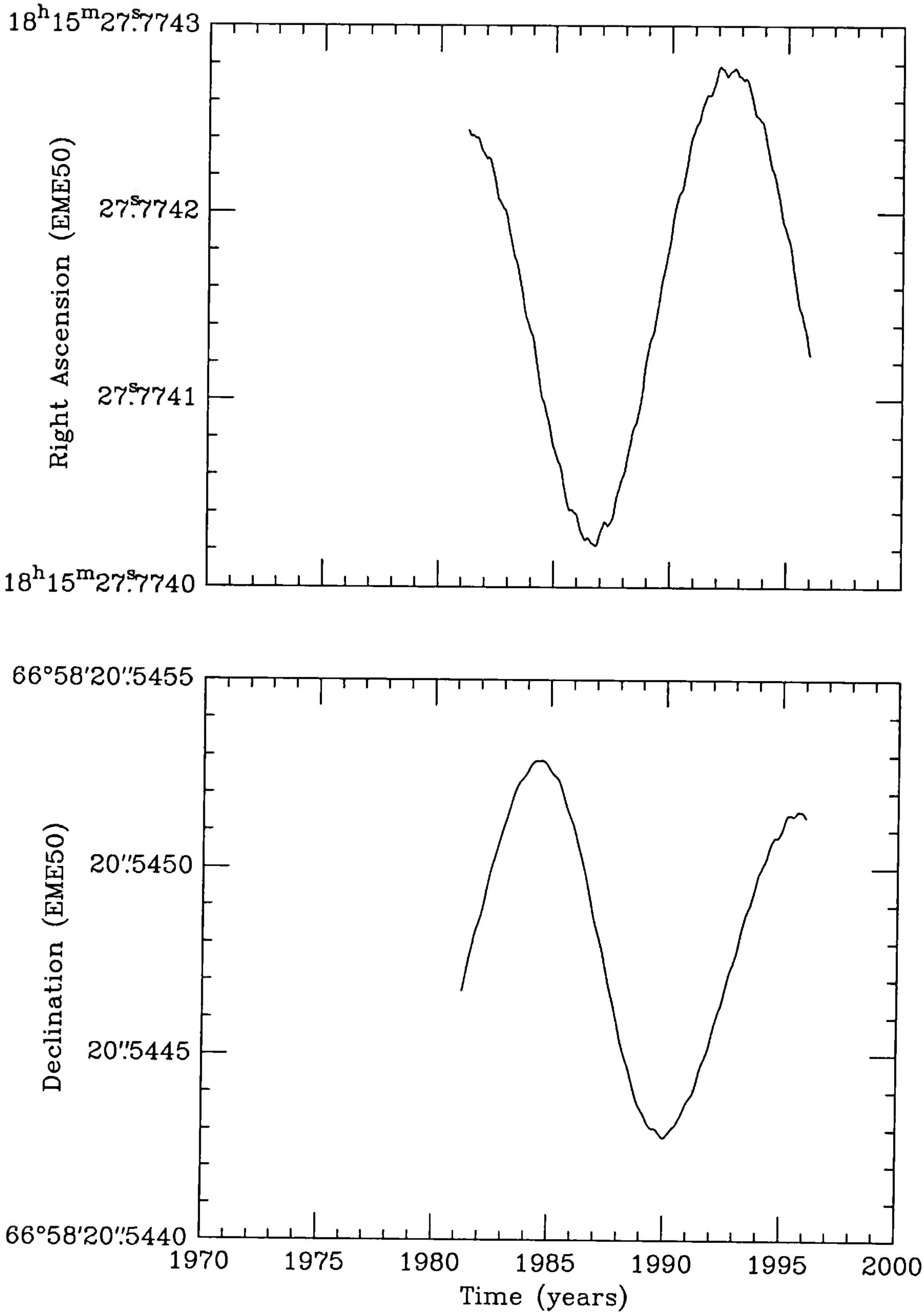

Figure 2–2. The Direction of the Total Angular Momentum (M04786 Ephemeris)

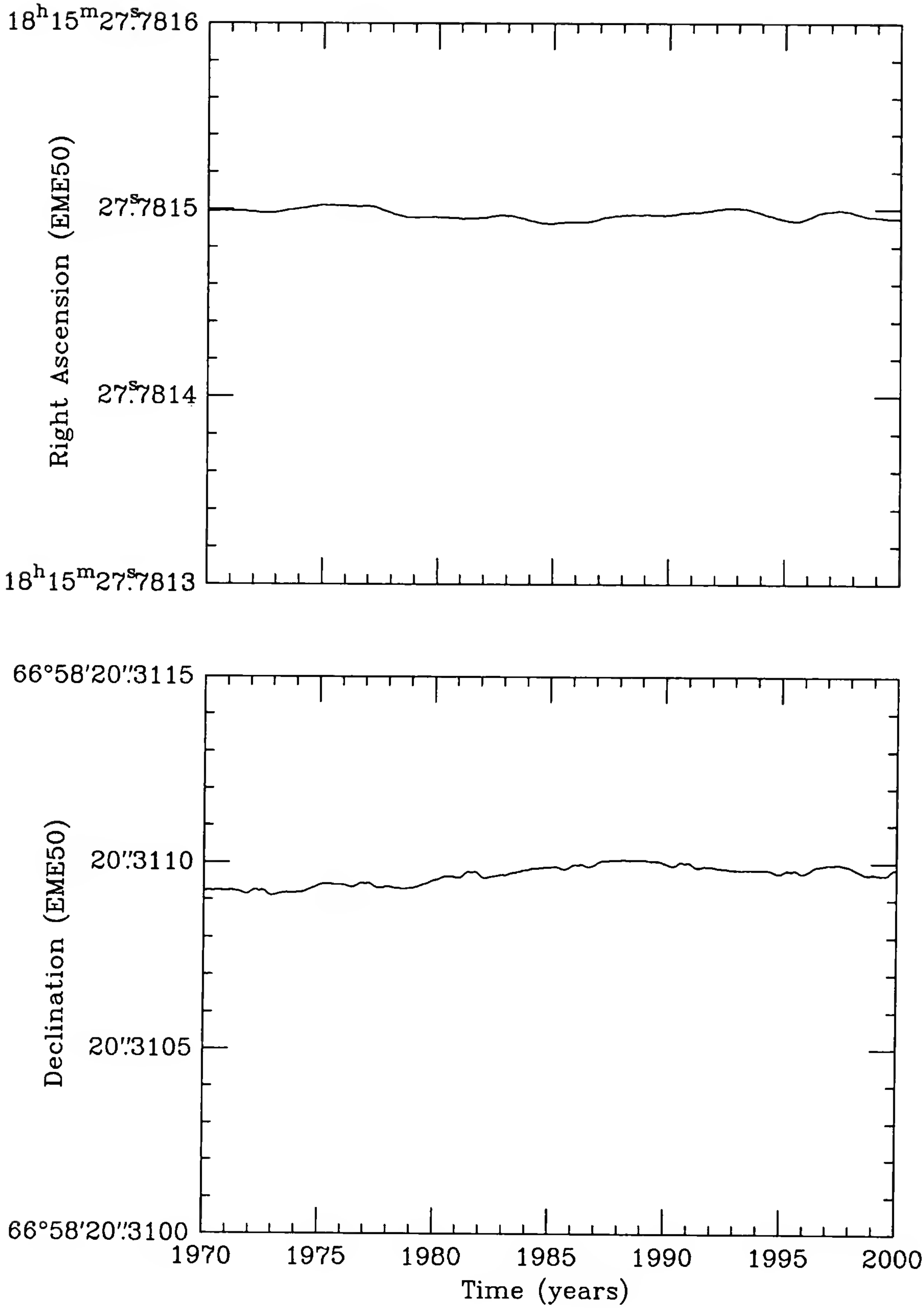

Figure 2–3. The Direction of the Total Angular Momentum (DE130 Ephemeris)

2.5. The Uncertainty in the Total Orbital Angular Momentum

The uncertainty in the total orbital angular momentum **h** of the Solar System, as found in the previous section, may be estimated through applying the standard formulas of error propagation (Bevington 1969) to equation (2–7). The SSDPS solves not for initial position and velocity, but rather for changes to the "Set III" orbital elements of Brouwer and Clemence (1961): $\Delta a/a$, Δe, Δp, Δq, $e\Delta\omega$, and $\Delta\omega + \Delta M$. These parameters represent the fractional change in semimajor axis; change in eccentricity; rotations of the orbital plane about the apsis, about the latus rectum, and about about the orbit normal; and the change in the mean argument of latitude. These are referred to each planet's osculating orbit at a standard epoch; the transformation from Set III elements into changes in the position and velocity vectors is well defined.

The orbital angular momentum for a planet is given by

$$\mathbf{h} = \mu\sqrt{(\mu_\odot + \mu)a(1-e^2)}\,\hat{\mathbf{h}}, \tag{2–21}$$

where $\hat{\mathbf{h}}$, the unit vector in the direction of the positive orbit normal, defines the third axis of the orbital coordinate system. The first and second unit vectors that define the orbital coordinate system are respectively the normalized "Laplace vector" $\hat{\mathbf{p}}$, directed toward the perihelion, and $\hat{\mathbf{q}} \equiv \hat{\mathbf{h}} \times \hat{\mathbf{p}}$. The rotation matrix from EME50 equatorial coordinates into the orbital system (p, q, h) is

$$\mathbf{M} = \mathbf{R}_3(\omega)\,\mathbf{R}_1(i)\,\mathbf{R}_3(\Omega). \tag{2–22}$$

Let the 7×7 covariance matrix from the SSDPS for a planet be denoted by $\mathbf{S}_{\mathrm{III}}$; the rows and columns correspond to μ_i followed by the six Set III parameters. Then the covariance matrix of **h**, expressed in orbital coordinates, is given by

$$\mathbf{S}_{pqh} = \mathbf{H}\,\mathbf{S}_{\mathrm{III}}\,\mathbf{H}^{\mathrm{T}}, \tag{2–23}$$

where the matrix $\mathbf{H}$ consists of partial derivatives of $\mathbf{h}$ with respect to the seven parameters:

$$\mathbf{H} = \frac{\partial\{h_p, h_q, h_h\}}{\partial\{\mu, \Delta a/a, \Delta e, \Delta p, \Delta q, e\Delta\omega, \Delta\omega + \Delta M\}}$$
$$= \begin{pmatrix} 0 & 0 & 0 & h & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & h & 0 & 0 \\ h/\mu + h/2(\mu_\odot + \mu) & h/2 & he/(1-e^2) & 0 & 0 & 0 & 0 \end{pmatrix}. \tag{2–24}$$

Finally, the uncertainty in $\mathbf{h}$ must be rotated into EME50 equatorial coordinates, giving

$$\mathbf{S}_{xyz} = \mathbf{M}^{\mathrm{T}}\,\mathbf{S}_{pqh}\,\mathbf{M}. \tag{2–25}$$

The orbital elements here can be taken to be the osculating elements at any epoch (which are easily accessible from converting position and velocity vectors) without introducing undue error in the calculation; after all, the goal is merely to determine the uncertainty in $\mathbf{h}$.

The major contribution to $\mathbf{S}_{xyz}$ will be from the uncertainties in the planetary masses. These uncertainties are usually expressed in units of inverse solar masses, so that a planet's inverse mass $\overline{m}_i \equiv \mu_\odot/\mu_i$. Accordingly

$$\frac{\sigma_{\mu_i}}{\mu_i} = \frac{\sigma_{\overline{m}_i}}{\overline{m}_i}. \tag{2–26}$$

Values for $\overline{m}_i$ and $\sigma_{\overline{m}_i}$, from Standish (1990*b*), are presented in Table 2–5, along with the resulting σ_{μ_i} from equation (2–26). Except for Pluto, the mass uncertainties are typically five or six orders of magnitude less than the masses themselves. By contrast, the uncertainty is Δp or Δq is typically $0\overset{''}{.}01$, at most $0\overset{''}{.}05$ for Pluto, or $O(10^{-7}$ rad). Although the uncertainties in the asteroids' masses is a sizeable fraction of their masses themselves, the contribution of the asteroids to $\mathbf{h}$ is small in any event; there is no harm in omitting the asteroids in this error analysis.

Table 2–5. Planetary Masses and Uncertainties

Body	$\overline{m}_i$	$\sigma_{\overline{m}_i}$	σ_{μ_i}
Mercury	6 023 600	250	0.91
Venus	408 523.7	0.2	0.16
Earth plus Moon	328 900.55	0.02	0.025
Mars system	3 098 708	10	0.14
Jupiter system	1 047.349	0.001	120
Saturn system	3 497.09	0.02	220
Uranus system	22 902.94	0.04	10
Neptune system	19 412.24	0.06	21
Pluto plus Charon	135 000 000	7 000 000	53

The covariance matrix $\mathbf{S}_{xyz}$ for each planet is given in Table 2–6; as expected, the major contributor is the uncertainty in the masses. The uncertainty in the total angular momentum of the Solar System is simply the sum of these matrices, given at the end of the table, as one can ignore correlations between parameters for one planet and those of the others. The inter-planet correlation coefficients for the outer planets (which carry the bulk of the angular momentum) are rarely above one percent. By contrast, the parameters for the inner planets are very highly correlated, but the inner planets' masses are so low and their contribution to $\mathbf{h}$ so small that these correlations can safely be ignored.

The final step is to transform the uncertainty in $\mathbf{h}$ into uncertainties in the adopted right ascension α and declination δ of the normal to the invariable plane. Since $\tan\alpha = h_y/h_x$ and $\sin\delta = h_z/h$, the Jacobian of the transformation is

$$\mathbf{D} = \frac{\partial\{h_x, h_y, h_z\}}{\partial\{\alpha, \delta\}}$$

$$= \begin{pmatrix} -h_y/(h_x^2+h_y^2) & h_x/(h_x^2+h_y^2) & 0 \\ -h_x h_z/h^2\sqrt{h_x^2+h_y^2} & -h_y h_z/h^2\sqrt{h_x^2+h_y^2} & \sqrt{h_x^2+h_y^2}/h^2 \end{pmatrix}. \qquad (2\text{–}27)$$

The resulting covariance matrix of α and δ is therefore

$$\mathbf{S}_{\alpha\delta} = \mathbf{D}\,\mathbf{S}_{xyz}\,\mathbf{D}^{\mathrm{T}} \qquad (2\text{–}28)$$

Table 2–6. Covariances of the Planets' Orbital Angular Momenta

Body		x (km^{10}/s^6)	y (km^{10}/s^6)	z (km^{10}/s^6)
Mercury	x	5.0008×10^{16}	-2.6084×10^{17}	4.8705×10^{17}
	y	-2.6084×10^{17}	1.3606×10^{18}	-2.5405×10^{18}
	z	4.8705×10^{17}	-2.5405×10^{18}	4.7436×10^{18}
Venus	x	1.2601×10^{15}	-8.5806×10^{15}	1.8988×10^{16}
	y	-8.5806×10^{15}	6.1258×10^{16}	-1.3555×10^{17}
	z	1.8988×10^{16}	-1.3555×10^{17}	3.0092×10^{17}
Earth	x	1.6179×10^{13}	2.8183×10^{12}	1.2981×10^{12}
	y	2.8183×10^{12}	2.0857×10^{15}	-4.1848×10^{15}
	z	1.2981×10^{12}	-4.1848×10^{15}	9.9230×10^{15}
Mars	x	3.3970×10^{14}	-5.8144×10^{15}	1.2667×10^{16}
	y	-5.8144×10^{15}	9.9629×10^{16}	-2.1703×10^{17}
	z	1.2667×10^{16}	-2.1703×10^{17}	4.7282×10^{17}
Jupiter	x	3.4311×10^{21}	-1.4203×10^{22}	3.0757×10^{22}
	y	-1.4203×10^{22}	2.3870×10^{23}	-5.4560×10^{23}
	z	3.0757×10^{22}	-5.4560×10^{23}	1.2766×10^{24}
Saturn	x	1.5633×10^{22}	-1.3513×10^{23}	3.2732×10^{23}
	y	-1.3513×10^{23}	1.2945×10^{24}	-3.1272×10^{24}
	z	3.2732×10^{23}	-3.1272×10^{24}	7.5630×10^{24}
Uranus	x	6.2578×10^{19}	-1.8624×10^{20}	4.6589×10^{20}
	y	-1.8624×10^{20}	6.3122×10^{21}	-1.4266×10^{22}
	z	4.6589×10^{20}	-1.4266×10^{22}	3.2619×10^{22}
Neptune	x	2.5811×10^{20}	-2.3328×10^{21}	5.7654×10^{21}
	y	-2.3328×10^{21}	3.8529×10^{22}	-9.3596×10^{22}
	z	5.7654×10^{21}	-9.3596×10^{22}	2.2868×10^{23}
Pluto	x	1.7137×10^{23}	-1.7859×10^{23}	5.6481×10^{23}
	y	-1.7859×10^{23}	1.8612×10^{23}	-5.8863×10^{23}
	z	5.6481×10^{23}	-5.8863×10^{23}	1.8616×10^{24}
Total—	x	1.9075×10^{23}	-3.3045×10^{23}	9.2912×10^{23}
	y	-3.3045×10^{23}	1.7642×10^{24}	-4.3693×10^{24}
	z	9.2912×10^{23}	-4.3693×10^{24}	1.0963×10^{25}

$$= \begin{pmatrix} 2.2940 \times 10^{-13} & 2.5407 \times 10^{-14} \\ 2.5407 \times 10^{-14} & 4.4279 \times 10^{-15} \end{pmatrix} \text{ radians}^2. \tag{2–29}$$

The resulting standard errors in the orientation of the total orbital angular momentum of the Solar System are

$$\sigma_\alpha \cos\delta = 0''.03865, \tag{2–30}$$

$$\sigma_\delta = 0''.01373. \tag{2–31}$$

The correlation between α and δ is $+0.7972$, indicating that the "error ellipse" is quite elongated; the semimajor and semiminor axes of the error ellipse are $0''.04166$ and $0''.01343$, and the major axis has a position angle of $73^\circ.53$. It is apparent that the effects of the Moon's nodal regression (Figure 2–1) and of the unbalanced Jupiter-Neptune couple (Figure 2–2) are both quite small in comparison to these errors.

The major obstacle to shrinking the error even further is the remaining uncertainty in Pluto's mass and to a lesser extent in its orbit. Hubble Space Telescope observations should resolve Pluto and Charon, thereby giving a reliable semimajor axis of their relative orbit and improving the estimate of the sum of their masses. Increases in the precision of Pluto's orbit estimate can come only after decades of additional observation. Nevertheless, before the Voyager 2 encounter with Neptune, the mass of Neptune was believed known to 0.2 percent; this alone would have produced an uncertainty of about $25''$ in the results. The improvement in the knowledge of the orientation of the invariable plane realized by the Voyager mission is evident.

2.6. The Rotational Angular Momentum of the Solar System

The rotational angular momentum $\mathbf{h}$ of a body about an adopted z-axis is given by integrating over the body's mass:

$$h = \int (x\dot{y} - y\dot{x})\, dm. \tag{2-32}$$

In the typical case of solid-body rotation about the z-axis, transformation into cylindrical coordinates (s, θ, z) yields

$$h = \int s^2 \Omega\, dm, \tag{2-33}$$

where $\Omega \equiv \dot{\theta}$ is the rotation rate. For spherically symmetric bodies of radius R, whose density ρ is a function only of the internal radius r, the integral takes the form

$$h = \int_0^R \tfrac{8}{3}\pi \rho r^4 \Omega\, dr; \tag{2-34}$$

for constant ρ,

$$h = \tfrac{8}{15}\pi \rho R^5 \Omega = \tfrac{2}{5} M R^2 \Omega \equiv I\Omega. \tag{2-35}$$

An evaluation of equation (2–32), the general case, will have the same functional form as equation (2–35), except that the constant $\frac{2}{5}$ will be replaced by a coefficient γ whose value depends on the body's density and angular speed as a function of position; R and Ω can be taken to be a reference radius and rotation rate, respectively.

Within the Solar System, the Sun has the largest rotational angular momentum; its enormous mass (over a thousand times Jupiter's) and radius (nearly ten times Jupiter's) more than compensate for its slow rotation rate. An approximate evaluation of equation (2–35), using the polar rotation rate and setting $\gamma = 0.2$ to account in part for central condensation, gives $h = 3.5 \times 10^{16}$ km^5/sec^3 for the solar rotational angular momentum. This is about one percent of the total orbital angular momentum of the planets and obviously should be included in the calculation of the orientation of the real invariable plane.

The uncertainty attached to this number is, however, unacceptably large. The mass distribution can in principle be obtained from a numerical integration of a stellar interior

model (*cf.* Iben 1967), but due to uncertainties in the opacity, energy generation, and convection models the results are probably good to not much more than three or four digits.

Furthermore, the photosphere of the Sun does not rotate uniformly; the sidereal rotation period varies from 26.2 days at the solar equator to 36.6 days at the poles (Howard and Harvey 1970). These rates can only be obtained through measurements of Doppler shifts at the solar limb; the more precise techniques applied to the planets (analysis of periodic radio emissions or landmark tracking) cannot be used. Although recent developments in helioseismology can in principle determine the rotation characteristics of the solar interior, results to date are in at best qualitative agreement (for example, Duvall *et al.* 1984, Brown 1985, and Duvall *et al.* 1986).

For both these reasons, the rotational angular momentum of the Sun is not known to the precision necessary to include it in a precise determination of the invariable plane. The prudent course of action is therefore to ignore all rotation and to define the "working model" of the invariable plane (as opposed to the "conceptual" definition) as that plane which contains the Solar System barycenter and is normal to the total orbital angular momentum of the Sun, planets, and largest asteroids.

This decision is also justified on the grounds that rotation (of the planets as well as of the Sun) does not enter into the equations of motion of the planetary barycenters as currently formulated in the SSDPS (Newhall *et al.* 1983). The Sun is assumed spherically symmetric; there is no detectable perturbation, even on Mercury's perihelion, due to solar oblateness. (Consequently there is also no mechanism whereby the Sun's spin axis can be made to precess noticeably.) While the Earth's rotation does figure conspicuously into the equations governing the Moon's motion, the actual integration is of the Earth-Moon barycenter and of the geocentric lunar orbit (Newhall *et al.* 1983). Therefore the "working

model" for **h** is expected to remain constant; and indeed, as seen in Section 2.4, this is reasonably true of a planetary ephemeris file.

The two theories (short-term and long-term) that are developed in the next two chapters of this work do not depend on the special physical nature of the invariable plane. Any plane that does not change its orientation relative to an inertial coordinate system will suffice. So there is no reason to insist on the inclusion of rotational angular momentum.

2.7. The Adopted Orientation of the Invariable Plane

Since rotational angular momentum is not included in the working definition of the invariable plane, the total orbital angular momentum found above becomes the normal vector to the invariable plane.

However, the vector **h** found above was specified in EME50 coordinates. In order to transform to J2000, one must not use the standard procedure (Aoki *et al.* 1983) for transforming star coordinates from the B1950 (FK4) system to the J2000 (FK5) system: because the M04786 ephemeris is already inertial, there is no need for the equinox drift term. Rather, Standish (1987) published a 3×3 rotation matrix **T** that transforms positions and velocities from the DE130 ephemeris (the predecessor of M04786) to the corresponding positions and velocities in DE202:

$$\mathbf{T} = \begin{pmatrix} +0.99992567955 09812 & -0.0111814782756923 & -0.0048590038154553 \\ +0.0111814782944231 & +0.9999374849486071 & -0.0000271625775175 \\ +0.0048590037723526 & -0.0000271702869124 & +0.9999881946023742 \end{pmatrix}. \qquad (2\text{–}36)$$

When the vector **h** from equation (2–18) is multiplied on the left by **T** above, the resulting vector will be expressed in J2000 coordinates. This is the result we seek:

$$\alpha_0 = 273^\circ\!.852572907 = 18^{\mathrm{h}}15^{\mathrm{m}}24^{\mathrm{s}}\!.617498; \qquad (2\text{–}37)$$

$$\delta_0 = \;\; 66^\circ\!.991111925 = 66^\circ 59' 28''\!.00293. \qquad (2\text{–}38)$$

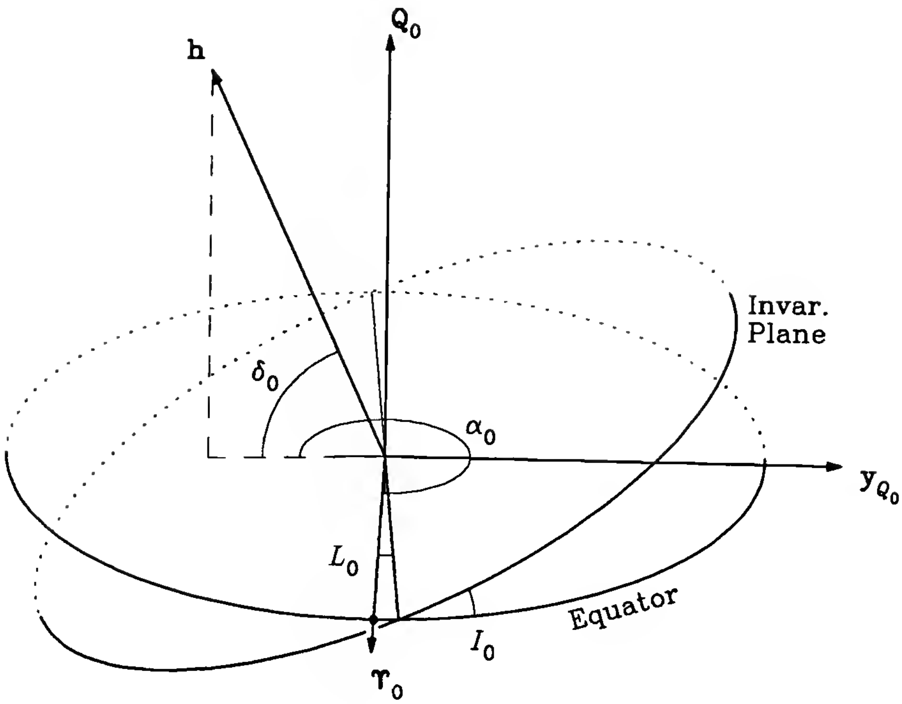

Figure 2–4. The Orientation of the Invariable Plane

The invariable plane can also be described by giving the right ascension L_0 of its ascending node on the Earth's mean equator of J2000 (which direction is defined by $\mathbf{n} = \mathbf{Q}_0 \times \mathbf{h}$, where $\mathbf{Q}_0 = (0, 0, 1)^{\mathrm{T}}$ in J2000 equatorial coordinates), and the inclination I_0 of the invariable plane to the Earth's mean equator of J2000. Figure 2–4 shows the angles I_0 and L_0 as well as the spherical coordinates of $\mathbf{h}$. These quantities are related by

$$L_0 = \alpha_0 + 6^{\mathrm{h}} \tag{2–39}$$

$$= \; 3^{\circ}\!.852572907 = 0^{\mathrm{h}}15^{\mathrm{m}}24^{\mathrm{s}}\!.617498; \tag{2–40}$$

$$I_0 = 90^{\circ} - \delta_0 \tag{2–41}$$

$$= 23^{\circ}\!.008888075 = 23^{\circ}00'31''\!.99707. \tag{2–42}$$

The standard error in these numbers is not appreciably changed from the EME50 values in equations (2–30) and (2–31). It is not meaningful to carry more digits than the

uncertainties permit; accordingly the above results for these angles will be rounded to the nearest $0''.001$, and the rounded values will be adopted for the orientation of the invariable plane:

$$\alpha_0 = 273°51'09''.262 \pm 0''.038; \tag{2–43}$$

$$\delta_0 = 66°59'28''.003 \pm 0''.013; \tag{2–44}$$

$$L_0 = 3°51'09''.262 \pm 0''.038; \tag{2–45}$$

$$I_0 = 23°00'31''.997 \pm 0''.013. \tag{2–46}$$

These values will be used throughout Chapters 3–5.

CHAPTER 3
THE SHORT-TERM THEORY

3.1. Introduction

The precession matrix $\mathbf{P}$, which transforms from the Q_0 system of epoch into the Q system of date, can be built up from elementary rotation matrices in several ways. Lieske *et al.* (1977), who present the currently-accepted short-term theory, construct $\mathbf{P}$ most easily in terms of the three angles ζ, θ, and z (Newcomb 1906; Andoyer 1911) by

$$\mathbf{P} = \mathbf{R}_3(-z)\,\mathbf{R}_2(\theta)\,\mathbf{R}_3(-\zeta). \tag{3–1}$$

These three angles are approximated in that paper by cubic polynomials, not in the initial and final times, but in the initial time T and the difference t between the initial and final times. When the initial time is the standard epoch J2000.0, Lieske *et al.* (1977) denote these angles by $\tilde{\zeta}_A$, $\tilde{\theta}_A$, and $\tilde{z}_A$, respectively. The coefficients of the various powers of time are denoted there as

$$\tilde{\zeta}_A = \zeta_1 T + \zeta_1' T^2 + \zeta_1'' T^3 \tag{3–2}$$

and similarly for the other two angles.

In this chapter, the tilde and subscript A will be suppressed for clarity: since an intermediate epoch does not enter until the end of the chapter, there is no ambiguity requiring diacritical marks; and since the rates as functions of time are not prominent, there is no need for the subscript to denote accumulated angles. The initial epoch (J2000.0) is taken to be the zero point of the time, and the final epoch (the "date") is denoted by T and

measured in Julian centuries past J2000. Furthermore, the system of primes and subscripts used in the paper by Lieske *et al.* is replaced by a system of subscripts only indicating the power of T. Thus within this chapter we will use the notation

$$\zeta = \zeta_1 T + \zeta_2 T^2 + \zeta_3 T^3 + \zeta_4 T^4 + O(T^5) \tag{3–3}$$

and analogously for the other angles.

Figure 3–1 shows the coordinate axes of both systems and the three "classical" angles. Reading the right-hand side of equation (3–1) from right to left, the rotation $\mathbf{R}_3(-\zeta)$ places the initial y-axis, marked $\mathbf{y}_{Q_0}$ in the figure, at the intersection of the two equators. The second rotation moves the pole from $\mathbf{Q}_0$ to $\mathbf{Q}$, and the third rotation puts the y-axis at its final location, along $\mathbf{y}_Q$. The ecliptics of epoch and of date are suppressed in the figure for clarity.

The precession matrix $\mathbf{P}$ can also be expressed by the following sequence of rotations:

$$\mathbf{P} = \mathbf{R}_3(-L)\,\mathbf{R}_1(-I)\,\mathbf{R}_3(-\Delta)\,\mathbf{R}_1(I_0)\,\mathbf{R}_3(L_0). \tag{3–4}$$

Figure 3–2 shows the coordinate axes of epoch and of date from the same perspective as Figure 3–1; now the invariable plane and the five angles of equation (3–4) appear. A magnified drawing of the region near the vernal equinoxes also appears as Figure 3–3. In equation (3–4), the first (rightmost) rotation moves the x-axis to the intersection of the equator of J2000.0 and the invariable plane. The second rotation puts the y-axis (and consequently the entire x-y plane) into the invariable plane. These two rotations depend only on the orientation of the invariable plane at the standard epoch. The last three rotations are functions of time: first a rotation about the pole of the invariable plane moves the x-axis onto the equator of date, then a rotation about the intermediate x-axis

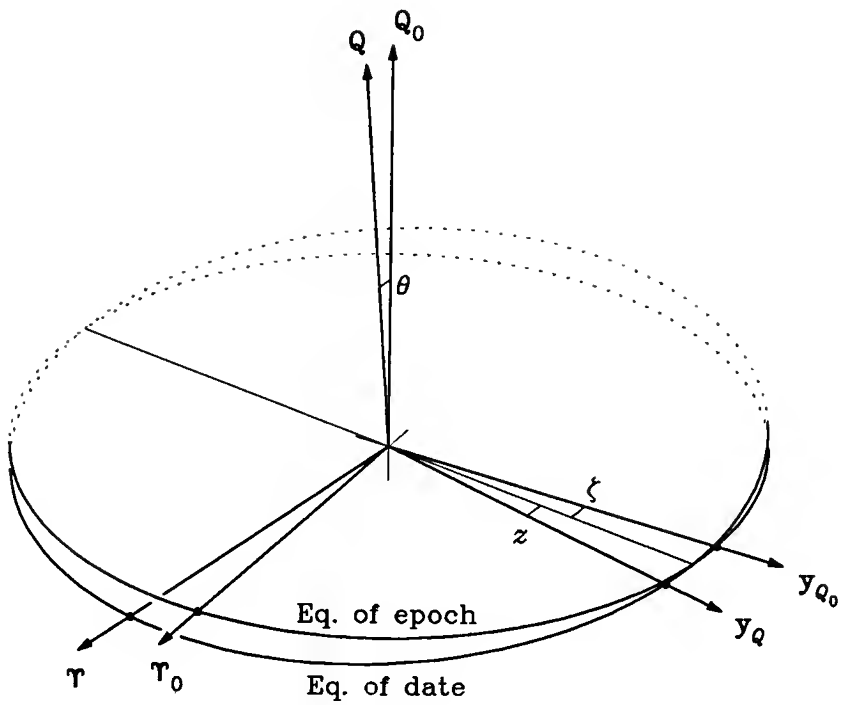

Figure 3–1. The Classical Precession Angles ζ, θ, and z

places the y-axis on the equator of date, and last a rotation about the vector **Q** (the final z-axis) positions the x-axis at the vernal equinox of date.

The purpose of the short-term theory is to provide analytical and numerical expressions for the coefficients of the angles I, L, and Δ. These are obtained by equating the expressions for **P** in equations (3–1) and (3–4) above; solving for I, L, and Δ in terms of I_0, L_0, ζ, θ, and z; and expanding the solutions in powers of time. The theory is developed to T^4 even though Lieske *et al.* (1977) go only as far as T^3. The extra term will give an indication of the sufficiency of the new theory to model precession with only cubic polynomials: if the coefficients of T^4 were large even in the absence of fourth-degree coefficients for ζ, θ, and z, then the theory would be inadequate.

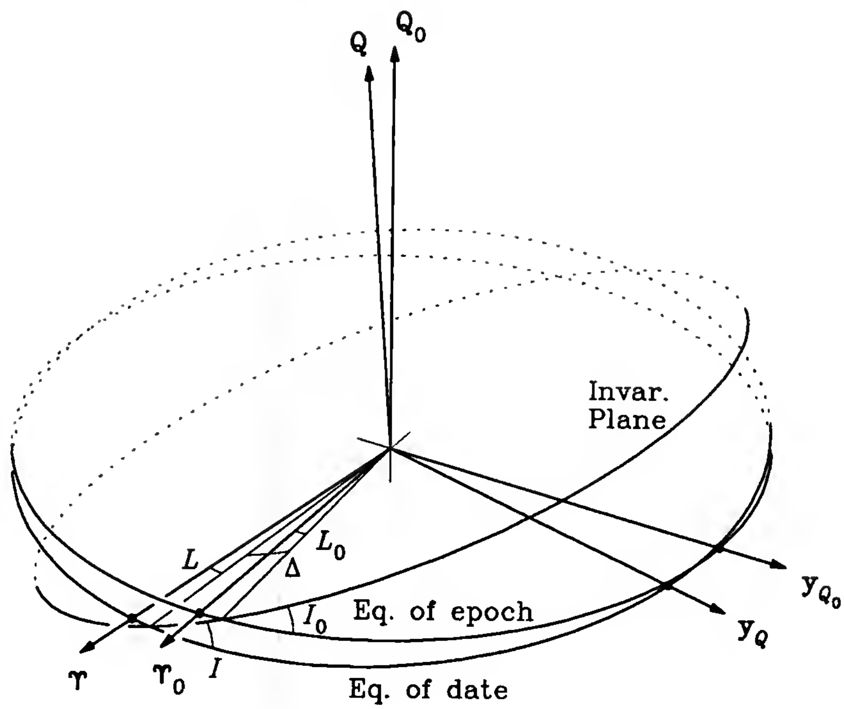

Figure 3–2. Precession Angles Using the Invariable Plane

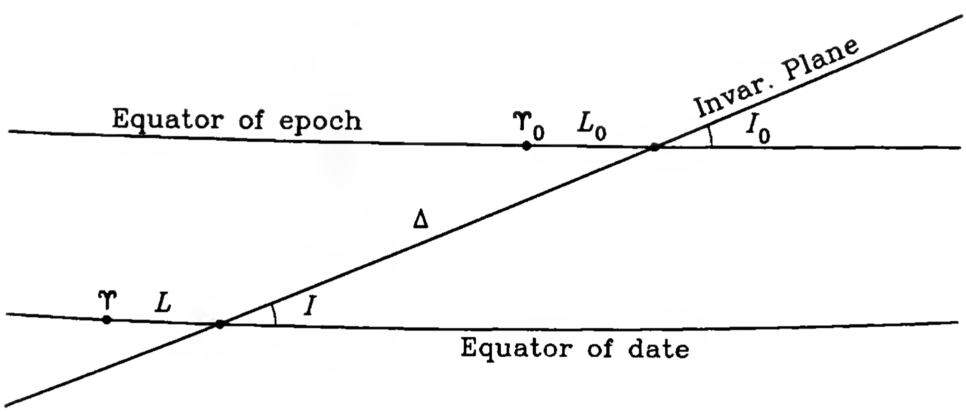

Figure 3–3. The Equators and the Invariable Plane

3.2. Analytic Formulas for I, L, and Δ

Equating the two expressions for the precession matrix above gives

$$\mathbf{R}_3(-z)\,\mathbf{R}_2(\theta)\,\mathbf{R}_3(-\zeta) = \mathbf{R}_3(-L)\,\mathbf{R}_1(-I)\,\mathbf{R}_3(-\Delta)\,\mathbf{R}_1(I_0)\,\mathbf{R}_3(L_0). \tag{3-5}$$

The desired angles may be isolated on the right-hand side of equation (3–5) by multiplying both sides on the right by $\mathbf{R}_3(-L_0)\,\mathbf{R}_1(-I_0)$, producing

$$\mathbf{R}_3(-z)\,\mathbf{R}_2(\theta)\,\mathbf{R}_3(-\zeta - L_0)\,\mathbf{R}_1(-I_0) = \mathbf{R}_3(-L)\,\mathbf{R}_1(-I)\,\mathbf{R}_3(-\Delta). \tag{3-6}$$

One can expand both sides of equation (3–6), obtaining the matrix elements $\sin\Delta\sin I$, $\cos\Delta\sin I$, $\sin L\sin I$, and $\cos L\sin I$ in terms of ζ, θ, z, I_0, and L_0. The angles themselves follow easily. The algebra is simplified considerably, however, if one first multiplies both sides of equation (3–6) on the left by $\mathbf{R}_3(z)$. This rotation combines with $\mathbf{R}_3(-L)$ on the right-hand side to yield

$$\mathbf{R}_2(\theta)\,\mathbf{R}_3[-(L_0+\zeta)]\,\mathbf{R}_1(-I_0) = \mathbf{R}_3[-(L-z)]\,\mathbf{R}_1(-I)\,\mathbf{R}_3(-\Delta). \tag{3-7}$$

Next the two sides are expanded. The left-hand side of equation (3–7) becomes

$$\begin{aligned}&\mathbf{R}_2(\theta)\,\mathbf{R}_3[-(L_0+\zeta)]\,\mathbf{R}_1(-I_0)\\ &= \begin{pmatrix} \cos\theta\cos(L_0+\zeta) & \begin{matrix}-\cos\theta\sin(L_0+\zeta)\cos I_0\\ -\sin\theta\sin I_0\end{matrix} & \begin{matrix}\cos\theta\sin(L_0+\zeta)\sin I_0\\ -\sin\theta\cos I_0\end{matrix} \\ \sin(L_0+\zeta) & \cos(L_0+\zeta)\cos I_0 & -\cos(L_0+\zeta)\sin I_0 \\ \sin\theta\cos(L_0+\zeta) & \begin{matrix}-\sin\theta\sin(L_0+\zeta)\cos I_0\\ +\cos\theta\sin I_0\end{matrix} & \begin{matrix}\sin\theta\sin(L_0+\zeta)\sin I_0\\ +\cos\theta\cos I_0\end{matrix}\end{pmatrix};\end{aligned} \tag{3-8}$$

the right-hand side is

$$\mathbf{R}_3[-(L-z)]\,\mathbf{R}_1(-I)\,\mathbf{R}_3(-\Delta)$$

$$= \begin{pmatrix} \begin{matrix}\cos(L-z)\cos\Delta \\ -\sin(L-z)\cos I\sin\Delta\end{matrix} & \begin{matrix}-\cos(L-z)\sin\Delta \\ -\sin(L-z)\cos I\cos\Delta\end{matrix} & \sin(L-z)\sin I \\ \begin{matrix}\sin(L-z)\cos\Delta \\ +\cos(L-z)\cos I\sin\Delta\end{matrix} & \begin{matrix}-\sin(L-z)\sin\Delta \\ +\cos(L-z)\cos I\cos\Delta\end{matrix} & -\cos(L-z)\sin I \\ \sin I\sin\Delta & \sin I\cos\Delta & \cos I \end{pmatrix}. \quad (3\text{–}9)$$

The angle I is then found from equating the (3,3) components:

$$I = \cos^{-1}[\cos\theta\cos I_0 + \sin\theta\sin(L_0+\zeta)\sin I_0]. \quad (3\text{–}10)$$

Then, assuming that $I \neq 0$ so that the factors $\sin I$ can be cancelled, the (1,3) and (2,3) components yield L by

$$L = \operatorname{plg}[\cos\theta\sin(L_0+\zeta)\sin I_0 - \sin\theta\cos I_0,\ \cos(L_0+\zeta)\sin I_0] + z \quad (3\text{–}11)$$

and the (3,1) and (3,2) components give Δ by

$$\Delta = \operatorname{plg}[\sin\theta\cos(L_0+\zeta),\ \cos\theta\sin I_0 - \sin\theta\sin(L_0+\zeta)\cos I_0], \quad (3\text{–}12)$$

where $\operatorname{plg}(y,x)$ is defined in Section 1–3 to be the four-quadrant arctangent. There is no sign ambiguity in either case because I is restricted to the first or second quadrants; therefore $\sin I > 0$ always. (The extreme case of $I = 0$ never occurs in practice, as the Earth's equator is always inclined at least 19° to the invariable plane.) Equations (3–10) through (3–12) are therefore rigorously correct for all possible values of ζ, θ, z, and all physically reasonable values for I_0 and L_0.

These definitions also reproduce correctly the desired behavior as T passes through zero. For $T < 0$, ζ, θ, and z are all negative by convention. The first argument in equation (3–12) will be negative and the second argument positive for reasonably small values of

ζ, θ, and z, guaranteeing that $\Delta < 0$. Similarly, for small positive values of ζ, θ, and z (corresponding to $T > 0$), both arguments are positive, and Δ will be positive as well. Equation (3–11) for L reduces to first order to $L_0 + \zeta + z$; therefore L is continuous near the time origin. Finally, if $T = 0$, one recovers $I = I_0$, $L = L_0$, and $\Delta = 0$.

3.3. Series Expansions for I, L, and Δ

Given that the angles I, L, and Δ are found by equations (3–10) through (3–12) above, their behavior with time depends on the behavior of the "classical" angles ζ, θ, and z that appear on the right-hand sides. (The angles I_0 and L_0 are constant.) If the classical angles are modeled as polynomials in time, then the new angles can also be so expressed.

Let the classical precession angles be approximated by the polynomials

$$\zeta = \sum_{k=1}^{4} \zeta_k T^k, \qquad \theta = \sum_{k=1}^{4} \theta_k T^k, \qquad z = \sum_{k=1}^{4} z_k T^k, \tag{3–13}$$

where T denotes time in Julian centuries from the standard J2000.0 epoch. These must be substituted into equations (3–10) through (3–12) and the trigonometric functions approximated by polynomials.

For the sine,

$$\begin{aligned}
\sin\theta &= \theta - \tfrac{1}{6}\theta^3 + O(\theta^5) \\
&= (\theta_1 T + \theta_2 T^2 + \theta_3 T^3 + \theta_4 T^4) - \tfrac{1}{6}(\theta_1 T + \theta_2 T^2)^3 + O(T^5) \\
&= \theta_1 T + \theta_2 T^2 + (\theta_3 - \tfrac{1}{6}\theta_1^3)T^3 + (\theta_4 - \tfrac{1}{2}\theta_1^2\theta_2)T^4 + O(T^5).
\end{aligned} \tag{3–14}$$

Similarly for the cosine,

$$\begin{aligned}
\cos\theta &= 1 - \tfrac{1}{2}\theta^2 + \tfrac{1}{24}\theta^4 + O(\theta^6) \\
&= 1 - \tfrac{1}{2}(\theta_1 T + \theta_2 T^2 + \theta_3 T^3)^2 + \tfrac{1}{24}(\theta_1 T)^4 + O(T^5) \\
&= 1 - \tfrac{1}{2}\theta_1^2 T^2 - \theta_1\theta_2 T^3 - (\theta_1\theta_3 + \tfrac{1}{2}\theta_2^2 - \tfrac{1}{24}\theta_1^4)T^4 + O(T^5).
\end{aligned} \tag{3–15}$$

Using these two expansions, the expansion for the functions of $(L_0 + \zeta)$ follows:

$$\begin{aligned}
\sin(L_0 + \zeta) &= \sin\zeta \cos L_0 + \cos\zeta \sin L_0 \\
&= \sin L_0 + (\zeta_1 \cos L_0)T + (\zeta_2 \cos L_0 - \tfrac{1}{2}\zeta_1^2 \sin L_0)T^2 \\
&\quad + [(\zeta_3 - \tfrac{1}{6}\zeta_1^3)\cos L_0 - \zeta_1\zeta_2 \sin L_0]T^3 \\
&\quad + [(\zeta_4 - \tfrac{1}{2}\zeta_1^2\zeta_2)\cos L_0 - (\zeta_1\zeta_3 + \tfrac{1}{2}\zeta_2^2 - \tfrac{1}{24}\zeta_1^4)\sin L_0]T^4 + O(T^5); \quad (3\text{–}16) \\
\cos(L_0 + \zeta) &= \cos\zeta \cos L_0 - \sin\zeta \sin L_0 \\
&= \cos L_0 - (\zeta_1 \sin L_0)T - (\tfrac{1}{2}\zeta_1^2 \cos L_0 + \zeta_2 \sin L_0)T^2 \\
&\quad - [\zeta_1\zeta_2 \cos L_0 + (\zeta_3 - \tfrac{1}{6}\zeta_1^3)\sin L_0]T^3 \\
&\quad - [(\zeta_1\zeta_3 + \tfrac{1}{2}\zeta_2^2 - \tfrac{1}{24}\zeta_1^4)\cos L_0 + (\zeta_4 - \tfrac{1}{2}\zeta_1^2\zeta_2)\sin L_0]T^4 + O(T^5). \quad (3\text{–}17)
\end{aligned}$$

The last series expansion that will be used more than once is for the arctangent function. Let $y_0 + \Delta y = \tan^{-1}(x_0 + \Delta x)$. A Taylor expansion of the right-hand side gives

$$\begin{aligned}
y_0 + \Delta y &= \tan^{-1} x_0 + \Delta x \left.\frac{d\tan^{-1} x}{dx}\right|_{x_0} + \frac{(\Delta x)^2}{2!} \left.\frac{d^2 \tan^{-1} x}{dx^2}\right|_{x_0} \\
&\quad + \frac{(\Delta x)^3}{3!} \left.\frac{d^3 \tan^{-1} x}{dx^3}\right|_{x_0} + \frac{(\Delta x)^4}{4!} \left.\frac{d^4 \tan^{-1} x}{dx^4}\right|_{x_0} + O[(\Delta x)^5]. \quad (3\text{–}18)
\end{aligned}$$

Since $x_0 = \tan y_0$, the first four derivatives can be expressed as

$$\left.\frac{d\tan^{-1} x}{dx}\right|_{x_0} = \frac{1}{1 + \tan^2 y_0} = \cos^2 y_0; \quad (3\text{–}19)$$

$$\left.\frac{d^2 \tan^{-1} x}{dx^2}\right|_{x_0} = \frac{-2\tan y_0}{(1 + \tan^2 y_0)^2} = -2\sin y_0 \cos^3 y_0; \quad (3\text{–}20)$$

$$\left.\frac{d^3 \tan^{-1} x}{dx^3}\right|_{x_0} = \frac{6\tan^2 y_0 - 2}{(1 + \tan^2 y_0)^3} = 6\sin^2 y_0 \cos^4 y_0 - 2\cos^6 y_0; \quad (3\text{–}21)$$

$$\left.\frac{d^4 \tan^{-1} x}{dx^4}\right|_{x_0} = \frac{24(\tan y_0 - \tan^3 y_0)}{(1 + \tan^2 y_0)^4} = 24(\sin y_0 \cos^7 y_0 - \sin^3 y_0 \cos^5 y_0). \quad (3\text{–}22)$$

Next let Δx be represented as a polynomial in time:

$$\Delta x \equiv x_1 T + x_2 T^2 + x_3 T^3 + x_4 T^4 + O(T^5). \tag{3-23}$$

The next three powers of Δx, complete to T^4, are then

$$(\Delta x)^2 = x_1^2 T^2 + 2x_1 x_2 T^3 + (x_2^2 + 2x_1 x_3) T^4 + O(T^5); \tag{3-24}$$

$$(\Delta x)^3 = x_1^3 T^3 + 3x_1^2 x_2 T^4 + O(T^5); \tag{3-25}$$

$$(\Delta x)^4 = x_1^4 T^4 + O(T^5). \tag{3-26}$$

When these expressions for $(\Delta x)^k$ and the values of the derivatives are inserted into the Taylor expansion, the resulting equation is of the form

$$\Delta y = y_1 T + y_2 T^2 + y_3 T^3 + y_4 T^4 + O(T^5) \tag{3-27}$$

with the coefficients y_k given by

$$y_1 = x_1 \cos^2 y_0; \tag{3-28}$$

$$y_2 = x_2 \cos^2 y_0 - x_1^2 \sin y_0 \cos^3 y_0; \tag{3-29}$$

$$y_3 = x_3 \cos^2 y_0 - 2x_1 x_2 \sin y_0 \cos^3 y_0 + x_1^3(\sin^2 y_0 \cos^4 y_0 - \tfrac{1}{3}\cos^6 y_0); \tag{3-30}$$

$$\begin{aligned} y_4 = {} & x_4 \cos^2 y_0 - (x_2^2 + 2x_1 x_3)\sin y_0 \cos^3 y_0 + 3x_1^2 x_2(\sin^2 y_0 \cos^4 y_0 - \tfrac{1}{3}\cos^6 y_0) \\ & + x_1^4(\sin y_0 \cos^7 y_0 - \sin^3 y_0 \cos^5 y_0). \end{aligned} \tag{3-31}$$

Additional expansions are required, but because they will be used but once, they will be developed below as the need arises.

3.3.1. The Expansion for I

The inclination I of the invariable plane to the equator of date is given by equation (3-10) above:

$$I = \cos^{-1}[\cos\theta \cos I_0 + \sin\theta \sin(L_0 + \zeta)\sin I_0]. \tag{3-10}$$

First the expressions for $\cos\theta$, $\sin\theta$, and $\sin(L_0+\zeta)$ must be substituted into the argument for the arccosine:

$$
\begin{aligned}
I = \cos^{-1}\Big(&[1 - \tfrac{1}{2}\theta_1^2T^2 - \theta_1\theta_2T^3 - (\theta_1\theta_3 + \tfrac{1}{2}\theta_2^2 - \tfrac{1}{24}\theta_1^4)T^4]\cos I_0 \\
&+ [\theta_1T + \theta_2T^2 + (\theta_3 - \tfrac{1}{6}\theta_1^3)T^3 + (\theta_4 - \tfrac{1}{2}\theta_1^2\theta_2)T^4] \\
&\quad \times \big\{\sin L_0 + (\zeta_1\cos L_0)T + (\zeta_2\cos L_0 - \tfrac{1}{2}\zeta_1^2\sin L_0)T^2 \\
&\qquad + [(\zeta_3 - \tfrac{1}{6}\zeta_1^3)\cos L_0 - \zeta_1\zeta_2\sin L_0]T^3 \\
&\qquad + [(\zeta_4 - \tfrac{1}{2}\zeta_1^2\zeta_2)\cos L_0 - (\zeta_1\zeta_3 + \tfrac{1}{2}\zeta_2^2 - \tfrac{1}{24}\zeta_1^4)\sin L_0]T^4\big\}\sin I_0 \\
&+ O(T^5)\Big)
\end{aligned}
\tag{3–32}
$$

$$
\begin{aligned}
= \cos^{-1}\Big(&[1 - \tfrac{1}{2}\theta_1^2T^2 - \theta_1\theta_2T^3 - (\theta_1\theta_3 + \tfrac{1}{2}\theta_2^2 - \tfrac{1}{24}\theta_1^4)T^4]\cos I_0 \\
&+ \big\{(\theta_1\sin L_0)T + (\theta_2\sin L_0 + \theta_1\zeta_1\cos L_0)T^2 \\
&\quad + [(\theta_3 - \tfrac{1}{6}\theta_1^3 - \tfrac{1}{2}\theta_1\zeta_1^2)\sin L_0 + (\theta_1\zeta_2 + \theta_2\zeta_1)\cos L_0]T^3 \\
&\quad + [(\theta_4 - \tfrac{1}{2}\theta_1^2\theta_2 - \tfrac{1}{2}\theta_2\zeta_1^2 - \theta_1\zeta_1\zeta_2)\sin L_0 \\
&\qquad + (\theta_1\zeta_3 + \theta_2\zeta_2 + \theta_3\zeta_1 - \tfrac{1}{6}\theta_1^3\zeta_1 - \tfrac{1}{6}\theta_1\zeta_1^3)\cos L_0]T^4\big\}\sin I_0 \\
&+ O(T^5)\Big)
\end{aligned}
\tag{3–33}
$$

$$
\begin{aligned}
= \cos^{-1}\big\{&\cos I_0 + (\theta_1\sin L_0\sin I_0)T \\
&+ (\theta_2\sin L_0\sin I_0 + \theta_1\zeta_1\cos L_0\sin I_0 - \tfrac{1}{2}\theta_1^2\cos I_0)T^2 \\
&+ [(\theta_3 - \tfrac{1}{6}\theta_1^3 - \tfrac{1}{2}\theta_1\zeta_1^2)\sin L_0\sin I_0 + (\theta_1\zeta_2 + \theta_2\zeta_1)\cos L_0\sin I_0 \\
&\quad - \theta_1\theta_2\cos I_0]T^3 \\
&+ [(\theta_4 - \tfrac{1}{2}\theta_1^2\theta_2 - \tfrac{1}{2}\theta_2\zeta_1^2 - \theta_1\zeta_1\zeta_2)\sin L_0\sin I_0 \\
&\quad + (\theta_1\zeta_3 + \theta_2\zeta_2 + \theta_3\zeta_1 - \tfrac{1}{6}\theta_1^3\zeta_1 - \tfrac{1}{6}\theta_1\zeta_1^3)\cos L_0\sin I_0 \\
&\quad + (\tfrac{1}{24}\theta_1^4 - \theta_1\theta_3 - \tfrac{1}{2}\theta_2^2)\cos I_0]T^4 + O(T^5)\big\}
\end{aligned}
\tag{3–34}
$$

$$
\equiv \cos^{-1}\left[\cos I_0 + c_1T + c_2T^2 + c_3T^3 + c_4T^4 + O(T^5)\right], \tag{3–35}
$$

where the c_k in equation (3–35) are defined by the various coefficients of T^k in equation (3–34).

Now equation (3–35) has the form

$$
I = \cos^{-1}(\cos I_0 + c) \tag{3–36}
$$

where

$$c = c_1T + c_2T^2 + c_3T^3 + c_4T^4 + O(T^5). \tag{3–37}$$

We therefore perform a Taylor expansion of the arccosine function, as was done above for the arctangent:

$$\begin{aligned} I = {} & \cos^{-1} x_0 + c\,\frac{d\cos^{-1} x}{dx}\bigg|_{x_0} + \frac{c^2}{2!}\,\frac{d^2\cos^{-1} x}{dx^2}\bigg|_{x_0} \\ & + \frac{c^3}{3!}\,\frac{d^3\cos^{-1} x}{dx^3}\bigg|_{x_0} + \frac{c^4}{4!}\,\frac{d^4\cos^{-1} x}{dx^4}\bigg|_{x_0} + O(c^5). \end{aligned} \tag{3–38}$$

Since $x_0 = \cos I_0$, the leading term is simply I_0. The first four derivatives become

$$\frac{d\cos^{-1} x}{dx}\bigg|_{x_0} = -\frac{1}{(1-x^2)^{1/2}} = -\frac{1}{\sin I_0}; \tag{3–39}$$

$$\frac{d^2\cos^{-1} x}{dx^2}\bigg|_{x_0} = -\frac{x}{(1-x^2)^{3/2}} = -\frac{\cos I_0}{\sin^3 I_0}; \tag{3–40}$$

$$\frac{d^3\cos^{-1} x}{dx^3}\bigg|_{x_0} = -\frac{1+2x^2}{(1-x^2)^{5/2}} = -\frac{1+2\cos^2 I_0}{\sin^5 I_0}; \tag{3–41}$$

$$\frac{d^4\cos^{-1} x}{dx^4}\bigg|_{x_0} = -\frac{9x+6x^3}{(1-x^2)^{7/2}} = -\frac{9\cos I_0 + 6\cos^3 I_0}{\sin^7 I_0}. \tag{3–42}$$

When one substitutes into equation (3–38) the expression in equation (3–37) for c, along with the values of the derivatives in equations (3–39) through (3–42), one obtains an equation giving the coefficients I_k of the various powers of T in terms of the c_k:

$$\begin{aligned} I = I_0 & - (c_1T + c_2T^2 + c_3T^3 + c_4T^4)\left(\frac{1}{\sin I_0}\right) \\ & - \tfrac{1}{2}(c_1T + c_2T^2 + c_3T^3 + c_4T^4)^2\left(\frac{\cos I_0}{\sin^3 I_0}\right) \\ & - \tfrac{1}{6}(c_1T + c_2T^2 + c_3T^3 + c_4T^4)^3\left(\frac{1+2\cos^2 I_0}{\sin^5 I_0}\right) \\ & - \tfrac{1}{24}(c_1T + c_2T^2 + c_3T^3 + c_4T^4)^4\left(\frac{9\cos I_0 + 6\cos^3 I_0}{\sin^7 I_0}\right) + O(T^5) \end{aligned} \tag{3–43}$$

$$= I_0 - \frac{c_1}{\sin I_0}T - \left(\frac{c_2}{\sin I_0} + \frac{c_1^2 \cos I_0}{2\sin^3 I_0}\right)T^2$$
$$- \left(\frac{c_3}{\sin I_0} + \frac{c_1 c_2 \cos I_0}{\sin^3 I_0} + \frac{c_1^3(1 + 2\cos^2 I_0)}{6\sin^5 I_0}\right)T^3$$
$$- \left(\frac{c_4}{\sin I_0} + \frac{(c_2^2 + 2c_1c_3)\cos I_0}{2\sin^3 I_0} + \frac{c_1^2 c_2(1 + 2\cos^2 I_0)}{2\sin^5 I_0} + \frac{c_1^4(9\cos I_0 + 6\cos^3 I_0)}{24\sin^7 I_0}\right)T^4$$
$$+ O(T^5) \tag{3–44}$$

$$\equiv I_0 + I_1T + I_2T^2 + I_3T^3 + I_4T^4 + O(T^5). \tag{3–45}$$

This last equation also shows, as expected, that $I \to I_0$ as $T \to 0$.

The last step is of course to obtain the coefficients I_k in terms of the coefficients θ_k and ζ_k in the approximation polynomials for θ and ζ, by substituting the various coefficients in equation (3–34) into equation (3–44). After some tedious algebra, one obtains

$$I_1 = -\frac{c_1}{\sin I_0} = -\frac{\theta_1 \sin L_0 \sin I_0}{\sin I_0}$$
$$= -\theta_1 \sin L_0; \tag{3–46}$$

$$I_2 = -\frac{c_2}{\sin I_0} - \frac{c_1^2 \cos I_0}{2\sin^3 I_0}$$
$$= -\frac{\theta_2 \sin L_0 \sin I_0 + \theta_1\zeta_1 \cos L_0 \sin I_0 - \frac{1}{2}\theta_1^2 \cos I_0}{\sin I_0}$$
$$- \frac{(\theta_1 \sin L_0 \sin I_0)^2 \cos I_0}{2\sin^3 I_0}$$
$$= -\theta_2 \sin L_0 - \theta_1\zeta_1 \cos L_0 + \tfrac{1}{2}\theta_1^2 \cos^2 L_0 \cot I_0; \tag{3–47}$$

$$I_3 = -\frac{c_3}{\sin I_0} - \frac{c_1 c_2 \cos I_0}{\sin^3 I_0} - \frac{c_1^3(1 + 2\cos^2 I_0)}{\sin^5 I_0}$$
$$= -\frac{(\theta_3 - \frac{1}{6}\theta_1^3 - \frac{1}{2}\theta_1\zeta_1^2)\sin L_0 \sin I_0 + (\theta_1\zeta_2 + \theta_2\zeta_1)\cos L_0 \sin I_0 - \theta_1\theta_2 \cos I_0}{\sin I_0}$$
$$- \frac{(\theta_1 \sin L_0 \sin I_0)(\theta_2 \sin L_0 \sin I_0 + \theta_1\zeta_1 \cos L_0 \sin I_0 - \frac{1}{2}\theta_1^2 \cos I_0)\cos I_0}{\sin^3 I_0}$$
$$- \frac{(\theta_1 \sin L_0 \sin I_0)^3(1 + 2\cos^2 I_0)}{6\sin^5 I_0}$$

$$
\begin{aligned}
&= (-\theta_3 + \tfrac{1}{6}\theta_1^3 + \tfrac{1}{2}\theta_1\zeta_1^2)\sin L_0 - (\theta_1\zeta_2 + \theta_2\zeta_1)\cos L_0 \\
&\quad + \cot I_0(\theta_1\theta_2\cos^2 L_0 - \theta_1^2\zeta_1 \sin L_0 \cos L_0) \\
&\quad + \cot^2 I_0[\theta_1^3(\tfrac{1}{2}\sin L_0 - \tfrac{1}{3}\sin^3 L_0)] - \csc^2 I_0(\tfrac{1}{6}\theta_1^3 \sin^3 L_0) \\
&= (-\theta_3 + \tfrac{1}{2}\theta_1\zeta_1^2)\sin L_0 - (\theta_1\zeta_2 + \theta_2\zeta_1)\cos L_0 + \tfrac{1}{6}\theta_1^3\cos^2 L_0 \sin L_0 \\
&\quad + \cot I_0(\theta_1\theta_2\cos^2 L_0 - \theta_1^2\zeta_1 \sin L_0 \cos L_0) \\
&\quad + \cot^2 I_0[\theta_1^3(\tfrac{1}{2}\sin L_0 \cos^2 L_0)];
\end{aligned} \tag{3-48}
$$

$$
\begin{aligned}
I_4 =& -\frac{c_4}{\sin I_0} - \frac{(c_2^2 + 2c_1c_3)\cos I_0}{2\sin^3 I_0} \\
& - \frac{c_1^2 c_2(1 + 2\cos^2 I_0)}{2\sin^5 I_0} - \frac{c_1^4(9\cos I_0 + 6\cos^3 I_0)}{24\sin^7 I_0} \\
=& -\big[(\theta_4 - \tfrac{1}{2}\theta_1^2\theta_2 - \tfrac{1}{2}\theta_2\zeta_1^2 - \theta_1\zeta_1\zeta_2)\sin L_0 \sin I_0 \\
&\quad + (\theta_1\zeta_3 + \theta_2\zeta_2 + \theta_3\zeta_1 - \tfrac{1}{6}\theta_1^3\zeta_1 - \tfrac{1}{6}\theta_1\zeta_1^3)\cos L_0 \sin I_0 \\
&\quad + (\tfrac{1}{24}\theta_1^4 - \theta_1\theta_3 - \tfrac{1}{2}\theta_2^2)\cos I_0\big]/\sin I_0 \\
& - \big[(\theta_2 \sin L_0 \sin I_0 + \theta_1\zeta_1 \cos L_0 \sin I_0 - \tfrac{1}{2}\theta_1^2\cos I_0)^2 \\
&\quad + 2(\theta_1 \sin L_0 \sin I_0)[(\theta_3 - \tfrac{1}{6}\theta_1^3 - \tfrac{1}{2}\theta_1\zeta_1^2)\sin L_0 \sin I_0 \\
&\qquad + (\theta_2\zeta_1 + \theta_1\zeta_2)\cos L_0 \sin I_0 - \theta_1\theta_2\cos I_0]\big]\cos I_0/(2\sin^3 I_0) \\
& - \big[(\theta_1 \sin L_0 \sin I_0)^2(\theta_2 \sin L_0 \sin I_0 + \theta_1\zeta_1\cos L_0 \sin I_0 - \tfrac{1}{2}\theta_1^2\cos I_0) \\
&\quad \times (1 + 2\cos^2 I_0)\big]/(2\sin^5 I_0) \\
& - \big[(\theta_1 \sin L_0 \sin I_0)^4(9\cos I_0 + 6\cos^3 I_0)\big]/(24\sin^7 I_0) \\
=& \big[-\theta_4 + \theta_1\zeta_1\zeta_2 + \tfrac{1}{2}\theta_2(\theta_1^2 + \zeta_1^2)\big]\sin L_0 \\
&+ \big[-\theta_1\zeta_3 - \theta_2\zeta_2 - \theta_3\zeta_1 + \tfrac{1}{6}\theta_1\zeta_1(\theta_1^2 + \zeta_1^2)\big]\cos L_0 \\
&+ \cot I_0\big[-\tfrac{1}{24}\theta_1^4 + (\tfrac{1}{6}\theta_1^4 + \tfrac{1}{2}\theta_1^2\zeta_1^2)\sin^2 L_0 + (\theta_1\theta_3 + \tfrac{1}{2}\theta_2^2 - \tfrac{1}{2}\theta_1^2\zeta_1^2)\cos^2 L_0 \\
&\quad - (2\theta_1\theta_2\zeta_1 + \theta_1^2\zeta_2)\sin L_0 \cos L_0\big] \\
&+ \cot^2 I_0\big[\theta_1^2\theta_2(\tfrac{3}{2}\sin L_0 - \sin^3 L_0) + \theta_1^3\zeta_1(\tfrac{1}{2}\cos L_0 - \sin^2 L_0 \cos L_0)\big] \\
&+ \cot^3 I_0\big[\theta_1^4(-\tfrac{1}{8} + \tfrac{1}{2}\sin^2 L_0 - \tfrac{1}{4}\sin^4 L_0)\big] \\
&+ \csc^2 I_0\big[-\tfrac{1}{2}\theta_1^2\sin^2 L_0(\theta_2 \sin L_0 + \theta_1\zeta_1 \cos L_0)\big] \\
&+ \cot I_0 \csc^2 I_0\big[\tfrac{1}{8}\theta_1^4 \sin^2 L_0(2 - 3\sin^2 L_0)\big] \\
=& (-\theta_4 + \theta_1\zeta_1\zeta_2 + \tfrac{1}{2}\theta_2\zeta_1^2)\sin L_0 + \tfrac{1}{2}\theta_1^2\theta_2 \sin L_0 \cos^2 L_0 - \tfrac{1}{2}\theta_1^3\zeta_1 \sin^2 L_0 \cos L_0 \\
&+ \big[-\theta_1\zeta_3 - \theta_2\zeta_2 - \theta_3\zeta_1 + \tfrac{1}{6}\theta_1\zeta_1(\theta_1^2 + \zeta_1^2)\big]\cos L_0 \\
&+ \cot I_0\big[\theta_1^4(-\tfrac{1}{24} + \tfrac{5}{12}\sin^2 L_0 - \tfrac{3}{8}\sin^4 L_0) + \tfrac{1}{2}\theta_1^2\zeta_1^2 \sin^2 L_0 \\
&\quad + (\theta_1\theta_3 + \tfrac{1}{2}\theta_2^2 - \tfrac{1}{2}\theta_1^2\zeta_1^2)\cos^2 L_0 - (2\theta_1\theta_2\zeta_1 + \theta_1^2\zeta_2)\sin L_0 \cos L_0\big] \\
&+ \cot^2 I_0\big[\tfrac{3}{2}\theta_1^2\theta_2 \sin L_0 \cos^2 L_0 + \theta_1^3\zeta_1(\tfrac{1}{2}\cos L_0 - \tfrac{3}{2}\sin^2 L_0 \cos L_0)\big]
\end{aligned}
$$

$$+ \cot^3 I_0 \big[\theta_1^4(-\tfrac{1}{8} + \tfrac{3}{4}\sin^2 L_0 - \tfrac{5}{8}\sin^4 L_0)\big]. \tag{3–49}$$

Equations (3–46) through (3–49) are the desired results for the coefficients of I.

3.3.2. The Expansion for L

The right ascension of the ascending node of the invariable plane on the equator of date, denoted by L, is given by equation (3–11):

$$L = \operatorname{plg}\left[\cos\theta\sin(L_0+\zeta)\sin I_0 - \sin\theta\cos I_0,\ \cos(L_0+\zeta)\sin I_0\right] + z \tag{3–11}$$
$$= \tan^{-1}\left(\frac{\cos\theta\sin(L_0+\zeta)\sin I_0 - \sin\theta\cos I_0}{\cos(L_0+\zeta)\sin I_0}\right) + z.$$

In order to derive the coefficients L_k such that

$$L = L_0 + L_1T + L_2T^2 + L_3T^3 + L_4T^4 + O(T^5), \tag{3–50}$$

one must develop both the numerator and denominator of the argument of the arctangent as approximation polynomials, obtain their quotient, and expand the arctangent itself.

Denote the numerator of the argument of the arctangent in equation (3–11) by n and the denominator by d. Then inserting the results of equations (3–14) through (3–17) gives the following:

$$\begin{aligned}
n = {} & \cos\theta\sin(L_0+\zeta)\sin I_0 - \sin\theta\cos I_0 \\
= {} & \left[1 - \tfrac{1}{2}\theta_1^2T^2 - \theta_1\theta_2T^3 - (\theta_1\theta_3 + \tfrac{1}{2}\theta_2^2 - \tfrac{1}{24}\theta_1^4)T^4\right] \\
& \times \Big\{\sin L_0 + (\zeta_1\cos L_0)T + (\zeta_2\cos L_0 - \tfrac{1}{2}\zeta_1^2\sin L_0)T^2 \\
& \quad + [(\zeta_3 - \tfrac{1}{6}\zeta_1^3)\cos L_0 - \zeta_1\zeta_2\sin L_0]T^3 \\
& \quad + [(\zeta_4 - \tfrac{1}{2}\zeta_1^2\zeta_2)\cos L_0 - (\zeta_1\zeta_3 + \tfrac{1}{2}\zeta_2^2 - \tfrac{1}{24}\zeta_1^4)\sin L_0]T^4\Big\}\sin I_0 \\
& - \left[1 - \tfrac{1}{2}\theta_1^2T^2 - \theta_1\theta_2T^3 - (\theta_1\theta_3 + \tfrac{1}{2}\theta_2^2 - \tfrac{1}{24}\theta_1^4)T^4\right]\cos I_0 + O(T^5) \\
= {} & \sin L_0\sin I_0 + (\zeta_1\cos L_0\sin I_0 - \theta_1\cos I_0)T \\
& + [\zeta_2\cos L_0\sin I_0 - \tfrac{1}{2}(\zeta_1^2 + \theta_1^2)\sin L_0\sin I_0 - \theta_2\cos I_0]T^2 \\
& + [(\zeta_3 - \tfrac{1}{6}\zeta_1^3 - \tfrac{1}{2}\theta_1^2\zeta_1)\cos L_0\sin I_0 - (\zeta_1\zeta_2 + \theta_1\theta_2)\sin L_0\sin I_0
\end{aligned}$$

$$- (\theta_3 - \tfrac{1}{6}\theta_1^3)\cos I_0]T^3$$
$$+ \{[\zeta_4 - \theta_1\theta_2\zeta_1 - \tfrac{1}{2}(\zeta_1^2 + \theta_1^2)\zeta_2]\cos L_0 \sin I_0$$
$$+ [-\theta_1\theta_3 - \zeta_1\zeta_3 - \tfrac{1}{2}\theta_2^2 + \tfrac{1}{2}\zeta_2^2 + \tfrac{1}{4}\theta_1^2\zeta_1^2 + \tfrac{1}{24}(\theta_1^4 + \zeta_1^4)]\sin L_0 \sin I_0$$
$$- (\theta_4 - \tfrac{1}{2}\theta_1^2\theta_2)\cos I_0\}T^4 + O(T^5); \tag{3–51}$$

$$\begin{aligned} d =\ & \cos(L_0 + \zeta)\sin I_0 \\ =\ & \{\cos L_0 - (\zeta_1 \sin L_0)T - (\tfrac{1}{2}\zeta_1^2\cos L_0 + \zeta_2 \sin L_0)T^2 \\ & - [\zeta_1\zeta_2\cos L_0 + (\zeta_3 - \tfrac{1}{6}\zeta_1^3)\sin L_0]T^3 \\ & - [(\zeta_1\zeta_3 + \tfrac{1}{2}\zeta_2^2 - \tfrac{1}{24}\zeta_1^4)\cos L_0 + (\zeta_4 - \tfrac{1}{2}\zeta_1^2\zeta_2)\sin L_0]T^4\}\sin I_0 + O(T^5) \\ =\ & \cos L_0 \sin I_0 - (\zeta_1 \sin L_0 \sin I_0)T - (\tfrac{1}{2}\zeta_1^2\cos L_0 + \zeta_2\sin L_0)\sin I_0 T^2 \\ & - [\zeta_1\zeta_2\cos L_0 + (\zeta_3 - \tfrac{1}{6}\zeta_1^3)\sin L_0]\sin I_0 T^3 \\ & - [(\zeta_1\zeta_3 + \tfrac{1}{2}\zeta_2^2 - \tfrac{1}{24}\zeta_1^4)\cos L_0 + (\zeta_4 - \tfrac{1}{2}\zeta_1^2\zeta_2)\sin L_0]\sin I_0 T^4 + O(T^5). \end{aligned} \tag{3–52}$$

If the leading terms in both n and d are factored out, the argument of the arctangent in equation (3–11) takes the form

$$\begin{aligned} \frac{n}{d} &= \frac{\sin L_0 \sin I_0[1 + n_1T + n_2T^2 + n_3T^3 + n_4T^4 + O(T^5)]}{\cos L_0 \sin I_0[1 - d_1T - d_2T^2 - d_3T^3 - d_4T^4 + O(T^5)]} \\ &= \tan L_0\left(\frac{1 + n_1T + n_2T^2 + n_3T^3 + n_4T^4 + O(T^5)}{1 - d_1T - d_2T^2 - d_3T^3 - d_4T^4 + O(T^5)}\right), \end{aligned} \tag{3–53}$$

since $\sin I_0 \neq 0$. (The minus signs in the denominator cancel some of the minus signs in equation (3–52) and also facilitate expansion of the quotient later.) Now the coefficients n_k and d_k are given by

$$n_1 = \zeta_1 \cot L_0 - \theta_1 \cot I_0 \csc L_0; \tag{3–54}$$

$$n_2 = \zeta_2 \cot L_0 - \tfrac{1}{2}(\theta_1^2 + \zeta_1^2) - \theta_2 \cot I_0 \csc L_0; \tag{3–55}$$

$$n_3 = (\zeta_3 - \tfrac{1}{6}\zeta_1^3 - \tfrac{1}{2}\theta_1^2\zeta_1)\cot L_0 - (\zeta_1\zeta_2 + \theta_1\theta_2) - (\theta_3 - \tfrac{1}{6}\theta_1^3)\cot I_0 \csc L_0; \tag{3–56}$$

$$\begin{aligned} n_4 = {} & [\zeta_4 - \tfrac{1}{2}\zeta_2(\theta_1^2 + \zeta_1^2) - \theta_1\theta_2\zeta_1]\cot L_0 \\ & - [\theta_1\theta_3 + \zeta_1\zeta_3 + \tfrac{1}{2}(\theta_2^2 + \zeta_2^2) - \tfrac{1}{4}\theta_1^2\zeta_1^2 - \tfrac{1}{24}(\theta_1^4 + \zeta_1^4)] \\ & - (\theta_4 - \tfrac{1}{2}\theta_1^2\theta_2)\cot I_0 \csc L_0; \end{aligned} \tag{3–57}$$

$$d_1 = \zeta_1 \tan L_0; \tag{3–58}$$

$$d_2 = \zeta_2 \tan L_0 + \tfrac{1}{2}\zeta_1^2; \tag{3–59}$$

$$d_3 = (\zeta_3 - \tfrac{1}{6}\zeta_1^3)\tan L_0 + \zeta_1\zeta_2; \tag{3–60}$$

$$d_4 = (\zeta_4 - \tfrac{1}{2}\zeta_1^2\zeta_2)\tan L_0 + \zeta_1\zeta_3 + \tfrac{1}{2}\zeta_2^2 - \tfrac{1}{24}\zeta_1^4. \tag{3–61}$$

The next step is to expand the quotient that forms the argument of the arctangent function in equation (3–11). Denote the quotient by q; one obtains by simple long division

$$q = \left(\frac{1 + n_1T + n_2T^2 + n_3T^3 + n_4T^4 + O(T^5)}{1 - d_1T - d_2T^2 - d_3T^3 - d_4T^4 + O(T^5)}\right) \tag{3–62}$$

$$\begin{aligned} &= 1 + (n_1 + d_1)T + (n_2 + d_2 + d_1n_1 + d_1^2)T^2 \\ &\quad + (n_3 + d_3 + 2d_1d_2 + d_2n_1 + d_1n_2 + d_1^2n_1 + d_1^3)T^3 \\ &\quad + (n_4 + d_4 + 2d_1d_3 + d_2n_2 + d_2^2 + 2d_1d_2n_1 + 3d_1^2d_2 \\ &\qquad + d_1n_3 + d_1^2n_2 + d_1^3n_1 + d_1^4)T^4 + O(T^5) \end{aligned} \tag{3–63}$$

$$\equiv 1 + q_1T + q_2T^2 + q_3T^3 + q_4T^4 + O(T^5). \tag{3–64}$$

This leaves equation (3–11) in the form

$$L = \tan^{-1}(q \tan L_0) + z \tag{3–65}$$

$$\begin{aligned} &= \tan^{-1}\left\{\tan L_0 + \tan L_0[q_1T + q_2T^2 + q_3T^3 + q_4T^4 + O(T^5)]\right\} \\ &\quad + z_1T + z_2T^2 + z_3T^3 + z_4T^4 + O(T^5). \end{aligned} \tag{3–66}$$

The expansion for the arctangent was given above in equations (3–27) through (3–31). Here L_0 plays the rôle of x_0, and $q_k \tan L_0$ are to be substituted for x_k. When z is added, the expansion becomes:

$$L_1 = (q_1 \tan L_0)\cos^2 L_0 + z_1; \tag{3–67}$$

$$L_2 = (q_2 \tan L_0)\cos^2 L_0 - (q_1 \tan L_0)^2 \sin L_0 \cos^3 L_0 + z_2; \tag{3–68}$$

$$\begin{aligned} L_3 &= (q_3 \tan L_0)\cos^2 L_0 - 2(q_1q_2 \tan^2 L_0)\sin L_0 \cos^3 L_0 \\ &\quad + (q_1 \tan L_0)^3(\sin^2 L_0 \cos^4 L_0 - \tfrac{1}{3}\cos^6 L_0) + z_3; \end{aligned} \tag{3–69}$$

$$
\begin{aligned}
L_4 = {} & (q_4 \tan L_0) \cos^2 L_0 - [(q_2^2 + 2q_1 q_3) \tan^2 L_0] \sin L_0 \cos^3 L_0 \\
& + 3q_1^2 q_2 \tan^3 L_0 (\sin^2 L_0 \cos^4 L_0 - \tfrac{1}{3} \cos^6 L_0) \\
& + (q_1 \tan L_0)^4 (\sin L_0 \cos^7 L_0 - \sin^3 L_0 \cos^5 L_0) + z_4.
\end{aligned} \tag{3-70}
$$

It is easiest to leave the $\tan L_0$ with the q_k.

Now replacing n_k and d_k with their values from equations (3–54) through (3–61) gives

$$
\begin{aligned}
q_1 \tan L_0 &= (n_1 + d_1) \tan L_0 \\
&= [\zeta_1 \cot L_0 - \theta_1 \cot I_0 \csc L_0 + \zeta_1 \tan L_0] \tan L_0 \\
&= \zeta_1 - \theta_1 \cot I_0 \sec L_0 + \zeta_1 \tan^2 L_0 \\
&= \zeta_1 \sec^2 L_0 - \theta_1 \cot I_0 \sec L_0;
\end{aligned} \tag{3-71}
$$

$$
\begin{aligned}
q_2 \tan L_0 &= (n_2 + d_2 + d_1 n_1 + d_1^2) \tan L_0 \\
&= \{[\zeta_2 \cot L_0 - \tfrac{1}{2}(\theta_1^2 + \zeta_1^2) - \theta_2 \cot I_0 \csc L_0] + (\zeta_2 \tan L_0 + \tfrac{1}{2}\zeta_1^2) \\
&\qquad + (\zeta_1 \cot L_0 - \theta_1 \cot I_0 \csc L_0)(\zeta_1 \tan L_0) + (\zeta_1 \tan L_0)^2\} \tan L_0 \\
&= [\zeta_2 - \tfrac{1}{2}(\theta_1^2 + \zeta_1^2) \tan L_0 - \theta_2 \cot I_0 \sec L_0] + (\zeta_2 \tan^2 L_0 + \tfrac{1}{2}\zeta_1^2 \tan L_0) \\
&\quad + (\zeta_1^2 \tan L_0 - \theta_1 \zeta_1 \cot I_0 \sin L_0 \tan L_0) + \zeta_1^2 \tan^3 L_0 \\
&= \zeta_2 \sec^2 L_0 - \theta_2 \cot I_0 \sec L_0 + \zeta_1^2 \tan L_0 \sec^2 L_0 \\
&\quad - \tfrac{1}{2}\theta_1^2 \tan L_0 - \theta_1 \zeta_1 \cot I_0 \tan L_0 \sec L_0;
\end{aligned} \tag{3-72}
$$

$$
\begin{aligned}
q_3 \tan L_0 &= [n_3 + n_2 d_1 + n_1(d_1^2 + d_2) + d_3 + d_1^3 + 2d_1 d_2] \tan L_0 \\
&= \{(\zeta_3 - \tfrac{1}{6}\zeta_1^3 - \tfrac{1}{2}\theta_1^2 \zeta_1) \cot L_0 - (\zeta_1 \zeta_2 + \theta_1 \theta_2) - (\theta_3 - \tfrac{1}{6}\theta_1^3) \cot I_0 \csc L_0 \\
&\qquad + [\zeta_2 \cot L_0 - \tfrac{1}{2}(\theta_1^2 + \zeta_1^2) - \theta_2 \cot I_0 \csc L_0](\zeta_1 \tan L_0) \\
&\qquad + (\zeta_1 \cot L_0 - \theta_1 \cot I_0 \csc L_0)[(\zeta_1 \tan L_0)^2 + (\zeta_2 \tan L_0 + \tfrac{1}{2}\zeta_1^2)] \\
&\qquad + [(\zeta_3 - \tfrac{1}{6}\zeta_1^3) \tan L_0 + \zeta_1 \zeta_2] + (\zeta_1 \tan L_0)^3 \\
&\qquad + 2(\zeta_1 \tan L_0)(\zeta_2 \tan L_0 + \tfrac{1}{2} z_1^2)\} \tan L_0 \\
&= [\zeta_3 - \tfrac{1}{6}\zeta_1^3 - \tfrac{1}{2}\theta_1^2 \zeta_1 - (\zeta_1 \zeta_2 + \theta_1 \theta_2) \tan L_0 - (\theta_3 - \tfrac{1}{6}\theta_1^3) \cot I_0 \sec L_0] \\
&\quad + [\zeta_1 \zeta_2 \tan L_0 - \tfrac{1}{2}\zeta_1(\theta_1^2 + \zeta_1^2) \tan L_0 - \theta_2 \zeta_1 \cot I_0 \tan L_0 \sec L_0] \\
&\quad + (\zeta_1^3 \tan^2 L_0 + \zeta_1 \zeta_2 \tan L_0 + \tfrac{1}{2}\zeta_1^3 - \theta_1 \zeta_1^2 \cot I_0 \tan^2 L_0 \sec L_0 \\
&\qquad - \theta_1 \zeta_2 \cot I_0 \tan L_0 \sec L_0 - \tfrac{1}{2}\theta_1 \zeta_1^2 \cot I_0 \sec L_0) \\
&\quad + [(\zeta_3 - \tfrac{1}{6}\zeta_1^3) \tan^2 L_0] + \zeta_1 \zeta_2 \tan L_0 + \zeta_1^3 \tan^4 L_0 \\
&\quad + (2\zeta_1 \zeta_2 \tan^3 L_0 + \zeta_1^3 \tan^2 L_0)
\end{aligned}
$$

$$= (\zeta_3 + \tfrac{1}{3}\zeta_1^3 - \tfrac{1}{2}\theta_1^2\zeta_1)\sec^2 L_0 + 2\zeta_1\zeta_2 \tan L_0 \sec^2 L_0 - \theta_1\theta_2 \tan L_0$$
$$+ \zeta_1^3 \tan^2 L_0 \sec^2 L_0$$
$$+ \cot I_0 \sec L_0[-\theta_3 + \tfrac{1}{6}\theta_1^3 - \tfrac{1}{2}\theta_1\zeta_1^2 - (\theta_2\zeta_1 + \theta_1\zeta_2)\tan L_0$$
$$- \theta_1\zeta_1^2 \tan^2 L_0]; \qquad (3\text{–}73)$$

$$q_4 \tan L_0 = (n_4 + d_4 + 2d_1d_3 + d_2n_2 + d_2^2 + 2d_1d_2n_1 + 3d_1^2d_2$$
$$+ d_1n_3 + d_1^2n_2 + d_1^3n_1 + d_1^4)\tan L_0$$

$$= \Big(\{[\zeta_4 - \tfrac{1}{2}\zeta_2(\theta_1^2 + \zeta_1^2) - \theta_1\theta_2\zeta_1]\cot L_0$$
$$- [\theta_1\theta_3 + \zeta_1\zeta_3 + \tfrac{1}{2}(\theta_2^2 + \zeta_2^2) - \tfrac{1}{4}\theta_1^2\zeta_1^2 - \tfrac{1}{24}(\theta_1^4 + \zeta_1^4)]$$
$$- (\theta_4 - \tfrac{1}{2}\theta_1^2\theta_2)\cot I_0 \csc L_0\}$$
$$+ [(\zeta_4 - \tfrac{1}{2}\zeta_1^2\zeta_2)\tan L_0 + \zeta_1\zeta_3 + \tfrac{1}{2}\zeta_2^2 - \tfrac{1}{24}\zeta_1^4]$$
$$+ 2(\zeta_1 \tan L_0)[(\zeta_3 - \tfrac{1}{6}\zeta_1^3)\tan L_0 + \zeta_1\zeta_2]$$
$$+ (\zeta_2 \tan L_0 + \tfrac{1}{2}\zeta_1^2)[\zeta_2 \cot L_0 - \tfrac{1}{2}(\theta_1^2 + \zeta_1^2) - \theta_2 \cot I_0 \csc L_0]$$
$$+ (\zeta_2 \tan L_0 + \tfrac{1}{2}\zeta_1^2)^2$$
$$+ 2(\zeta_1 \tan L_0)(\zeta_2 \tan L_0 + \tfrac{1}{2}\zeta_1^2)(\zeta_1 \cot L_0 - \theta_1 \cot I_0 \csc L_0)$$
$$+ 3(\zeta_1 \tan L_0)^2(\zeta_2 \tan L_0 + \tfrac{1}{2}\zeta_1^2)$$
$$+ (\zeta_1 \tan L_0)[(\zeta_3 - \tfrac{1}{6}\zeta_1^3 - \tfrac{1}{2}\theta_1^2\zeta_1)\cot L_0$$
$$- (\zeta_1\zeta_2 + \theta_1\theta_2) - (\theta_3 - \tfrac{1}{6}\theta_1^3)\cot I_0 \csc L_0]$$
$$+ (\zeta_1 \tan L_0)^2[\zeta_2 \cot L_0 - \tfrac{1}{2}(\theta_1^2 + \zeta_1^2) - \theta_2 \cot I_0 \csc L_0]$$
$$+ (\zeta_1 \tan L_0)^3(\zeta_1 \cot L_0 - \theta_1 \cot I_0 \csc L_0) + (\zeta_1 \tan L_0)^4\Big)\tan L_0$$

$$= [\zeta_4 - \tfrac{1}{2}\zeta_2(\theta_1^2 + \zeta_1^2) - \theta_1\theta_2\zeta_1$$
$$- [\theta_1\theta_3 + \zeta_1\zeta_3 + \tfrac{1}{2}(\theta_2^2 + \zeta_2^2) - \tfrac{1}{4}\theta_1^2\zeta_1^2 - \tfrac{1}{24}(\theta_1^4 + \zeta_1^4)]\tan L_0$$
$$- (\theta_4 - \tfrac{1}{2}\theta_1^2\theta_2)\cot I_0 \sec L_0]$$
$$+ [(\zeta_4 - \tfrac{1}{2}\zeta_1^2\zeta_2)\tan^2 L_0 + (\zeta_1\zeta_3 + \tfrac{1}{2}\zeta_2^2 - \tfrac{1}{24}\zeta_1^4)\tan L_0]$$
$$+ (2\zeta_1\zeta_3 \tan^3 L_0 - \tfrac{1}{3}\zeta_1^4 \tan^3 L_0 + 2\zeta_1^2\zeta_2 \tan^2 L_0)$$
$$+ [\zeta_2^2 \tan L_0 - \tfrac{1}{2}\zeta_2(\theta_1^2 + \zeta_1^2)\tan^2 L_0 - \theta_2\zeta_2 \cot I_0 \tan L_0 \sec L_0$$
$$+ \tfrac{1}{2}\zeta_1^2\zeta_2 - \tfrac{1}{4}\zeta_1^2(\theta_1^2 + \zeta_1^2)\tan L_0 - \tfrac{1}{2}\theta_2\zeta_1^2 \cot I_0 \sec L_0]$$
$$+ (\zeta_2^2 \tan^3 L_0 + \zeta_1^2\zeta_2 \tan^2 L_0 + \tfrac{1}{4}\zeta_1^2 \tan L_0)$$
$$+ (2\zeta_1^2\zeta_2 \tan^2 L_0 + \zeta_1^4 - 2\theta_1\zeta_1\zeta_2 \cot I_0 \tan^2 L_0 \sec L_0 - \theta_1\zeta_1^3 \cot I_0 \sec L_0)$$
$$+ (3\zeta_1^2\zeta_2 \tan^4 L_0 + \tfrac{3}{2}\zeta_1^4 \tan_3 L_0)$$
$$+ [(\zeta_1\zeta_3 - \tfrac{1}{6}\zeta_1^4 - \tfrac{1}{2}\theta_1^2\zeta_1^2)\tan L_0 - (\zeta_1^2\zeta_2 + \theta_1\theta_2\zeta_1)\tan^2 L_0$$
$$- (\theta_3\zeta_1 - \tfrac{1}{6}\theta_1^3\zeta_1)\cot I_0 \tan L_0 \sec L_0]$$
$$+ [\zeta_1^2\zeta_2 \tan^2 L_0 - \tfrac{1}{2}\zeta_1^2(\theta_1^2 + \zeta_1^2)\tan^3 L_0 - \theta_2\zeta_1^2 \cot I_0 \tan^2 L_0 \sec L_0]$$

$$
\begin{aligned}
&+ (\zeta_1^4 \tan^3 L_0 - \theta_1 \zeta_1^3 \cot I_0 \tan^3 L_0 \sec L_0) + \zeta_1^4 \tan^5 L_0 \\
&= (\zeta_4 + \zeta_1^2 \zeta_2 - \theta_1 \theta_2 \zeta_1 - \tfrac{1}{2}\theta_1^2 \zeta_2) \sec^2 L_0 \\
&\quad + (\tfrac{2}{3}\zeta_1^4 + \zeta_2^2 + 2\zeta_1\zeta_3 - \tfrac{1}{2}\theta_1^2 \zeta_1^2) \tan L_0 \sec^2 L_0 \\
&\quad + 3\zeta_1^2 \zeta_2 \tan^2 L_0 \sec^2 L_0 + \zeta_1^4 \tan^3 L_0 \sec^2 L_0 - (\tfrac{1}{2}\theta_2^2 + \theta_1\theta_3 - \tfrac{1}{24}\theta_1^4) \tan L_0 \\
&\quad + \cot I_0 \sec L_0 [(\tfrac{1}{2}\theta_1^2 \theta_2 - \theta_4 - \tfrac{1}{2}\theta_2 \zeta_1^2 - \theta_1 \zeta_1 \zeta_2) \\
&\qquad + (\tfrac{1}{6}\theta_1^3 \zeta_1 - \tfrac{5}{6}\theta_1 \zeta_1^3 - \theta_1 \zeta_3 - \theta_2 \zeta_2 - \theta_3 \zeta_1) \tan L_0 \\
&\qquad - (\theta_2 \zeta_1^2 + 2\theta_1 \zeta_1 \zeta_2) \tan^2 L_0 - \theta_1 \zeta_1^3 \tan^3 L_0]. \qquad (3\text{–}74)
\end{aligned}
$$

The final step is to substitute these values for the $q_k \tan L_0$ into equations (3–67) through (3–70). For L_1, we obtain

$$
\begin{aligned}
L_1 &= (q_1 \tan L_0) \cos^2 L_0 + z_1 \qquad (3\text{–}67) \\
&= (\zeta_1 \sec^2 L_0 - \theta_1 \cot I_0 \sec L_0) \cos^2 L_0 + z_1 \\
&= \zeta_1 + z_1 - \theta_1 \cot I_0 \cos L_0. \qquad (3\text{–}75)
\end{aligned}
$$

Next, we get for L_2:

$$
\begin{aligned}
L_2 &= (q_2 \tan L_0) \cos^2 L_0 - (q_1 \tan L_0)^2 \sin L_0 \cos^3 L_0 + z_2 \qquad (3\text{–}68) \\
&= [\zeta_2 \sec^2 L_0 - \theta_2 \cot I_0 \sec L_0 + \zeta_1^2 \tan L_0 \sec^2 L_0 \\
&\qquad - \tfrac{1}{2}\theta_1^2 \tan L_0 - \theta_1 \zeta_1 \cot I_0 \tan L_0 \sec L_0] \cos^2 L_0 \\
&\quad - (\zeta_1 \sec^2 L_0 - \theta_1 \cot I_0 \sec L_0)^2 \sin L_0 \cos^3 L_0 + z_2 \\
&= \zeta_2 + z_2 - \theta_2 \cot I_0 \cos L_0 + \zeta_1^2 \tan L_0 \\
&\quad - \tfrac{1}{2}\theta_1^2 \sin L_0 \cos L_0 - \theta_1 \zeta_1 \cot I_0 \sin L_0 \\
&\quad - (\zeta_1^2 \tan L_0 - 2\theta_1 \zeta_1 \cot I_0 \sin L_0 + \theta_1^2 \cot^2 I_0 \sin L_0 \cos L_0) \\
&= \zeta_2 + z_2 - \theta_2 \cot I_0 \cos L_0 + \theta_1 \zeta_1 \cot I_0 \sin L_0 \\
&\quad - \theta_1^2 \sin L_0 \cos L_0 (\cot^2 I_0 + \tfrac{1}{2}). \qquad (3\text{–}76)
\end{aligned}
$$

Note that the terms involving $\tan L_0$ cancel. Heuristic reasoning suggests as much, since there must be no singularity involving L_0 regardless of which quadrant it happens to occupy. The same will also be seen to hold for L_3 and L_4.

The right-hand side of equation (3–69) for L_3 has three terms involving q_k, plus z_3. Let us include z_3 in the first term (which will contain ζ_3); the three terms become, in order:

$$\begin{aligned}
L_{3a} &= (q_3 \tan L_0)\cos^2 L_0 + z_3\\
&= \{(\zeta_3 + \tfrac{1}{3}\zeta_1^3 - \tfrac{1}{2}\theta_1^2\zeta_1)\sec^2 L_0 + 2\zeta_1\zeta_2 \tan L_0 \sec^2 L_0 - \theta_1\theta_2 \tan L_0\\
&\qquad + \zeta_1^3 \tan^2 L_0 \sec^2 L_0\\
&\qquad + \cot I_0 \sec L_0[-\theta_3 + \tfrac{1}{6}\theta_1^3 - \tfrac{1}{2}\theta_1\zeta_1^2 - (\theta_2\zeta_1 + \theta_1\zeta_2)\tan L_0\\
&\qquad - \theta_1\zeta_1^2 \tan^2 L_0]\}\cos^2 L_0 + z_3\\
&= \zeta_3 + z_3 + \tfrac{1}{3}\zeta_1^3 - \tfrac{1}{2}\theta_1^2\zeta_1 + 2\zeta_1\zeta_2 \tan L_0 - \theta_1\theta_2 \sin L_0 \cos L_0 + \zeta_1^3 \tan^2 L_0\\
&\quad + \cot I_0[(-\theta_3 + \tfrac{1}{6}\theta_1^3 - \tfrac{1}{2}\theta_1\zeta_1^2)\cos L_0 - (\theta_2\zeta_1 + \theta_1\zeta_2)\sin L_0\\
&\quad - \theta_1\zeta_1^2 \sin L_0 \tan L_0];
\end{aligned} \tag{3-77}$$

$$\begin{aligned}
L_{3b} &= -2(q_1 q_2 \tan^2 L_0)\sin L_0 \cos^3 L_0\\
&= -2(\zeta_1 \sec^2 L_0 - \theta_1 \cot I_0 \sec L_0)\\
&\qquad \times (\zeta_2 \sec^2 L_0 - \theta_2 \cot I_0 \sec L_0 + \zeta_1^2 \tan L_0 \sec^2 L_0 - \tfrac{1}{2}\theta_1^2 \tan L_0\\
&\qquad - \theta_1\zeta_1 \cot I_0 \tan L_0 \sec L_0)\\
&\qquad \times \sin L_0 \cos^3 L_0\\
&= -2\zeta_1\zeta_2 \tan L_0 - 2\zeta_1^3 \tan^2 L_0 - \theta_1^2\zeta_1 \sin^2 L_0\\
&\quad + \cot I_0[2(\theta_1\zeta_2 + \theta_2\zeta_1)\sin L_0 + 4\theta_1\zeta_1^2 \sin L_0 \tan L_0 - \theta_1^3 \sin^2 L_0 \cos L_0]\\
&\quad + \cot^2 I_0(-2\theta_1\theta_2 \sin L_0 \cos L_0 - 2\theta_1^2\zeta_1 \sin^2 L_0);
\end{aligned} \tag{3-78}$$

$$\begin{aligned}
L_{3c} &= (q_1 \tan L_0)^3(\sin^2 L_0 \cos^4 L_0 - \tfrac{1}{3}\cos^6 L_0)\\
&= (\zeta_1 \sec^2 L_0 - \theta_1 \cot I_0 \sec L_0)^3(\sin^2 L_0 \cos^4 L_0 - \tfrac{1}{3}\cos^6 L_0)\\
&= \zeta_1^3 \tan^2 L_0 - \tfrac{1}{3}\zeta_1^3 + \cot I_0(-3\theta_1\zeta_1^2 \sin L_0 \tan L_0 + 3\theta_1\zeta_1^2 \cos L_0)\\
&\quad + \cot^2 I_0(3\theta_1^2\zeta_1 \sin^2 L_0 - \theta_1^2\zeta_1 \cos^2 L_0)\\
&\quad + \cot^3 I_0(-\theta_1^3 \sin^2 L_0 \cos L_0 + \tfrac{1}{3}\theta_1^3 \cos^3 L_0).
\end{aligned} \tag{3-79}$$

The desired coefficient L_3 is the sum of these three:

$$\begin{aligned}
L_3 &= \zeta_3 + z_3 - \tfrac{1}{2}\theta_1^2\zeta_1 - \theta_1\theta_2 \sin L_0 \cos L_0 + \zeta_1\theta_1^2 \sin^2 L_0\\
&\quad + \cot I_0[(-\theta_3 + \tfrac{1}{6}\theta_1^3 + \tfrac{1}{2}\theta_1\zeta_1^2)\cos L_0 + (\theta_1\zeta_2 + \theta_2\zeta_1)\sin L_0\\
&\quad - \theta_1^3 \sin^2 L_0 \cos L_0]
\end{aligned}$$

$$+ \cot^2 I_0[\theta_1^2\zeta_1(\sin^2 L_0 - \cos^2 L_0) - 2\theta_1\theta_2 \sin L_0 \cos L_0]$$
$$+ \cot^3 I_0[\theta_1^3(\tfrac{1}{3}\cos^3 L_0 - \sin^2 L_0 \cos L_0)]. \tag{3-80}$$

Finally, equation (3-70) for L_4 contains four terms on its right-hand side:

$$\begin{aligned}
L_{4a} &= (q_4 \tan L_0)\cos^2 L_0 + z_4 \\
&= \{(\zeta_4 + \zeta_1^2\zeta_2 - \theta_1\theta_2\zeta_1 - \tfrac{1}{2}\theta_1^2\zeta_2)\sec^2 L_0 \\
&\quad + (\tfrac{2}{3}\zeta_1^4 + \zeta_2^2 + 2\zeta_1\zeta_3 - \tfrac{1}{2}\theta_1^2\zeta_1^2)\tan L_0 \sec^2 L_0 \\
&\quad + 3\zeta_1^2\zeta_2 \tan^2 L_0 \sec^2 L_0 + \zeta_1^4 \tan^3 L_0 \sec^2 L_0 - (\tfrac{1}{2}\theta_2^2 + \theta_1\theta_3 - \tfrac{1}{24}\theta_1^4)\tan L_0 \\
&\quad + \cot I_0 \sec L_0[(\tfrac{1}{2}\theta_1^2\theta_2 - \theta_4 - \tfrac{1}{2}\theta_2\zeta_1^2 - \theta_1\zeta_1\zeta_2) \\
&\qquad + (\tfrac{1}{6}\theta_1^3\zeta_1 - \tfrac{5}{6}\theta_1\zeta_1^3 - \theta_1\zeta_3 - \theta_2\zeta_2 - \theta_3\zeta_1)\tan L_0 \\
&\qquad - (\theta_2\zeta_1^2 + 2\theta_1\zeta_1\zeta_2)\tan^2 L_0 - \theta_1\zeta_1^3 \tan^3 L_0]\}\cos^2 L_0 + z_4 \\
&= \zeta_4 + z_4 + \zeta_1^2\zeta_2 - \theta_1\theta_2\zeta_1 - \tfrac{1}{2}\theta_1^2\zeta_2 - (\tfrac{1}{2}\theta_2^2 + \theta_1\theta_3 - \tfrac{1}{24}\theta_1^4)\sin L_0 \cos L_0 \\
&\quad + (\tfrac{2}{3}\zeta_1^4 + \zeta_2^2 + 2\zeta_1\zeta_3 - \tfrac{1}{2}\theta_1^2\zeta_1^2)\tan L_0 + 3\zeta_1^2\zeta_2 \tan^2 L_0 + \zeta_1^4 \tan^3 L_0 \\
&\quad + \cot I_0[(\tfrac{1}{2}\theta_1^2\theta_2 - \theta_4 - \tfrac{1}{2}\theta_2\zeta_1^2 - \theta_1\zeta_1\zeta_2)\cos L_0 \\
&\qquad + (\tfrac{1}{6}\theta_1^3\zeta_1 - \tfrac{5}{6}\theta_1\zeta_1^3 - \theta_1\zeta_3 - \theta_2\zeta_2 - \theta_3\zeta_1)\sin L_0 \\
&\qquad - (\theta_2\zeta_1^2 + 2\theta_1\zeta_1\zeta_2)\sin L_0 \tan L_0 - \theta_1\zeta_1^3 \sin L_0 \tan^2 L_0];
\end{aligned} \tag{3-81}$$

$$\begin{aligned}
L_{4b} &= -\,[(q_2^2 + 2q_1q_3)\tan^2 L_0]\sin L_0 \cos^3 L_0 \\
&= -\,\big([\zeta_2 \sec^2 L_0 - \theta_2 \cot I_0 \sec L_0 + \zeta_1^2 \tan L_0 \sec^2 L_0 - \tfrac{1}{2}\theta_1^2 \tan L_0 \\
&\qquad - \theta_1\zeta_1 \cot I_0 \tan L_0 \sec L_0]^2 \\
&\quad + 2(\zeta_1 \sec^2 L_0 - \theta_1 \cot I_0 \sec L_0) \\
&\quad \times \{(\zeta_3 + \tfrac{1}{3}\zeta_1^3 - \tfrac{1}{2}\theta_1\zeta_1^2)\sec^2 L_0 \\
&\qquad + 2\zeta_1\zeta_2 \tan L_0 \sec^2 L_0 - \theta_1\theta_2 \tan L_0 + \zeta_1^3 \tan^2 L_0 \sec^2 L_0 \\
&\quad + \cot I_0 \sec L_0[-\theta_3 + \tfrac{1}{6}\theta_1^3 - \tfrac{1}{2}\theta_1^2\zeta_1 \\
&\qquad - (\theta_2\zeta_1 + \theta_1\zeta_2)\tan L_0 - \theta_1\zeta_1^2 \tan^2 L_0]\}\big)\sin L_0 \cos^3 L_0 \\
&= -\,\big(\zeta_2^2 \tan L_0 + \theta_2^2 \cot^2 I_0 \sin L_0 \cos L_0 + \zeta_1^4 \tan^3 L_0 + \tfrac{1}{4}\theta_1^4 \sin^3 L_0 \cos L_0 \\
&\quad + \theta_1^2\zeta_1^2 \cot^2 I_0 \sin^2 L_0 \tan L_0 \\
&\quad - 2\theta_2\zeta_2 \cot I_0 \sin L_0 + 2\zeta_1^2\zeta_2 \tan^2 L_0 - \theta_1^2\zeta_2 \sin^2 L_0 \\
&\quad - 2\theta_1\zeta_1\zeta_2 \cot I_0 \sin L_0 \tan L_0 - 2\theta_2\zeta_1^2 \cot I_0 \sin L_0 \tan L_0 \\
&\quad + \theta_1^2\theta_2 \cot I_0 \sin^2 L_0 \cos L_0 + 2\theta_1\theta_2\zeta_1 \cot^2 I_0 \sin^2 L_0 \\
&\quad - \theta_1^2\zeta_1^2 \sin^2 L_0 \tan L_0 - 2\theta_1\zeta_1^3 \cot I_0 \sin L_0 \tan^2 L_0 + \theta_1^3\zeta_1 \cot I_0 \sin^3 L_0\big) \\
&\quad - 2\{(\zeta_1\zeta_3 + \tfrac{1}{3}\zeta_1^4 - \tfrac{1}{2}\theta_1^2\zeta_1^2)\tan L_0 + 2\zeta_1^2\zeta_1 \tan^2 L_0 - \theta_1\theta_2\zeta_1 \sin^2 L_0
\end{aligned}$$

$$
\begin{aligned}
&+ \zeta_1^4 \tan^3 L_0 \\
&+ \cot I_0[(-\theta_3\zeta_1 + \tfrac{1}{6}\theta_1\zeta_1^3 - \tfrac{1}{2}\theta_1\zeta_1^3 + \theta_1\zeta_1\zeta_2)\sin L_0 \\
&\quad - (\theta_1\zeta_1^2 + \theta_1\zeta_1\zeta_2)\sin L_0 \tan L_0 - \theta_1\zeta_1^3 \sin L_0 \tan^2 L_0] \\
&- \cot I_0[(\theta_1\zeta_3 + \tfrac{1}{3}\theta_1\zeta_1^3 - \tfrac{1}{2}\theta_1^3\zeta_1)\sin L_0 + 2\theta_1\zeta_1\zeta_2 \sin L_0 \tan L_0 \\
&\quad - \theta_1^2\theta_2 \sin^2 L_0 \cos L_0 + \theta_1\zeta_1^3 \tan^2 L_0 \sin L_0] \\
&- \cot^2 I_0[(-\theta_1\theta_3 + \tfrac{1}{6}\theta_1^4 - \tfrac{1}{2}\theta_1^2\zeta_1^2)\sin L_0 \cos L_0 \\
&\quad - (\theta_1\theta_2\zeta_1 + \theta_1^2\zeta_2)\sin^2 L_0 - \theta_1^2\zeta_1^2 \sin^2 L_0 \tan L_0]\}
\end{aligned}
$$

$$
\begin{aligned}
&= (\theta_1^2\zeta_2 + 2\theta_1\theta_2\zeta_1)\sin^2 L_0 - \tfrac{1}{4}\theta_1^4 \sin^3 L_0 \cos L_0 \\
&\quad + (-\zeta_2^2 - 2\zeta_1\zeta_3 - \tfrac{2}{3}\zeta_1^4 + \theta_1^2\zeta_1^2)\tan L_0 + \theta_1^2\zeta_1^2 \sin^2 L_0 \tan L_0 - 6\zeta_1^2 \tan^2 L_0 \\
&\quad - 3\zeta_1^4 \tan^3 L_0 \\
&\quad + \cot I_0 \sin L_0[2\theta_1\zeta_3 + 2\theta_2\zeta_2 + 2\theta_3\zeta_1 - \tfrac{4}{3}\theta_1^3\zeta_1 + \tfrac{5}{3}\theta_1\zeta_1^3 - 3\theta_1^2\theta_2 \sin L_0 \cos L_0 \\
&\quad - \theta_1^3\zeta_1 \sin^2 L_0 + (8\theta_1\zeta_1\zeta_2 + 4\theta_2\zeta_1^2)\tan L_0 + 6\theta_1\zeta_1^3 \tan^2 L_0] \\
&\quad + \cot^2 I_0 \sin L_0[(-\theta_2^2 + \tfrac{1}{3}\theta_1^4 - 2\theta_1\theta_3 - \theta_1^2\zeta_1^2)\cos L_0 \\
&\quad - (4\theta_1\theta_2\zeta_1 + 2\theta_1^2\zeta_2)\sin L_0 - 3\theta_1^2\zeta_1^2 \sin L_0 \tan L_0]; \qquad (3\text{–}82)
\end{aligned}
$$

$$
L_{4c} = 3q_1^2 q_2 \tan^3 L_0(\sin^2 L_0 \cos^4 L_0 - \tfrac{1}{3}\cos^6 L_0)
$$

$$
\begin{aligned}
&= 3(\zeta_1 \sec^2 L_0 - \theta_1 \cot I_0 \sec L_0)^2 \\
&\quad \times (\zeta_2 \sec^2 L_0 - \theta_2 \cot I_0 \sec L_0 + \zeta_1^2 \tan L_0 \sec^2 L_0 - \tfrac{1}{2}\theta_1^2 \tan L_0 \\
&\quad - \theta_1\zeta_1 \cot I_0 \tan L_0 \sec L_0) \\
&\quad \times (\sin^2 L_0 \cos^4 L_0 - \tfrac{1}{3}\cos^6 L_0)
\end{aligned}
$$

$$
\begin{aligned}
&= (\zeta_1 - \theta_1 \cot I_0 \cos L_0)^2 \\
&\quad \times (\zeta_2 - \theta_2 \cot I_0 \cos L_0 + \zeta_1^2 \tan L_0 - \tfrac{1}{2}\theta_1^2 \sin L_0 \cos L_0 - \theta_1\zeta_1 \cot I_0 \sin L_0) \\
&\quad \times (3\tan^2 L_0 - 1)
\end{aligned}
$$

$$
\begin{aligned}
&= [\zeta_1^2\zeta_2 - \tfrac{1}{2}\theta_1^2\zeta_1^2 \sin L_0 \cos L_0 + \zeta_1^4 \tan L_0 \\
&\quad + \cot I_0(-\theta_2\zeta_1^2 \cos L_0 - \theta_1\zeta_1^3 \sin L_0 - 2\theta_1\zeta_1\zeta_2 \cos L_0 + \theta_1^3 \sin L_0 \cos^2 L_0 \\
&\qquad - 2\theta_1\zeta_1^3 \sin L_0) \\
&\quad + \cot^2 I_0(2\theta_1\theta_2\zeta_1 \cos^2 L_0 + 2\theta_1^2\zeta_1^2 \sin L_0 \cos^2 L_0 - 2\theta_1\zeta_1^3 \sin L_0) \\
&\quad + \cot^3 I_0(-\theta_1^2\theta_2 \cos^3 L_0 - \theta_1^3\zeta_1 \sin L_0 \cos^2 L_0)](3\tan^2 L_0 - 1)
\end{aligned}
$$

$$
\begin{aligned}
&= -\zeta_1^2\zeta_2 + \tfrac{1}{2}\theta_1^2\zeta_1^2 \sin L_0 \cos L_0 - \zeta_1^4 \tan L_0 - \tfrac{3}{2}\theta_1^2\zeta_1^2 \sin^2 L_0 \tan L_0 \\
&\quad + 3\zeta_1^2\zeta_2 \tan^2 L_0 + 3\zeta_1^4 \tan^3 L_0 \\
&\quad + \cot I_0[(\theta_2\zeta_1^2 + 2\theta_1\zeta_1\zeta_2)\cos L_0 + 3\theta_1\zeta_1^3 \sin L_0 - \theta_1^3\zeta_1 \sin L_0 \cos^2 L_0 \\
&\qquad + 3\theta_1^3\zeta_1 \sin^3 L_0 - (3\theta_2\zeta_1^2 + 6\theta_1\zeta_1\zeta_2)\sin L_0 \tan L_0 - 9\theta_1\zeta_1^3 \sin L_0 \tan^2 L_0]
\end{aligned}
$$

$$
\begin{aligned}
&+ \cot^2 I_0[(-2\theta_1\theta_2\zeta_1 - \theta_1^2\zeta_2)\cos^2 L_0 + \tfrac{1}{2}\theta_1^4 \sin L_0 \cos^3 L_0 - 3\theta_1^2\zeta_1^2 \sin L_0 \cos L_0 \\
&\qquad + (6\theta_1\theta_2\zeta_1 + 3\theta_1^2\zeta_2)\sin^2 L_0 - \tfrac{3}{2}\theta_1^4 \sin_3 L_0 \cos L_0 + 9\theta_1^2\zeta_1^2 \sin^2 L_0 \tan L_0] \\
&+ \cot^3 I_0[\theta_1^2\theta_2 \cos^3 L_0 + \theta_1^3\zeta_1 \sin L_0 \cos^2 L_0 \\
&\qquad - 3\theta_1^2 \sin^2 L_0 \cos L_0 - 3\theta_1^3\zeta_1 \sin^3 L_0]; \qquad (3\text{–}83)
\end{aligned}
$$

$$
\begin{aligned}
L_{4d} &= (q_1 \tan L_0)^4(\sin L_0 \cos^7 L_0 - \sin^3 L_0 \cos^5 L_0) \\
&= (\zeta_1 \sec^2 L_0 - \theta_1 \cot I_0 \sec L_0)^4(\sin L_0 \cos^7 L_0 - \sin^3 L_0 \cos^5 L_0) \\
&= (\zeta_1 - \theta_1 \cot I_0 \cos L_0)^4(\tan L_0 - \tan^3 L_0) \\
&= \zeta_1^4(\tan L_0 - \tan^3 L_0) \\
&\quad + \cot I_0[4\theta_1\zeta_1^3(\sin L_0 \tan^2 L_0 - \sin L_0)] \\
&\quad + \cot^2 I_0[6\theta_1^2\zeta_1^2(\sin L_0 \cos L_0 - \sin^2 L_0 \tan L_0)] \\
&\quad + \cot^3 I_0[4\theta_1^3\zeta_1(\sin^3 L_0 - \sin L_0 \cos^2 L_0)] \\
&\quad + \cot^4 I_0[\theta_1^4(\sin L_0 \cos^3 L_0 - \sin^3 L_0 \cos L_0)]. \qquad (3\text{–}84)
\end{aligned}
$$

Their sum gives L_4:

$$
\begin{aligned}
L_4 &= \zeta_4 + z_4 - \tfrac{1}{2}\theta_1^2\zeta_2 + (\theta_1^2\zeta_1^2 - \tfrac{1}{2}\theta_2^2 - \theta_1\theta_3 + \tfrac{1}{24}\theta_1^4)\sin L_0 \cos L_0 \\
&\quad - \theta_1\theta_2\zeta_1(\cos^2 L_0 - \sin^2 L_0) + \theta_1^2\zeta_2 \sin^2 L_0 - \tfrac{1}{4}\theta_1^4 \sin^3 L_0 \cos L_0 \\
&\quad + \cot I_0\big\{[-\theta_4 + \theta_1\zeta_1\zeta_2 + \tfrac{1}{2}\theta_2\zeta_1^2 + \theta_1^2\theta_2(\tfrac{1}{2} - 3\sin^2 L_0)]\cos L_0 \\
&\qquad + [\theta_1\zeta_3 + \theta_2\zeta_2 + \theta_3\zeta_1 - \tfrac{1}{6}(\theta_1\zeta_1^3 + \theta_1^3\zeta_1) + \theta_1^3\zeta_1(\sin^2 L_0 - 2\cos^2 L_0)]\sin L_0\big\} \\
&\quad + \cot^2 I_0[(2\theta_1^2\zeta_1^2 - \theta_2^2 + \tfrac{1}{3}\theta_1^4 - 2\theta_1\theta_3)\sin L_0 \cos L_0 \\
&\qquad - (2\theta_1\theta_2\zeta_1 + \theta_1^2\zeta_2)(\cos^2 L_0 \sin^2 L_0) \\
&\qquad + \tfrac{1}{2}\theta_1^4 \sin L_0 \cos L_0(\cos^2 L_0 - 3\sin^2 L_0)] \\
&\quad + \cot^3 I_0[\theta_1^2\theta_2 \cos L_0(\cos^2 L_0 - 3\sin^2 L_0) - \theta_1^3\zeta_1 \sin L_0(3\cos^2 L_0 - \sin^2 L_0)] \\
&\quad + \cot^4 I_0[\theta_1^4 \sin L_0 \cos L_0(\cos^2 L_0 - \sin^2 L_0)]. \qquad (3\text{–}85)
\end{aligned}
$$

Equations (3–75), (3–76), (3–80), and (3–85) therefore contain the final results for the first four coefficients of the approximation polynomial for L. No attempt has been made here to use common trigonometric identities to simplify subexpressions such as $(\cos^2 L_0 - \sin^2 L_0)$, because in practice one would not wish to compute $\cos 2L_0$ directly: sines and cosines are

relatively time-consuming on a computer, and there will be no significant cancellation in the subtraction.

3.3.3. The Expansion for Δ

The angle Δ is measured along the invariable plane; its origin is the ascending node of the invariable plane on the mean equator of J2000.0, and its endpoint is the ascending node of the invariable plane on the mean equator of date. The exact expression for Δ was found at the beginning of this section:

$$\Delta = \operatorname{plg}\left[\cos(L_0+\zeta)\sin\theta,\ \sin I_0\cos\theta - \cos I_0\sin\theta\sin(L_0+\zeta)\right]. \tag{3–12}$$

We will expand the right-hand side of this equation in a manner similar to the expansion of L in the previous subsection, resulting in an expression of the form

$$\Delta = \Delta_1 T + \Delta_2 T^2 + \Delta_3 T^3 + \Delta_4 T^4 + O(T^5). \tag{3–86}$$

As in the previous subsection, denote the two arguments of the plg function in equation (3–12) by n and d respectively, as these are the numerator and denominator of the argument of the implied arctangent. (Although the notation is the same, the values of the symbols are obviously quite different in this subsection.) Substituting the expansions for the sine and cosine of θ and $(L_0+\zeta)$ from equations (3–14) through (3–17) gives:

$$\begin{aligned}
n =\ & \cos(L_0+\zeta)\sin\theta \\
=\ & \big[\cos L_0 - (\zeta_1\sin L_0)T - (\tfrac{1}{2}\zeta_1^2\cos L_0 + \zeta_2\sin L_0)T^2 \\
& \quad - [\zeta_1\zeta_2\cos L_0 + (\zeta_3 - \tfrac{1}{6}\zeta_1^3)\sin L_0]T^3 \\
& \quad - [(\zeta_1\zeta_3 + \tfrac{1}{2}\zeta_2^2 - \tfrac{1}{24}\zeta_1^4)\cos L_0 + (\zeta_4 - \tfrac{1}{2}\zeta_1^2\zeta_2)\sin L_0]T^4 + O(T^5)\big] \\
& \times \big[\theta_1 T + \theta_2 T^2 + (\theta_3 - \tfrac{1}{6}\theta_1^3)T^3 + (\theta_4 - \tfrac{1}{2}\theta_1^2\theta_2)T^4 + O(T^5)\big] \\
=\ & (\theta_1\cos L_0)T + (\theta_2\cos L_0 - \theta_1\zeta_1\sin L_0)T^2 \\
& + [(\theta_3 - \tfrac{1}{6}\theta_1^3 - \tfrac{1}{2}\theta_1\zeta_1^2)\cos L_0 - (\theta_1\zeta_2 + \theta_2\zeta_1)\sin L_0]T^3
\end{aligned}$$

$$+ [(\theta_4 - \tfrac{1}{2}\theta_1^2\theta_2 - \tfrac{1}{2}\theta_2\zeta_1^2 - \theta_1\zeta_1\zeta_2)\cos L_0$$
$$- (\theta_1\zeta_3 + \theta_2\zeta_2 + \theta_3\zeta_1 - \tfrac{1}{6}\theta_1^3\zeta_1 - \tfrac{1}{6}\theta_1\zeta_1^3)\sin L_0]T^4 + O(T^5); \qquad (3\text{–}87)$$

$$\begin{aligned}
d = {} & \sin I_0 \cos\theta - \cos I_0 \sin\theta \sin(L_0 + \zeta) \\
= {} & \sin I_0 \big[1 - \tfrac{1}{2}\theta_1^2 T^2 - \theta_1\theta_2 T^3 - (\theta_1\theta_3 + \tfrac{1}{2}\theta_2^2 - \tfrac{1}{24}\theta_1^4)T^4 + O(T^5)\big] \\
& - \cos I_0 \big[\theta_1 T + \theta_2 T^2 + (\theta_3 - \tfrac{1}{6}\theta_1^3)T^3 + (\theta_4 - \tfrac{1}{2}\theta_1^2\theta_2)T^4 + O(T^5)\big] \\
& \times \big\{ \sin L_0 + (\zeta_1 \cos L_0)T + (\zeta_2 \cos L_0 - \tfrac{1}{2}\zeta_1^2 \sin L_0)T^2 \\
& + [(\zeta_3 - \tfrac{1}{6}\zeta_1^3)\cos L_0 - \zeta_1\zeta_2 \sin L_0]T^3 \\
& + [(\zeta_4 - \tfrac{1}{2}\zeta_1^2\zeta_2)\cos L_0 - (\zeta_1\zeta_3 + \tfrac{1}{2}\zeta_2^2 - \tfrac{1}{24}\zeta_1^4)\sin L_0]T^4 + O(T^5)\big\} \\
= {} & \sin I_0 - (\theta_1 \sin L_0 \cos I_0)T \\
& - (\theta_2 \sin L_0 \cos I_0 + \theta_1\zeta_1 \cos L_0 \cos I_0 + \tfrac{1}{2}\theta_1^2 \sin I_0)T^2 \\
& - [(\theta_3 - \tfrac{1}{6}\theta_1^3 - \tfrac{1}{2}\theta_1\zeta_1^2)\sin L_0 \cos I_0 + (\theta_1\zeta_2 + \theta_2\zeta_1)\cos L_0 \cos I_0 + \theta_1\theta_2 \sin I_0]T^3 \\
& - [(\theta_4 - \tfrac{1}{2}\theta_1^2\theta_2 - \tfrac{1}{2}\theta_2\zeta_1^2 - \theta_1\zeta_1\zeta_2)\sin L_0 \cos I_0 \\
& + (\theta_1\zeta_3 + \theta_2\zeta_2 + \theta_3\zeta_1 - \tfrac{1}{6}\theta_1\zeta_1^3 - \tfrac{1}{6}\theta_1^3\zeta_1)\cos L_0 \sin I_0 \\
& + (\theta_1\theta_3 + \tfrac{1}{2}\theta_2^2 - \tfrac{1}{24}\theta_1^4)\sin I_0]T^4 + O(T^5). \qquad (3\text{–}88)
\end{aligned}$$

The absence of a constant term in the numerator is consistent with the statement made earlier that $\Delta = 0$ at $T = 0$. The algebra is simplified, however, if we divide both numerator and denominator by the the constant term in the denominator, namely $\sin I_0$; this is permissible because (as stated before) $I_0 \neq 0$. This leaves the argument of the arctangent in equation (3–12) in the form

$$\frac{n}{d} = \frac{n_1 T + n_2 T^2 + n_3 T^3 + n_4 T^4 + O(T^5)}{1 - d_1 T - d_2 T^2 - d_3 T^3 - d_4 T^4 + O(T^5)} \qquad (3\text{–}89)$$

with the coefficients n_k and d_k given by

$$n_1 = \theta_1 \cos L_0 \csc I_0; \qquad (3\text{–}90)$$

$$n_2 = (\theta_2 \cos L_0 - \theta_1\zeta_1 \sin L_0)\csc I_0; \qquad (3\text{–}91)$$

$$n_3 = [(\theta_3 - \tfrac{1}{6}\theta_1^3 - \tfrac{1}{2}\theta_1\zeta_1^2)\cos L_0 - (\theta_1\zeta_2 + \theta_2\zeta_1)\sin L_0]\csc I_0; \qquad (3\text{–}92)$$

$$n_4 = [(\theta_4 - \tfrac{1}{2}\theta_1^2\theta_2 - \tfrac{1}{2}\theta_2\zeta_1^2 - \theta_1\zeta_1\zeta_2)\cos L_0$$

$$- (\theta_1\zeta_3 + \theta_2\zeta_2 + \theta_3\zeta_1 - \tfrac{1}{6}\theta_1^3\zeta_1 - \tfrac{1}{6}\theta_1\zeta_1^3) \sin L_0] \csc I_0; \tag{3–93}$$

$$d_1 = \theta_1 \sin L_0 \cot I_0; \tag{3–94}$$

$$d_2 = \theta_2 \sin L_0 \cot I_0 + \theta_1\zeta_1 \cos L_0 \cot I_0 + \tfrac{1}{2}\theta_1^2; \tag{3–95}$$

$$d_3 = (\theta_3 - \tfrac{1}{6}\theta_1^3 - \tfrac{1}{2}\theta_1\zeta_1^2) \sin L_0 \cot I_0 + (\theta_1\zeta_2 + \theta_2\zeta_1) \cos L_0 \cot I_0 + \theta_1\theta_2; \tag{3–96}$$

$$\begin{aligned} d_4 = {} & (\theta_4 - \tfrac{1}{2}\theta_1^2\theta_2 - \tfrac{1}{2}\theta_2\zeta_1^2 - \theta_1\zeta_1\zeta_2) \sin L_0 \cot I_0 \\ & + (\theta_1\zeta_3 + \theta_2\zeta_2 + \theta_3\zeta_1 - \tfrac{1}{6}\theta_1\zeta_1^3 - \tfrac{1}{6}\theta_1^3\zeta_1) \cos L_0 \cot I_0 \\ & + (\theta_1\theta_3 + \tfrac{1}{2}\theta_2^2 - \tfrac{1}{24}\theta_1^4). \end{aligned} \tag{3–97}$$

The next step, as before, is to expand the quotient that forms the argument of the arctangent function in (3–12). Once again we shall denote the quotient by q. By an even simpler long division than before,

$$q = \left(\frac{n_1 T + n_2 T^2 + n_3 T^3 + n_4 T^4 + O(T^5)}{1 - d_1 T - d_2 T^2 - d_3 T^3 - d_4 T^4 + O(T^5)} \right) \tag{3–98}$$

$$\begin{aligned} &= 1 + n_1 T + (n_2 + d_1 n_1) T^2 + (n_3 + d_1 n_2 + d_2 n_1 + d_1^2 n_1) T^3 \\ &\quad + (n_4 + d_1 n_3 + d_2 n_2 + d_3 n_1 + d_1^2 n_2 + 2 d_1 d_2 n_1 + d_1^3 n_1) T^4 + O(T^5) \end{aligned} \tag{3–99}$$

$$\equiv 1 + q_1 T + q_2 T^2 + q_3 T^3 + q_4 T^4 + O(T^5). \tag{3–100}$$

The expressions for the q_k are given by:

$$\begin{aligned} q_1 &= n_1 \\ &= \theta_1 \cos L_0 \csc I_0; \end{aligned} \tag{3–101}$$

$$\begin{aligned} q_2 &= n_2 + d_1 n_1 \\ &= [(\theta_2 \cos L_0 - \theta_1\zeta_1 \sin L_0) \csc I_0] + (\theta_1 \sin L_0 \cot I_0)(\theta_1 \cos L_0 \csc I_0) \\ &= [\theta_2 \cos L_0 - \theta_1\zeta_1 \sin L_0 + \cot I_0(\theta_1^2 \sin L_0 \cos L_0)] \csc I_0; \end{aligned} \tag{3–102}$$

$$\begin{aligned} q_3 &= n_3 + d_1 n_2 + d_2 n_1 + d_1^2 n_1 \\ &= [(\theta_3 - \tfrac{1}{6}\theta_1^3 - \tfrac{1}{2}\theta_1\zeta_1^2) \cos L_0 - (\theta_1\zeta_2 + \theta_2\zeta_1) \sin L_0] \csc I_0 \\ &\quad + (\theta_1 \sin L_0 \cot I_0)[(\theta_2 \cos L_0 - \theta_1\zeta_1 \sin L_0) \csc I_0] \\ &\quad + (\theta_2 \sin L_0 \cot I_0 + \theta_1\zeta_1 \cos L_0 \cot I_0 + \tfrac{1}{2}\theta_1^2)(\theta_1 \cos L_0 \csc I_0) \end{aligned}$$

$$
\begin{aligned}
&+ (\theta_1 \sin L_0 \cot I_0)^2(\theta_1 \cos L_0 \csc I_0) \\
&= \big\{(\theta_3 + \tfrac{1}{3}\theta_1^3 - \tfrac{1}{2}\theta_1\zeta_1^2)\cos L_0 - (\theta_1\zeta_2 + \theta_2\zeta_1)\sin L_0 \\
&\quad + \cot I_0[2\theta_1\theta_2 \sin L_0 \cos L_0 + \theta_1^2\zeta_1(\cos^2 L_0 - \sin^2 L_0)] \\
&\quad + \cot^2 I_0(\theta_1^3 \sin^2 L_0 \cos L_0)\big\} \csc I_0; \qquad (3\text{–}103)
\end{aligned}
$$

$$
\begin{aligned}
q_4 &= n_4 + d_1 n_3 + d_2 n_2 + d_3 n_1 + d_1^2 n_2 + 2d_1 d_2 n_1 + d_1^3 n_1 \\
&= [(\theta_4 - \tfrac{1}{2}\theta_1^2\theta_2 - \tfrac{1}{2}\theta_2\zeta_1^2 - \theta_1\zeta_1\zeta_2)\cos L_0 \\
&\quad - (\theta_1\zeta_3 + \theta_2\zeta_2 + \theta_3\zeta_1 - \tfrac{1}{6}\theta_1^3\zeta_1 - \tfrac{1}{6}\theta_1\zeta_1^3)\sin L_0]\csc I_0 \\
&\quad + (\theta_1 \sin L_0 \cot I_0)\big\{[(\theta_3 - \tfrac{1}{6}\theta_1^3 - \tfrac{1}{2}\theta_1\zeta_1^2)\cos L_0 - (\theta_1\zeta_2 + \theta_2\zeta_1)\sin L_0]\csc I_0\big\} \\
&\quad + (\theta_2 \sin L_0 \cot I_0 + \theta_1\zeta_1 \cos L_0 \cot I_0 + \tfrac{1}{2}\theta_1^2)[(\theta_2 \cos L_0 - \theta_1\zeta_1 \sin L_0)\csc I_0] \\
&\quad + \big[(\theta_3 - \tfrac{1}{6}\theta_1^3 - \tfrac{1}{2}\theta_1\zeta_1^2)\sin L_0 \cot I_0 + (\theta_1\zeta_2 + \theta_2\zeta_1)\cos L_0 \cot I_0 + \theta_1\theta_2\big] \\
&\qquad \times (\theta_1 \cos L_0 \csc I_0) \\
&\quad + (\theta_1 \sin L_0 \cot I_0)^2[(\theta_2 \cos L_0 - \theta_1\zeta_1 \sin L_0)\csc I_0] \\
&\quad + 2(\theta_1 \sin L_0 \cot I_0)(\theta_2 \sin L_0 \cot I_0 + \theta_1\zeta_1 \cos L_0 \cot I_0 + \tfrac{1}{2}\theta_1^2) \\
&\qquad \times (\theta_1 \cos L_0 \csc I_0) \\
&\quad + (\theta_1 \sin L_0 \cot I_0)^3(\theta_1 \cos L_0 \csc I_0) \\
&= \big\{(\theta_4 + \theta_1^2\theta_2 - \tfrac{1}{2}\theta_2\zeta_1^2 - \theta_1\zeta_1\zeta_2)\cos L_0 \\
&\quad - (\theta_1\zeta_3 + \theta_2\zeta_2 + \theta_3\zeta_1 + \tfrac{1}{3}\theta_1^3\zeta_1 - \tfrac{1}{6}\theta_1\zeta_1^3)\sin L_0 \\
&\quad + \cot I_0[(2\theta_1\theta_3 - 2\theta_1^2\zeta_1^2 + \theta_2^2 + \tfrac{2}{3}\theta_1^4)\sin L_0 \cos L_0 \\
&\qquad + (2\theta_1\theta_2\zeta_1 + \theta_1^2\zeta_2)(\cos^2 L_0 - \sin^2 L_0)] \\
&\quad + \cot^2 I_0[3\theta_1^2\theta_2 \sin^2 L_0 \cos L_0 + \theta_1^3\zeta_1 \sin L_0(2\cos^2 L_0 - \sin^2 L_0)] \\
&\quad + \cot^3 I_0(\theta_1^4 \sin^3 L_0 \cos L_0)\big\}\csc I_0. \qquad (3\text{–}104)
\end{aligned}
$$

This leaves equation (3–12) in the simple form

$$
\begin{aligned}
\Delta &= \tan^{-1} q \qquad (3\text{–}105) \\
&= \tan^{-1}[q_1 T + q_2 T^2 + q_3 T^3 + q_4 T^4 + O(T^5)]. \qquad (3\text{–}106)
\end{aligned}
$$

Once again we employ equations (3–27) through (3–31). This time, however, x_0 is zero, which greatly simplifies the expansion: the q_k here replace the x_k; and the y_k become the Δ_k, which are the desired coefficients directly. Since all terms containing $\sin x_0$ disappear, the result is:

$$\Delta_1 = q_1 \tag{3–107}$$

$$= \theta_1 \cos L_0 \csc I_0; \tag{3–108}$$

$$\Delta_2 = q_2 \tag{3–109}$$

$$= [\theta_2 \cos L_0 - \theta_1\zeta_1 \sin L_0 + \cot I_0(\theta_1^2 \sin L_0 \cos L_0)] \csc I_0; \tag{3–110}$$

$$\Delta_3 = q_3 - \tfrac{1}{3}q_1^3 \tag{3–111}$$

$$\begin{aligned}
&= \big\{(\theta_3 + \tfrac{1}{3}\theta_1^3 - \tfrac{1}{2}\theta_1\zeta_1^2) \cos L_0 - (\theta_1\zeta_2 + \theta_2\zeta_1) \sin L_0 \\
&\qquad + \cot I_0[2\theta_1\theta_2 \sin L_0 \cos L_0 + \theta_1^2\zeta_1(\cos^2 L_0 - \sin^2 L_0)] \\
&\qquad + \cot^2 I_0(\theta_1^3 \sin^2 L_0 \cos L_0)\big\} \csc I_0 \\
&\quad - \tfrac{1}{3}\theta_1^3 \cos^3 L_0 \csc^3 I_0
\end{aligned}$$

$$\begin{aligned}
&= \big\{(\theta_3 - \tfrac{1}{2}\theta_1\zeta_1^2 + \tfrac{1}{3}\theta_1^3 \cos^2 L_0) \cos L_0 - (\theta_1\zeta_2 + \theta_2\zeta_1) \sin L_0 \\
&\qquad + \cot I_0[2\theta_1\theta_2 \sin L_0 \cos L_0 + \theta_1^2\zeta_1(\cos^2 L_0 - \sin^2 L_0)] \\
&\qquad + \cot^2 I_0[\theta_1^3(\sin^2 L_0 \cos L_0 - \tfrac{1}{3}\cos^3 L_0)]\big\} \csc I_0;
\end{aligned} \tag{3–112}$$

$$\Delta_4 = q_4 - q_1^2 q_2 \tag{3–113}$$

$$\begin{aligned}
&= \big\{(\theta_4 + \theta_1^2\theta_2 - \tfrac{1}{2}\theta_2\zeta_1^2 - \theta_1\zeta_1\zeta_2) \cos L_0 \\
&\qquad\quad - (\theta_1\zeta_3 + \theta_2\zeta_2 + \theta_3\zeta_1 + \tfrac{1}{3}\theta_1^3\zeta_1 - \tfrac{1}{6}\theta_1\zeta_1^3) \sin L_0 \\
&\qquad + \cot I_0[(2\theta_1\theta_3 - 2\theta_1^2\zeta_1^2 + \theta_2^2 + \tfrac{2}{3}\theta_1^4) \sin L_0 \cos L_0 \\
&\qquad\quad + (2\theta_1\theta_2\zeta_1 + \theta_1^2\zeta_2)(\cos^2 L_0 - \sin^2 L_0)] \\
&\qquad + \cot^2 I_0[3\theta_1^2\theta_2 \sin^2 L_0 \cos L_0 + \theta_1^3\zeta_1 \sin L_0(2\cos^2 L_0 - \sin^2 L_0)] \\
&\qquad + \cot^3 I_0(\theta_1^4 \sin^3 L_0 \cos L_0)\big\} \csc I_0 \\
&\quad - (\theta_1^2\theta_2 \cos^3 L_0 - \theta_1^3\zeta_1 \cos^2 L_0 \sin L_0 + \theta_1^4 \sin L_0 \cos^3 L_0 \cot I_0) \csc^3 I_0
\end{aligned}$$

$$\begin{aligned}
&= \big\{(\theta_4 - \tfrac{1}{2}\theta_2\zeta_1^2 - \theta_1\zeta_1\zeta_2 + \theta_1^2\theta_2 \sin^2 L_0) \cos L_0 \\
&\qquad - (\theta_1\zeta_3 + \theta_2\zeta_2 + \theta_3\zeta_1 + \tfrac{1}{3}\theta_1^3\zeta_1 - \tfrac{1}{6}\theta_1\zeta_1^3) \sin L_0 + \theta_1^3\zeta_1 \cos^2 L_0 \sin L_0 \\
&\qquad + \cot I_0[(2\theta_1\theta_3 - 2\theta_1^2\zeta_1^2 + \theta_2^2 + \tfrac{2}{3}\theta_1^4) \sin L_0 \cos L_0 \\
&\qquad\quad + (2\theta_1\theta_2\zeta_1 + \theta_1^2\zeta_2)(\cos^2 L_0 - \sin^2 L_0) - \theta_1^4 \sin L_0 \cos^3 L_0] \\
&\qquad + \cot^2 I_0[\theta_1^2\theta_2 \cos L_0(3s^2 L_0 - \cos^2 L_0) \\
&\qquad\quad + \theta_1^3\zeta_1 \sin L_0(3\cos^2 L_0 - \sin^2 L_0)] \\
&\qquad + \cot^3 I_0[\theta_1^4 \sin L_0 \cos L_0(\sin^2 L_0 - \cos^2 L_0)]\big\} \csc I_0.
\end{aligned} \tag{3–114}$$

Equations (3–108), (3–110), (3–112), and (3–114) contain the expressions for Δ_k in the expansion

$$\Delta = \Delta_1 T + \Delta_2 T^2 + \Delta_3 T^3 + \Delta_4 T^4 + O(T^5). \tag{3-86}$$

These expressions, together with the corresponding equations for I_k and L_k that were developed in the previous two subsections, complete the analytical development of the short-term theory of general precession relative to the invariable plane of the Solar System.

3.4. Numerical Results and Verification

The right ascension α_0 and declination δ_0 of the angular momentum vector of the Solar System were found in Chapter 2 to be

$$\alpha_0 = 273°51'09''.262; \tag{2-43}$$

$$\delta_0 = 66°59'28''.003. \tag{2-44}$$

From these angles, the right ascension of the ascending node of the invariable plane on the mean equator of J2000.0 is

$$L_0 = \alpha_0 - 270° = 3°51'09''.262, \tag{2-45}$$

and the inclination of the invariable plane to the mean equator of J2000.0 is

$$I_0 = 90° - \delta_0 = 23°00'31''.997. \tag{2-46}$$

Now in order to obtain numerical values for the coefficients for the polynomials in L, I, and Δ, we also require numerical values for the coefficients of ζ, θ, and z. The IAU currently recommends the values of these quantities which were published by Lieske *et al.* (1977) in their Table 5. However, these coefficients have been rounded, and using their rounded values might produce an error in the last decimal place. To avoid this possibility, I obtained from Lieske (private communication) a computer printout of the run that generated the numbers in their Table 5. The values for ζ_k, θ_k, and z_k (my notation, not

Table 3–1. Full-Precision Values of Current Precessional Constants

ζ_1	$2306''.2181082828$	θ_1	$2004''.3109489144$	z_1	$2306''.2181082828$
ζ_2	$+0''.30187986821595$	θ_2	$-0''.42665200261276$	z_2	$+1''.0946778621317$
ζ_3	$+0''.017997590049701$	θ_3	$-0''.041832591413886$	z_3	$+0''.018202974269150$

Lieske's) from that printout are presented in Table 3–1. These values are given to sixteen digits, representing essentially the full precision of a Univac double-precision number.

The formulas for the coefficients in the polynomial expansions of L, I, and Δ were found in the previous section in terms of the corresponding coefficients of the precession angles ζ, θ, and z and of the initial angles L_0 and I_0. Given the numerical values in Table 3–1 and the above values for L_0 and I_0, the coefficients L_k, I_k, and Δ_k have the values listed in Table 3–2.

Table 3–2. Constants in the Short-Term Theory

L_0	$3°51'09''.262$	I_0	$23°00'31''.997$	Δ_0	$0''.000$
L_1	$-96''.7230$	I_1	$-134''.6685$	Δ_1	$5116''.1809$
L_2	$-1''.94824$	I_2	$0''.49754$	Δ_2	$2''.92466$
L_3	$0''.006539$	I_3	$0''.006173$	Δ_3	$-0''.005636$
L_4	$0''.0000881$	I_4	$-0''.0000188$	Δ_4	$-0''.0000736$

It must be noted here that the numbers in the last row of Table 3–2 were obtained under the assumption that $\zeta_4 = \theta_4 = z_4 = 0$, as the theory developed by Lieske *et al.* (1977) only goes to the third power of time. There is no contribution from these coefficients until the fourth power of time in any event. The values for L_4, I_4, and Δ_4 arise solely from products of coefficients with subscripts 1 to 3. The results in the last row of Table 3–2 can be said to be "deterministic" in that they appear as a consequence of the transformation to the invariable plane; their true values cannot be found without first having values for ζ_4, θ_4, and z_4. Since these are not given by Lieske *et al.*, they cannot appear in a theory that is to duplicate theirs. Nevertheless, the magnitude of these terms gives an indication of the ability of three terms in each angle to produce the same results as classical theory.

While L_0 and I_0 are intended to represent the orientation of the invariable plane in the Q_0 system, there is no such restriction in the equations themselves. Any nonrotating plane will do. In particular, if L_0 is set to zero and I_0 is set to ε_0, the obliquity at epoch, the invariable plane is replaced by the ecliptic of J2000.0. In this instance, the resulting $L(T)$ is none other than the negative of the accumulated planetary precession $\tilde{\chi}_A(T)$ (in Lieske's notation); $I(T)$ is the angle between the mean equator of date and the ecliptic of J2000, denoted $\tilde{\omega}_A(T)$ by Lieske; and $\Delta(T)$ is the accumulated luni-solar precession, or $\tilde{\psi}_A(T)$. When this test case was run, the output coefficients duplicated Lieske's results for these angles to better than a microarcsecond. This increases one's confidence that both the theory itself and the computer program that calculates the coefficients are correct—at least in those terms independent of $\sin L_0$.

A further verification is possible by comparing the angles obtained by evaluating the rigorous formulas for L, I, and Δ in equations (3–10) through (3–12) with the results obtained from evaluating the time polynomials. (Since the "rigorous" equations depend on ζ, θ, and z, their absolute accuracy depends on the accuracy with which these angles themselves are modeled.) If these comparisons are made at regular time intervals on either side of the origin, the differences between the rigorous angles and the polynomial approximations should not show any systematic trend except proportional to the fifth or higher powers of time. This is indeed the case for various random values of L_0 and I_0 in addition to the two cases mentioned above.

For the adopted invariable plane, the differences δL, δI, and $\delta\Delta$ between the rigorous angles and polynomial approximations are presented in Table 3–3. Even after two centuries before or after J2000.0, the largest difference amounts to barely $0\overset{''}{.}0001$. It is apparent from this table that the dominant term in each column is indeed proportional to T^5.

Table 3–3. Comparison of Rigorous Angles and Polynomial Approximations

T	δL	δI	$\delta\Delta$
−2.0	0″.0000078626	−0″.0000034404	0″.0000103361
−1.5	0″.0000018587	−0″.0000008204	0″.0000024600
−1.0	0″.0000002438	−0″.0000001086	0″.0000003249
−0.5	0″.0000000076	−0″.0000000034	0″.0000000102
0.0	0″.0000000000	0″.0000000000	0″.0000000000
0.5	−0″.0000000075	0″.0000000034	−0″.0000000102
1.0	−0″.0000002398	0″.0000001107	−0″.0000003290
1.5	−0″.0000018127	0″.0000008443	−0″.0000025064
2.0	−0″.0000076039	0″.0000035746	−0″.0000105968

3.5. Precession Between Two Arbitrary Times

The short-term theory as it has been presented thus far provides only for precessing from the standard epoch (J2000.0) to another date. In this section there will be developed expressions for precession from one arbitrary time to another.

For the sake of clarity, the matrix **P** from equation (3–4) will now be denoted by

$$\mathbf{P}(0 \rightarrow T) \equiv \mathbf{R}_3[-L(T)]\,\mathbf{R}_1[-I(T)]\,\mathbf{R}_3[-\Delta(T)]\,\mathbf{R}_1(I_0)\,\mathbf{R}_3(L_0). \tag{3–115}$$

In this way the initial and final time arguments of **P** are given explicitly, as is the dependence of L, I, and Δ on the final time T.

The precession matrix from one arbitrary time T_1 to a second time T_2 can be thought of as first precessing from T_1 back to J2000.0, then from J2000.0 to T_2. Therefore

$$\mathbf{P}(T_1 \rightarrow T_2) = \mathbf{P}(0 \rightarrow T_2)\,\mathbf{P}(T_1 \rightarrow 0). \tag{3–116}$$

Because the precession matrix is orthogonal, this is equivalent to

$$\mathbf{P}(T_1 \rightarrow T_2) = \mathbf{P}(0 \rightarrow T_2)\,\mathbf{P}(0 \rightarrow T_1)^{\mathrm{T}}. \tag{3–117}$$

Expanding the right-hand side gives

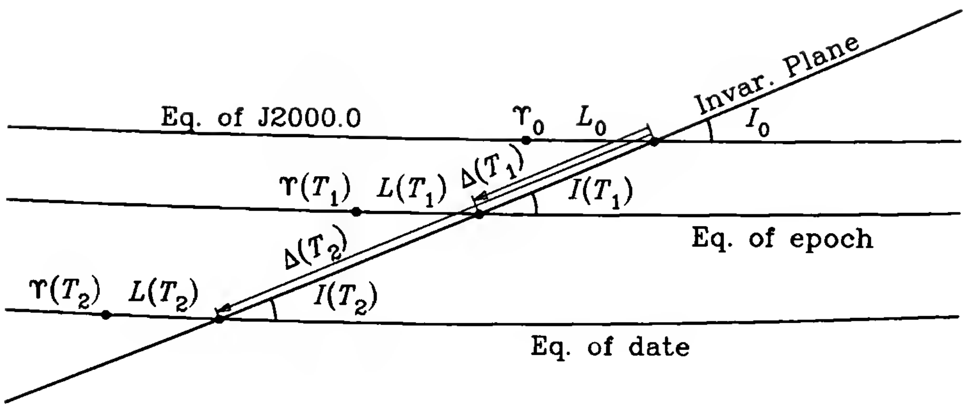

Figure 3–4. Precession Between Two Arbitrary Times

$$
\begin{aligned}
\mathbf{P}(T_1 \to T_2) &= \{\mathbf{R}_3[-L(T_2)]\, \mathbf{R}_1[-I(T_2)]\, \mathbf{R}_3[-\Delta(T_2)]\, \mathbf{R}_1(I_0)\, \mathbf{R}_3(L_0)\} \\
&\quad \times \{\mathbf{R}_3[-L(T_1)]\, \mathbf{R}_1[-I(T_1)]\, \mathbf{R}_3[-\Delta(T_1)]\, \mathbf{R}_1(I_0)\, \mathbf{R}_3(L_0)\}^{\mathrm{T}} \\
&= \{\mathbf{R}_3[-L(T_2)]\, \mathbf{R}_1[-I(T_2)]\, \mathbf{R}_3[-\Delta(T_2)]\, \mathbf{R}_1(I_0)\, \mathbf{R}_3(L_0)\} \\
&\quad \times \{\mathbf{R}_3(-L_0)\, \mathbf{R}_1(-I_0)\, \mathbf{R}_3[\Delta(T_1)]\, \mathbf{R}_1[I(T_1)]\, \mathbf{R}_3[L(T_1)]\} \\
&= \mathbf{R}_3[-L(T_2)]\, \mathbf{R}_1[-I(T_2)]\, \mathbf{R}_3[\Delta(T_1) - \Delta(T_2)]\, \mathbf{R}_1[I(T_1)]\, \mathbf{R}_3[L(T_1)], \qquad (3\text{–}118)
\end{aligned}
$$

which is geometrically evident from Figure 3–4. Equation (3–118) in fact has the same structure as equation (3–4), except that L_0 and I_0 are replaced by their values at T_1, and Δ is replaced by the accumulated angle from T_1 to T_2. If T_1 is zero, equation (3–118) reduces to equation (3–4) exactly. Also, if T_1 and T_2 be exchanged, equation (3–118) shows that resulting matrix $\mathbf{P}(T_2 \to T_1)$ is the transpose of $\mathbf{P}(T_1 \to T_2)$, as in fact it must be.

The most obvious problem with equation (3–118) as it stands is a subtraction of two nearly equal quantities when forming $\Delta(T_2) - \Delta(T_1)$, the negative of which is the argument of the third rotation matrix. Define $t \equiv T_2 - T_1$; then from equation (3–86)

$$\begin{aligned}\Delta(T_2) - \Delta(T_1) &= (\Delta_1 T_2 + \Delta_2 T_2^2 + \Delta_3 T_2^3 + \Delta_4 T_2^4) \\ &\quad - (\Delta_1 T_1 + \Delta_2 T_1^2 + \Delta_3 T_1^3 + \Delta_4 T_1^4) \\ &= [\Delta_1(T_1 + t) + \Delta_2(T_1 + t)^2 + \Delta_3(T_1 + t)^3 + \Delta_4(T_1 + t)^4] \\ &\quad - (\Delta_1 T_1 + \Delta_2 T_1^2 + \Delta_3 T_1^3 + \Delta_4 T_1^4) \\ &= \Delta_1 t + \Delta_2(2T_1 t + t^2) + \Delta_3(3T_1^2 t + 3T_1 t^2 + t^3) \\ &\quad + \Delta_4(4T_1^3 t + 6T_1 t^2 + 4T_1 t^3 + t^4) \\ &= (\Delta_1 + 2\Delta_2 T_1 + 3\Delta_3 T_1^2 + 4\Delta_4 T_1^3)t \\ &\quad + (\Delta_2 + 3\Delta_3 T_1 + 6\Delta_4 T_1^2)t^2 \\ &\quad + (\Delta_3 + 4\Delta_4 T_1)t^3 + \Delta_4 t^4. \end{aligned} \tag{3–119}$$

The last form of equation (3–119) is in the traditional two-argument form used by Lieske *et al.* (1977), with T_1 replacing their T inside the parentheses.

Numerically, with the expansion restricted to the third power of time,

$$\begin{aligned}\Delta(T_2) - \Delta(T_1) = (5116''.1809 + 5''.89432T_1 - 0''.016908T_1^2)t \\ + (2''.92466 - 0''.016908T_1)t^2 - 0''.005636t^3. \end{aligned} \tag{3–120}$$

A more subtle numerical problem that occurs as $t \to 0$ involves near cancellation in the off-diagonal elements of **P**. Clearly, in the limit as $t \to 0$, **P** must approach the identity matrix. However, the individual terms in any one element will not vanish; rather, they will tend to cancel. Certainly the right-hand side of equation (3–118) can be multiplied out to give the individual terms of the **P**; by judicious applications of the identity

$$\cos x = 1 - 2\sin^2(\tfrac{1}{2}x) \tag{3–121}$$

the gross cancellations can be avoided. However, the result is a dramatic increase in the number of terms required to express each element of **P**. These terms are not guaranteed to be all of the same sign, and so the problem of cancellation may not in fact be resolved. The

author believes that a straightforward mechanization of (3–118), using (3–120) to compute $\Delta(T_2) - \Delta(T_1)$, is probably to be preferred.

3.6. Summary and Discussion

This chapter has developed the short-term aspect of precession theory using the invariable plane of the Solar System as a reference plane. The classical precession matrix that rotates from the Earth mean equator and equinox of epoch (T_1 centuries past J2000) to the Earth mean equator and equinox of date ($T_2 \equiv T_1 + t$ centuries past J2000) is the product of five elementary rotations:

$$\mathbf{P} = \mathbf{R}_3[-L(T_2)]\,\mathbf{R}_1[-I(T_2)]\,\mathbf{R}_3[\Delta(T_1) - \Delta(T_2)]\,\mathbf{R}_1[I(T_1)]\,\mathbf{R}_3[L(T_1)], \qquad (3\text{–}118)$$

where the angles on the right-hand side of the equation are computed by:

$$L(T) = 3°51'09''.262 - 92''.7230T - 1''.94824T^2 + 0''.006539T^3; \qquad (3\text{–}122)$$

$$I(T) = 23°00'31''.997 - 134''.6685T + 0''.49754T^2 + 0''.006173T^3; \qquad (3\text{–}123)$$

$$\begin{aligned}\Delta(T_2) - \Delta(T_1) = {} & (5116''.1809 + 5''.89432T_1 - 0''.016908T_1^2)t \\ & + (2''.92466 - 0''.016908T_1)t^2 - 0''.005636t^3. \end{aligned} \qquad (3\text{–}120)$$

This version of the matrix $\mathbf{P}$ agrees with the classical matrix (Lieske 1979) to better than $0''.0001$ for $|T| < 1$ century, with virtually all the difference due to the neglected "deterministic" fourth-order terms.

The accuracy with which either theory tracks the actual precessional motion of the Earth depends on the time span over which the model for the precession angles ζ_A, θ_A, and z_A is valid. Laskar (1986) found a fourth-order term in the accumulated general precession in longitude, p_A, in the amount of $-0''.0000235316T^4$, and a small term in the obliquity of date, $-0''.0000005138T^4$. If these numbers are indicative of the behavior of the other angles, the current polynomials in T^3 will begin to show errors on the order of $0''.001$ after

only 2.5 centuries on either side of J2000.0. This topic will be addressed in more detail in Chapter 5, after the long-term theory will have been presented.

Equation (3–118) for the matrix **P** is unnecessarily complicated, since any rotation matrix can be expressed as the product of three, not five, elementary rotations. The best way to eliminate the extra two rotations would be to refer the origin for the right ascension system not to the vernal equinox, as has been done by European astronomers since the Renaissance, but instead to the intersection of the Earth mean equator and the invariable plane. The precession matrix would then be written as

$$\mathbf{P}' = \mathbf{R}_1[-I(T_2)]\,\mathbf{R}_3[\Delta(T_1) - \Delta(T_2)]\,\mathbf{R}_1[I(T_1)]; \tag{3–124}$$

now only equations (3–120) and (3–123) are required to evaluate the matrix.

This change would cause all right ascensions to be decreased by $L(T)$ and all annual variations in right ascension to be decreased by dL/dT. (There would be no change to the declination system.) In particular, at the standard epoch J2000.0 one would subtract $L_0 = 15^{\mathrm{m}}24\overset{\mathrm{s}}{.}6175$ from all right ascensions, and add $-L_1 = 6\overset{\mathrm{s}}{.}18153$/century to all centennial variations in right ascension.

Such a change would involve a radical redefinition of the standard coordinate systems that have been used in astronomy for centuries. The rôle of the vernal equinox as the origin for right ascensions would be lost; in effect the ecliptic itself would be replaced by the invariable plane. The origin of right ascension would still be defined by the intersection of two planes; however, only one of them (the equator) would be moving. The resulting theory, when put into practice, is simpler, because only two different polynomials are needed, and because only one of those two has two arguments. The classical theory uses three polynomials, all of which have two arguments.

This is not to imply that the concept of the ecliptic has completely outworn its usefulness. Any analytical theory of precession must use the ecliptic. When suitably averaged over time, both the Sun and Moon are found in the ecliptic. The secular torque that causes luni-solar precession will therefore be a function of the location of the ecliptic and of the obliquity.

One of the major objections to using the invariable plane in this fashion will be: "But we can't observe the invariable plane, and we can observe the ecliptic directly!" Both halves of this claim cannot simultaneously be true if identical assumptions are used to evaluate their truth.

Can we observe the invariable plane? There is no flashing beacon in the sky that marks the plane, true, but that is also true of the ecliptic. We *do* observe the apparent locations of the planets; centuries of observations have furnished reliable mean motions and mean orbits. Spacecraft encounters have given us both reliable initial conditions and dramatic improvements in our estimates of the planetary masses. So while we do not observe the invariable plane *per se*, we certainly do have enough information at hand, all gleaned from observation, to enable us to determine its orientation.

Can we actually observe the ecliptic? The Sun itself does *not* lie in the ecliptic. What we observe is the apparent direction to the Sun. This is affected by the light-time correction; by annual, monthly, and diurnal aberration; and by diurnal and monthly parallax. (The monthly effects are caused by the motion of the Earth about the Earth-Moon barycenter.) A glance at an ephemeris of the Sun as expressed in ecliptic coordinates will confirm this: the Sun's geocentric latitude can reach almost one arcsecond in magnitude. All these effects can be removed mathematically if we know the motion of the Earth, and of the observer relative to the geocenter, to the required precision. Even so, the result is simply the inferred direction from the Earth-Moon barycenter to the Sun. A set of such observations

will determine an osculating orbit, to be sure; but this is not the ecliptic, as the ecliptic is defined as the *mean* orbit (whatever that is) of the Earth-Moon barycenter. One must therefore account somehow for planetary perturbations in deriving the orientation and motion of the ecliptic from observations.

Therefore both planes require for their determination the entire system of planetary masses and orbits as well as models for reducing apparent positions to true ones. No object (as a general rule) lies in either plane. The details of the reduction are of course different, but the conclusion is unavoidable: if the ecliptic can be said to be observable, then so is the invariable plane; and if the invariable plane is said to be unobservable, then so is the ecliptic.

The only reason therefore to prefer one to the other is convenience. Is the relative simplicity of the precession theory using the invariable plane outweighed by centuries of observations (not to mention tradition) referred to the vernal equinox? The answer to this question must be given by the astronomical community as a whole.

CHAPTER 4
THE LONG-TERM THEORY

4.1. Overview

As pointed out by Simon Newcomb (1906) and reinforced by Lieske *et al.* (1977), it is not possible to develop rigorous analytic formulas for the precession angles that are valid for all time. The reasoning behind this statement is that the equations of motion of both the ecliptic and the mean equator cannot be integrated analytically in their most general form. Lieske *et al.* took the velocity of the ecliptic pole vector to be parabolic in time; they obtained it by fitting a second-degree polynomial through the values of the perturbation function at three epochs. This provided an approximate expression, as a cubic in time, for the ecliptic pole. The cubic polynomial for the ecliptic pole was next substituted into Newcomb's equation of motion for the mean celestial pole; another analytic integration supplied an approximation for the location of the mean celestial pole; and the locations of both poles yielded the desired precession angles.

Accurate precession angles over long periods of time must, however, be obtained by numerical integration. The orientation of the ecliptic pole—that is, of the vector normal to the mean orbit of the Earth-Moon barycenter—has already been obtained by Laskar (1990) over an interval of $\pm 500,000$ years from the standard epoch J2000.0, as part of a revision of his "Numerical General Theory" of the Solar System (Laskar 1985, 1986, 1988). His results, obtained by integrating some 153,824 terms, are expressed as sets of components at 1000-year intervals, from which the ecliptic pole can be obtained at any time by interpolation.

Given the ecliptic pole and the eccentricity of the Earth's orbit, the angular speed of the celestial pole can be found. This work uses the equations of motion in the form developed by Kinoshita (1975, 1977), a more complete accounting of the torques on the Earth than Newcomb's model. Kinoshita's result for the speed of luni-solar precession is then used to integrate the motion of the celestial pole. The equations are close enough to linear that a simple fourth-order Runge-Kutta integrator (Abramowitz and Stegun 1964, Section 25.5.10), with a constant step size, sufficed to give numerical accuracies on the order of a nanoarcsecond after 500,000 years. Coordinates of both poles were written to a file at regular intervals.

Given the poles of the ecliptic and equator, it is a straightforward matter to obtain the desired precession angles. These are found, not by the historical developments of spherical trigonometry, but by the simpler methods of vector algebra. All of the classical precession angles, as well as the angles L, I, and Δ using the invariable plane, were determined at each output point.

Since the resulting output file is quite large (one output point per century for a million years), the results were condensed by fitting Chebyshev polynomials through the values for each angle, in a manner similar to that employed in the production of JPL planetary ephemerides (Newhall 1989). These Chebyshev coefficients form the final result of the long-term theory.

The individual sections of this chapter describe the various facets of this procedure in detail: Laskar's theory for the ecliptic; the development of the equations of motion of the celestial pole, including Kinoshita's equation for the speed of luni-solar precession; their subsequent integration; the equations for the precession angles; and the Chebyshev fitting process. A final section estimates the magnitude of the integration errors, which are shown to be far less important than the uncertainty in the initial conditions.

4.2. The Motion of the Ecliptic

The problem of finding the orientation of the ecliptic, and indeed of the orbits of all the planets, is an old one in the field of celestial mechanics. The work of Jacques Laskar builds on the strong French tradition started by Laplace, Le Verrier, and Lagrange and continuing today with Duriez (1977, 1979) and Bretagnon (1982).

Laskar's "Numerical General Theory" (NGT) uses a combination of analytical and numerical methods to evaluate the mean orbital elements of the first eight planets (Laskar 1985). First the disturbing function was developed to second order in the planetary masses and fifth degree in eccentricity and inclination. This process yielded some 153,824 monomial terms in the differential equations for the eccentricity and inclination of the planetary orbits. These equations were then integrated numerically. The initial orbital elements in Laskar's work were based on Bretagnon's (1982) VSOP theory; the planetary masses are those currently recommended by the International Astronomical Union (1976).

A subsequent paper by Laskar (1988) isolated the most prominent frequencies in the eccentricity and inclination variables for the planets. In this technique, Laskar took a Fast Fourier Transform of each component under study, found the frequency with the strongest amplitude, subtracted that term from the original, and repeated the process to find the next most significant term. His work stopped after 80 terms for the inner planets and 50 for the outer planets. Such a formulation would have been very useful for this work, but for the fact that Laskar's Fourier representation does not preserve the initial conditions.

I therefore requested from Laskar a file of the eccentricity and inclination variables themselves. He very graciously sent me such a file, not from the original NGT but from a more recent (and as yet unpublished) integration. This file spans one million years (500,000 years on either side of J2000.0), at 1000-year intervals; thus it consists of 1001 records. Each record contains the quantities

$$h = e \sin \varpi, \tag{4-1}$$

$$k = e \cos \varpi, \tag{4-2}$$

$$p = \sin \tfrac{1}{2}\pi_A \sin \Pi_A, \tag{4-3}$$

$$q = \sin \tfrac{1}{2}\pi_A \cos \Pi_A, \tag{4-4}$$

where e is the eccentricity, ϖ the longitude of perihelion, $|\pi_A|$ the inclination, and Π_A the longitude of the ascending node (of the descending node, if $T < 0$) of the mean orbit of the Earth-Moon barycenter, all reckoned with respect to the E_0 system. Therefore p and q are identically zero at J2000.0 ($T = 0$).

The standard orbital elements may be retrieved from these by

$$e = \sqrt{h^2 + k^2}; \tag{4-5}$$

$$\varpi = \operatorname{plg}(h, k); \tag{4-6}$$

$$\pi_A = 2 \operatorname{sgn} T \sin^{-1} \sqrt{p^2 + q^2}; \tag{4-7}$$

$$\Pi_A = \operatorname{plg}(p \operatorname{sgn} T, q \operatorname{sgn} T). \tag{4-8}$$

In equations (4–6) and (4–8), the notation follows Eichhorn (1987/88), as the quadrant of ϖ and of Π_A is sensitive to the signs of the numerator and denominator; this is the ATAN2 function of Fortran. The factors $\operatorname{sgn} T$ in equations (4–7) and (4–8) provide continuity for π_A and Π_A as T passes through zero, as p and q change signs then. (Otherwise the derivative of π_A would change sign instantaneously, and Π_A itself would suffer a discontinuity of 180°.) Equation (4–8) cannot be evaluated at $T = 0$; then Π_A assumes its limiting value.

The equations of motion will turn out to be evaluated most easily if the unit vector directed to the ecliptic pole is expressed in terms of its rectangular coordinates. Relative to the ecliptic and mean equinox of J2000.0, the ecliptic pole $\mathbf{E}$ is given by

$$\mathbf{E} = \begin{pmatrix} \sin \pi_A \sin \Pi_A \\ -\sin \pi_A \cos \Pi_A \\ \cos \pi_A \end{pmatrix} = \begin{pmatrix} 2p\sqrt{1 - p^2 - q^2} \\ -2q\sqrt{1 - p^2 - q^2} \\ 1 - 2(p^2 + q^2) \end{pmatrix} \equiv \begin{pmatrix} s \\ -c \\ \sqrt{1 - s^2 - c^2} \end{pmatrix}. \tag{4-9}$$

In order to save computer time during the integrations, the p and q values for each record were transformed at the start into s and c. These were then stored along with h and k in 1001-element arrays, which were interpolated as needed. The interpolating polynomial was selected to be the unique eleventh-order polynomial passing through twelve consecutive points bracketing the interpolation time. (To improve the numerical stability of the process, the Chebyshev representation was chosen in lieu of the simple polynomial representation.) Except at the very beginning or end of the file, six points out of the twelve preceded the interpolation time and six followed it. This scheme guarantees that the interpolated result will be the tabular value at each point in the table; the discontinuity between régimes enters only at the first derivative, and this should be small due to the high degree of the fit. Subroutines DPFIT and DCPVAL, of JPL's MATH77 library (JPL Applied Mathematics Group 1987), performed the polynomial construction and evaluation, respectively.

4.3. The Equations of Motion for the Celestial Pole

The development of the equations of motion for the mean celestial pole (the vector normal to the mean equator of date) proceeds in several logical steps. One begins with the equation of motion in vector form. From this relatively simple equation the derivatives of the components of the pole vector are found, since it is the components that are actually integrated. Finally Kinoshita's expression for the rate of luni-solar precession is adopted, and the appropriate constants of his theory are found.

4.3.1. The Vector Equation of Motion

The principal source of the motion of the celestial pole is a torque produced by the gravitational attraction of the Sun and Moon on the oblate Earth. The periodic part of the torque (that is, the part whose periods are less than 18.6 years) gives rise to nutation, while the time-averaged part produces luni-solar precession. The character of luni-solar

precession may be obtained by considering the Sun and Moon to be smeared into a ring around the ecliptic, and the oblateness of the Earth likewise to be concentrated into a ring around the equator. These two rings have a mutual inclination equal to the obliquity of the ecliptic, ε. The resulting torque on the equatorial ring is in the direction of the line of intersection of the two planes (the vernal equinox), and its magnitude is proportional to $\sin 2\varepsilon$. A detailed derivation is given in chapter 8 of Woolard and Clemence (1966) and need not be repeated here.

The second contribution to the motion of the celestial pole is the so-called "geodesic precession." Although de Sitter (1916) first showed that general relativity predicts that spinning bodies in a gravitational field will undergo a forced precession, only two decades later did de Sitter and Brouwer (1938) include its effect in luni-solar precession. The effect of geodesic precession is also a torque in the direction of the vernal equinox, but its magnitude is proportional to $\sin\varepsilon$ instead of $\sin 2\varepsilon$.

Since $\sin 2\varepsilon = 2\sin\varepsilon\cos\varepsilon$, the magnitudes of the two torques may be combined into an expression of the form

$$\left|\frac{d\mathbf{Q}}{dT}\right| = (P\cos\varepsilon - p_g)\sin\varepsilon, \tag{4–10}$$

where $\mathbf{Q}$ is the celestial pole vector, a unit vector; P is Newcomb's "Precessional Constant" (Newcomb 1906, p. 228); and p_g is the rate of geodesic precession. Neither P nor p_g is quite constant, as both depend slightly on the eccentricity of the Earth's orbit, which varies over time.

Now the dot product of two unit vectors gives the cosine of the angle between them, and the magnitude of the cross product of two unit vectors is the (positive) sine of the angle between them. The factors $\cos\varepsilon$ and $\sin\varepsilon$ in equation (4–10) can therefore be replaced by $\mathbf{Q}\cdot\mathbf{E}$ and $|\mathbf{Q}\times\mathbf{E}|$, where $\mathbf{E}$ is the unit vector in the direction toward the north ecliptic pole. But since the torque is already directed toward the vernal equinox, *i.e.*, toward $\mathbf{Q}\times\mathbf{E}$, we

can use the cross product itself rather than merely its magnitude. Thus equation (4–10) is expressed in vector notation by

$$\frac{d\mathbf{Q}}{dT} = [P(\mathbf{Q}\cdot\mathbf{E}) - p_g]\mathbf{Q}\times\mathbf{E}. \tag{4–11}$$

This form of the equation was given by Fabri (1980).

Further simplification is possible. From equation (22) of Lieske *et al.* (1977), the value of P at J2000.0, denoted P_0, can be expressed as

$$P_0 = (p_1 + p_g)\sec\varepsilon_0 + \chi_1, \tag{4–12}$$

where p_1 is the speed of general precession in longitude (a fundamental constant), χ_1 is the rate of planetary precession, and ε_0 is the obliquity, all at J2000.0. Further, since the rate of luni-solar precession at J2000.0, ψ_1, is also given by the same set of equations as

$$\psi_1 = p_1 + \chi_1\cos\varepsilon_0, \tag{4–13}$$

the quantity in brackets in equation (4–11) can be expressed as

$$\begin{aligned} P\cos\varepsilon - p_g &= [(p + p_g)\sec\varepsilon + \chi]\cos\varepsilon - p_g \\ &= p + \chi\cos\varepsilon \\ &= \psi. \end{aligned} \tag{4–14}$$

This equation applies for all times T, not merely at the initial epoch; therefore the subscripts denoting the J2000 quantities have been dropped. The equation of motion for the celestial pole thus becomes

$$\frac{d\mathbf{Q}}{dt} = \psi(T)\mathbf{Q}\times\mathbf{E}; \tag{4–15}$$

this equation shows explicitly the dependence on T of the rate of luni-solar precession, measured relative to the ecliptic of date.

4.3.2. The Equation of Motion in Component Form

Equation (4–15) is a vector equation and thus valid in any inertial coordinate system. The choice of coordinate systems is therefore arbitrary, as is the choice of representations of the components (rectangular or spherical) of the vector **Q**. Accordingly, the coordinate system that makes the numerical integration the most stable and permits the easiest computation of the derivatives is the proper one to choose.

Laskar's results for the motion of the ecliptic were expressed relative to the inertial E_0 system. His variables p and q can be transformed easily into s and c by equation (4–9), giving the rectangular components of **E** referred to this same coordinate system. For this reason alone the E_0 system would be a good choice for the orientation of the coordinate system in which the integration is performed.

The choice between rectangular and spherical coordinates for representing **Q** also reveals that a coordinate system based on the ecliptic is to be preferred. Since the derivative of **Q** is perpendicular to **Q** itself, the magnitude of **Q** will not change. If spherical coordinates (radius r, colatitude angle ω, longitude angle ϕ; Figure 4–1) are adopted for **Q**, r is therefore constant, and the number of components to be integrated is only two: ω and ϕ.

Finally, if **E** were stationary, and if ψ were constant, equation (4–15) shows that **Q** would describe a right circular cone whose axis is parallel to **E**; both the angular rate of **Q** and the obliquity would be constant. In this "zeroth-order" approximation, ω would be constant, and ϕ would be a linear function of time. For these reasons, the integration is best carried out using spherical polar coordinates for **Q**, referred to the E_0 system.

The derivatives of ω and ϕ are therefore required. Denote the rectangular components of **Q** by $(Q_x, Q_y, Q_z)^{\mathrm{T}}$, and similarly for **E**. Then equation (4–15) becomes, in rectangular coordinates,

$$\begin{pmatrix} \dot{Q}_x \\ \dot{Q}_y \\ \dot{Q}_z \end{pmatrix} = \psi \begin{pmatrix} Q_y E_z - Q_z E_y \\ Q_z E_x - Q_x E_z \\ Q_x E_y - Q_y E_x \end{pmatrix} . \tag{4–16}$$

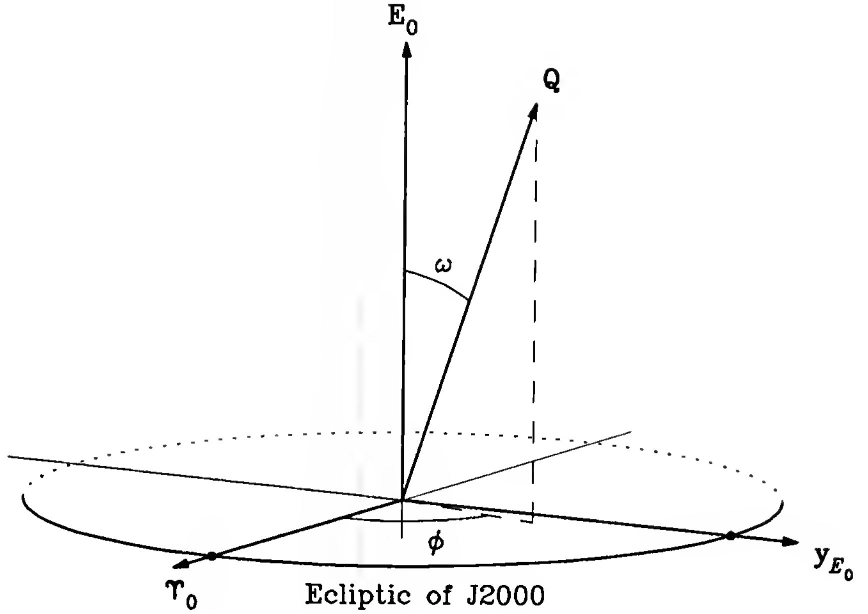

Figure 4–1. Spherical Coordinates for $\mathbf{Q}$ in the E_0 System

Now

$$\omega = \cos^{-1} Q_z. \tag{4–17}$$

Differentiating this gives

$$\begin{aligned}
\dot{\omega} &= -\frac{1}{\sqrt{1 - Q_z^2}} \dot{Q}_z. \\
&= \frac{\psi}{\sin \omega}(Q_y E_x - Q_x E_y). && (4\text{–}18) \\
&= \frac{\psi}{\sin \omega}[(\sin \omega \sin \phi)(\sin \pi_A \sin \Pi_A) - (\sin \omega \cos \phi)(-\sin \pi_A \cos \Pi_A)] \\
&= \psi \sin \pi_A \cos(\Pi_A - \phi). && (4\text{–}19)
\end{aligned}$$

Similarly, since

$$\phi = \operatorname{plg}(Q_y, Q_x), \tag{4–20}$$

the derivative is

$$\dot{\phi} = \frac{Q_x \dot{Q}_y - Q_y \dot{Q}_x}{Q_x^2 + Q_y^2}$$

$$= \frac{\psi}{\sin^2\omega}[Q_x(Q_z E_x - Q_x E_z) - Q_y(Q_y E_z - Q_z E_y)]$$

$$= \psi \left[\frac{Q_z(Q_x E_x + Q_y E_y)}{\sin^2\omega} - E_z \right] \tag{4–21}$$

$$= \psi \left[\frac{\cos\omega[(\sin\omega\cos\phi)(\sin\pi_A \sin\Pi_A) + (\sin\omega\sin\phi)(-\sin\pi_A\cos\Pi_A)]}{\sin^2\omega} - \cos\pi_A \right]$$

$$= \psi[\cot\omega \sin\pi_A \sin(\Pi_A - \phi) - \cos\pi_A]. \tag{4–22}$$

Clearly the colatitude angle ω is just the separation between the celestial pole of date and the ecliptic pole of J2000, or equivalently the inclination of the equator of date to the ecliptic of J2000. Similarly, the longitude angle ϕ is 90° ahead of the longitude of the descending node of the equator of date on the ecliptic of J2000; this node is the point marked by $\overline{\Upsilon}_1$ in Figure 1 of Lieske *et al.* (1977). The angles ω and ϕ are related to angles in that paper by

$$\omega = \tilde{\omega}_A \tag{4–23}$$

and

$$\phi = 90° - \tilde{\psi}_A. \tag{4–24}$$

One aspect of equation (4–22) deserves further comment. At $T = 0$, when $\pi_A = 0$, $\dot{\phi} = -\psi$, intuitively consistent with equation (4–24). But when $\pi_A \neq 0$, this simple relation does not hold. The explanation is that ψ actually represents the instantaneous rate of luni-solar precession relative to the ecliptic of date; it is not equal to $d\tilde{\psi}_A/dT$, which is measured along the ecliptic of J2000.

4.3.3. Kinoshita's Expression for Luni-solar Precession

As we have seen above, the rate of luni-solar precession ψ can be expressed, following Lieske *et al.* (1977), as

$$\psi = P\cos\varepsilon - p_g. \tag{4–14}$$

Over the short term, it is sufficient to model P as $P_0 + P_1 T$ (Lieske *et al.* use $P_1 = -0\overset{''}{.}00369$ per century), and p_g can be regarded as constant since it is a relatively small quantity ($1\overset{''}{.}92$ per century). For an integration extending over a million years, more precise expressions of these quantities are needed.

A better formulation for $P\cos\varepsilon$ was given by Kinoshita (1975, 1977). Kinoshita's original version (equation (10.1) of his 1977 paper) has a sign error; the correct version (equation (24) of Laskar 1986) is

$$\begin{aligned} R(\varepsilon) = \frac{3k^2 m_☾}{a_☾^3 \Omega}\frac{2C-A-B}{2C}\bigg[&(M_0 - \tfrac{1}{2}M_2)\cos\varepsilon + M_1\frac{\cos 2\varepsilon}{\sin\varepsilon} \\ &- M_3 \frac{m_☾}{m_\oplus + m_☾}\frac{n_☾^2}{\Omega n_☊}\frac{2C-A-B}{2C}(6\cos^2\varepsilon - 1)\bigg] \\ &+ \frac{3k^2 m_\odot}{a_\oplus^3\Omega}\frac{2C-A-B}{2C}[(S_0 - \tfrac{1}{2}S_2)\cos\varepsilon]. \end{aligned} \tag{4–25}$$

Here, k is the Gaussian gravitational constant; the m's are masses, in units of the solar mass; the a's are the mean orbital semimajor axes; Ω is the Earth's rotation rate; A, B, and C are the Earth's principal moments of inertia (with C being the polar moment); $n_☾$ is the Moon's mean motion; $n_☊$ is the Moon's nodal rate, a negative quantity; and the M_i and S_i are numerical factors determined by Kinoshita. The resulting rate R is primarily a function of the obliquity, although the Earth's eccentricity enters into S_0.

If M_1 and M_3 be neglected, equation (4–25) takes the form

$$R \propto \cos\varepsilon \tag{4–26}$$

where the constant of proportionality contains all the astronomical and geophysical constants above. A comparison of this with equation (4–14) shows clearly that Kinoshita's rate is not $\dot{\psi}$ itself but rather $P\cos\varepsilon$, since p_g is not present in his equation. Thus, from equation (4–12),

$$R\Big|_{T=0} = p_1 + p_g + \chi_1 \cos\varepsilon_0. \tag{4–27}$$

Of all the constants appearing in equation (4–25), only the ratio of the moments of inertia, $H \equiv (2C - A - B)/2C$, is not known. Equations (4–25) and (4–27) thus combine to yield a quadratic equation for H:

$$aH^2 + bH + c = 0 \tag{4–28}$$

with

$$a = \frac{3k^2 m_☾}{a_☾^3 \Omega} M_3 \frac{m_☾}{m_\oplus + m_☾} \frac{n_☾^2}{\Omega n_☊} (6\cos^2\varepsilon_0 - 1) \tag{4–29}$$

$$\begin{aligned} b = &- \frac{3k^2 m_☾}{a_☾^3 \Omega} \left[(M_0 - \tfrac{1}{2}M_2)\cos\varepsilon_0 + M_1 \frac{\cos 2\varepsilon_0}{\sin\varepsilon_0} \right] \\ &- \frac{3k^2 m_\odot}{a_\oplus^3 \Omega} [(S_0 - \tfrac{1}{2}S_2)\cos\varepsilon_0] \end{aligned} \tag{4–30}$$

$$c = p_1 + p_g + \chi_1 \cos\varepsilon_0 \tag{4–31}$$

The solution of equation (4–28) is the usual

$$H = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}. \tag{4–32}$$

The physically real solution demands the negative sign before the square root. Since b is both dominant and negative, equation (4–32) as it stands contains a near cancellation in the numerator. The equation is instead reliably evaluated (Press *et al.* 1986, Section 5.5) by

$$H = \frac{2c}{\sqrt{b^2 - 4ac} - b}. \tag{4-33}$$

It still remains to express p_g and S_0 in terms of the eccentricity of the Earth's orbit. Barker and O'Connell (1970) give

$$p_g = \frac{3\mu_\odot n_\oplus}{2c^2 a_\oplus (1 - e_\oplus^2)}, \tag{4-34}$$

where $\mu_\odot$ is G times the Sun's mass, $n_\oplus$ is the Earth's mean motion, c is the speed of light, and $a_\oplus$ and $e_\oplus$ are the semimajor axis and eccentricity of the Earth's orbit.

As for S_0, equation (28) of Laskar (1986) reads:

$$S_0 = \tfrac{1}{2}(1 - e_\oplus^2)^{-3/2} - 0.422 \times 10^{-6}. \tag{4-35}$$

The constant term was added by Laskar to reconcile the first term on the right-hand side with Kinoshita's value $S_0 = 0.5002101$ at $T = 0$; it accounts for the departure of the Earth's orbit from Keplerian motion.

The rate of planetary precession at the J2000 epoch, χ_1, is (from equation (22) of Lieske *et al.* 1977)

$$\chi_1 = \left.\frac{ds}{dT}\right|_{T=0} \csc\varepsilon_0. \tag{4-36}$$

The derivative was found by fitting a tenth-order polynomial through Laskar's central eleven points for s, in a manner similar to that described in the previous section, and then differentiating the polynomial and evaluating it at $T = 0$.

Now the value of H can be found. Equations (4–35) and (4–36) give p_g at epoch J2000.0 and χ_1, which appear in the right-hand side of equation (4–31). The remaining quantities in equations (4–29) through (4–31) are known; in particular, the obliquity at epoch and the speed of general precession in longitude, ε_0 and p respectively, are set to

their currently-accepted values. With a, b, and c evaluated, equation (4–33) can be solved for H.

Numerical values of the various constants appearing within this section are collected, with their sources, in Table 4–1. These values yield

$$\left.\frac{ds}{dT}\right|_{T=0} = 4''\!.2010367/\text{century}; \tag{4–37}$$

$$\chi_1 = 10''\!.5612820/\text{century}; \tag{4–38}$$

$$\left.p_g\right|_{T=0} = 1''\!.9193475/\text{century}; \tag{4–39}$$

$$H = 0.0032800411. \tag{4–40}$$

The constants $k_☾$ and $k_\odot$ in Kinoshita (1977) are then

$$k_☾ \equiv \frac{3k^2}{\Omega}\frac{m_☾}{a_☾^3}H = 7561''\!.2323660/\text{century}; \tag{4–41}$$

$$k_\odot \equiv \frac{3k^2}{\Omega}\frac{m_\odot}{a_\oplus^3}H = 3481''\!.8511638/\text{century}; \tag{4–42}$$

These last two values are somewhat different from Kinoshita's results, which are 7567″.72161 and 3475″.39770 per century, respectively, probably due to his use of older values for the various masses. Their sum agrees quite well; this indicates a slight difference in dividing the cause of luni-solar precession between the Sun and the Moon.

4.4. The Numerical Integration of the Motion of the Celestial Pole

The numerical integration of **Q** is straightforward. The integration itself involves but two dependent variables, ω and ϕ; the two differential equations, (4–19) and (4–22), are of the first order. Furthermore, $\dot{\omega}$ is fairly small, since one of the factors is $\sin \pi_A$ and π_A itself is not large; and $\dot{\phi}$ is nearly constant, since the leading term in $\dot{\phi}$ is $\cos \pi_A$.

Table 4–1. Astronomical Constants for Kinoshita's Luni-solar Precession Model

Quantity	Value	Source
k	0.01720209895 $AU^{3/2}$/day	IAU (1976)
AU	149597870.66 km	IAU (1976)
$m_\odot$	1	—
$m_\oplus + m_☾$	1/328900.5	IAU (1976)
$m_\oplus$	1/332946.0	IAU (1976)
$a_\oplus$	1.000001015726665 AU	Bretagnon (1982)
$a_☾$	384747.980645 km	Chapront-Touzé and Chapront (1983)
$n_\oplus$	628.30662287852 rad/century	Bretagnon (1982)
$n_☾$	1732559343.″18/century	Chapront-Touzé and Chapront (1983)
$n_☊$	−6967919.″36222/century	Chapront-Touzé and Chapront (1983)
Ω	15.″04106717866910/sec	Aoki *et al.* (1982)
M_0	0.4963033	Kinoshita (1977)
M_1	−0.0000207	Kinoshita (1977)
M_2	−0.0000001	Kinoshita (1977)
M_3	0.0030202	Kinoshita (1977)
S_2	< 0.0000001	Kinoshita (1977)
ε_0	23°26′21.″448	IAU (1976)
p_1	5029.″0966/century	IAU (1976)

Fancy integration techniques are not necessary to integrate so simple a set of equations. Rather, since output is desired at equally spaced time intervals, a fixed-step integrator is indicated. Of these, the fourth-order Runge-Kutta method (Abramowitz and Stegun 1964, Section 25.5.10) was selected as it combines reliable results with ease of programming.

The derivatives $\dot{\omega}$ and $\dot{\phi}$ are obtained in several steps. First Laskar's tables provide by interpolation the variables h and k, which yield the eccentricity of the Earth's orbit by equation (4–5). This value of $e_\oplus$ was substituted into equations (4–34) and (4–35) for the rate of geodesic precession and the value of Kinoshita's coefficient S_0. Laskar's tables were again interpolated, this time for c and s. These yielded both rectangular and spherical coordinates for the vector $\mathbf{E}$ by equation (4–9). The dot product of $\mathbf{E}$ and $\mathbf{Q}$, which is $\cos\varepsilon$, was then found, and $\sin\varepsilon$ and $\sin 2\varepsilon$ were calculated from the cosine. Equation (4–25) gave Kinoshita's rate R, and equation (4–14) yielded ψ. Finally, $\dot{\omega}$ and $\dot{\phi}$ were found from equations (4–19) and (4–22). This process was performed four times per integration step.

The initial conditions are straightforward: at $T = 0$,

$$\omega_0 = \varepsilon_0, \tag{4–43}$$

$$\phi_0 = 90^\circ. \tag{4–44}$$

The integration extended forward from $T = 0$ to $T = 5000$ centuries, then backward from $T = 0$ to $T = -5000$ centuries.

The step size was determined by examining trial 200-year integrations in which the ecliptic pole and rate of luni-solar precession were those of Lieske *et al.* (1977). The precession angles inferred from the integrated celestial pole were examined as the step size was varied from one century to one year; even with a one-century step size, the integration error after two steps was at the μarcsecond level. A step size of $\frac{1}{8}$ century was chosen to be safe.

4.5. The Determination of the Precession Angles

The output from the numerical integrator, consisting of the coordinates of the poles $\mathbf{Q}$ and $\mathbf{E}$ referred to the mean ecliptic and equinox of J2000, provides sufficient information for the calculation of both the full set of classical precession angles and the angles I, L, and Δ involving the invariable plane. These angles can all be expressed as the latitude or longitude angles of specific vectors with respect to coordinate systems that can be immediately realized from $\mathbf{Q}$ and $\mathbf{E}$.

The obliquity of the ecliptic—the angle between the ecliptic of date and the equator of date—is given by either of

$$\varepsilon = \sin^{-1} |\mathbf{Q} \times \mathbf{E}| = \cos^{-1}(\mathbf{Q} \cdot \mathbf{E}). \tag{4–45}$$

The formulation in terms of the cross product and arcsine is more stable numerically since $|\varepsilon| < 45^\circ$.

The accumulated precession angles ζ_A, θ_A, and z_A are found by decomposing the standard rotation matrix $\mathbf{P}$ which transforms from the Q_0 system to the Q system. (All these angles, being measured from the standard epoch, are given tildes in Lieske *et al.* (1977); for clarity the tildes are omitted here.) In order to obtain $\mathbf{P}$, it is first necessary to calculate the positions of the coordinate axes of the Q system. The z-axis of this system is already at hand; it is $\mathbf{Q}$. The x-axis is the vernal equinox of date, Υ, given by

$$\Upsilon = (\mathbf{Q} \times \mathbf{E}) / \sin \varepsilon. \tag{4–46}$$

The y-axis completes the triad:

$$\mathbf{y}_Q = \mathbf{Q} \times \Upsilon. \tag{4–47}$$

Since the components of three vectors are still specified in the E_0 system, the matrix $\mathbf{A}$ formed by

$$\mathbf{A} = \begin{pmatrix} \Upsilon^{\mathrm{T}} \\ \mathbf{y}_Q^{\mathrm{T}} \\ \mathbf{Q}^{\mathrm{T}} \end{pmatrix} \tag{4–48}$$

will transform from E_0 into Q. The matrix $\mathbf{P}$ follows from this by

$$\mathbf{P} = \mathbf{A}\,\mathbf{R}_1(\varepsilon_0) \tag{4–49}$$

because $\mathbf{R}_1(\varepsilon_0)$ rotates from Q_0 into E_0.

Given $\mathbf{P}$, it is a simple matter to reconstruct the accumulated precession angles ζ_A, θ_A, and z_A. By definition,

$$\mathbf{P} = \mathbf{R}_3(-z_A)\,\mathbf{R}_2(\theta_A)\,\mathbf{R}_3(-\zeta_A). \tag{2–1}$$

Writing out the product of the three elementary rotation matrices gives

$$\mathbf{P} = \begin{pmatrix} \cos z_A \cos\theta_A \cos\zeta_A \\ -\sin z_A \sin\zeta_A & -\cos z_A \cos\theta_A \sin\zeta_A \\ -\sin z_A \cos\zeta_A & -\cos z_A \sin\theta_A \\ \sin z_A \cos\theta_A \cos\zeta_A \\ +\cos z_A \sin\zeta_A & -\sin z_A \cos\theta_A \sin\zeta_A \\ +\cos z_A \cos\zeta_A & -\sin z_A \sin\theta_A \\ \sin\theta_A \cos\zeta_A & -\sin\theta_A \sin\zeta_A & \cos\theta_A \end{pmatrix}. \tag{4-50}$$

Immediately θ_A is given by

$$\theta_A = \cos^{-1} P_{33}. \tag{4-51}$$

Since $\cos\theta_A = \cos(-\theta_A)$, there is a sign ambiguity. This ambiguity extends to the other two angles as well, since the product

$$\mathbf{R}_3(180° - z_A)\,\mathbf{R}_2(-\theta_A)\,\mathbf{R}_3(180° - \zeta_A)$$

yields exactly the same matrix as the right-hand side of equation (4–50). The ambiguity is resolved by the convention that θ_A (like π_A above) must have the same sign as T. This convention is followed implicitly in the current precession theories, since the polynomial for θ_A gives negative values for $T < 0$ and positive values for $T > 0$. Therefore

$$\theta_A = \operatorname{sgn} T \, \cos^{-1} P_{33}. \tag{4-52}$$

The principal value of the arccosine function is to be used here. The angles ζ_A and z_A follow from

$$\zeta_A = \begin{cases} \operatorname{plg}(+P_{32},\, -P_{31}), & \text{if } T < 0; \\ 0, & \text{if } T = 0; \\ \operatorname{plg}(-P_{32},\, +P_{31}), & \text{if } T > 0; \end{cases} \tag{4-53}$$

$$z_A = \begin{cases} \operatorname{plg}(+P_{23},\, +P_{13}), & \text{if } T < 0; \\ 0, & \text{if } T = 0; \\ \operatorname{plg}(-P_{23},\, -P_{13}), & \text{if } T > 0. \end{cases} \tag{4-54}$$

Multiples of 360° are to be added as necessary: as with θ_A, the angles ζ_A and z_A are negative by convention if $T < 0$.

An alternative way of constructing the matrix $\mathbf{P}$ is to express it as

$$\mathbf{P} = \mathbf{R}_3(\chi_A)\,\mathbf{R}_1(-\omega_A)\,\mathbf{R}_3(-\psi_A)\,\mathbf{R}_1(\varepsilon_0), \tag{4–55}$$

where χ_A is the accumulated planetary precession from J2000 to date, ω_A is the inclination of the equator of date to the ecliptic of J2000, and ψ_A is the accumulated luni-solar precession from J2000 to date. A comparison of equations (4–55) and (4–49) reveals that the product of the three leftmost rotation matrices on the right-hand side of equation (4–55) is simply the matrix $\mathbf{A}$ defined by equation (4–48). Multiplying these out yields

$$\mathbf{A} = \mathbf{R}_3(\chi_A)\,\mathbf{R}_1(-\omega_A)\,\mathbf{R}_3(-\psi_A) \tag{4–56}$$

$$= \begin{pmatrix} \begin{array}{c}\cos\chi_A\cos\psi_A \\ +\sin\chi_A\cos\omega_A\sin\psi_A\end{array} & \begin{array}{c}-\cos\chi_A\sin\psi_A \\ +\sin\chi_A\cos\omega_A\cos\psi_A\end{array} & -\sin\chi_A\sin\omega_A \\ \begin{array}{c}-\sin\chi_A\cos\psi_A \\ +\cos\chi_A\cos\omega_A\sin\psi_A\end{array} & \begin{array}{c}\sin\chi_A\sin\psi_A \\ +\cos\chi_A\cos\omega_A\cos\psi_A\end{array} & -\cos\chi_A\sin\omega_A \\ \sin\omega_A\sin\psi_A & \sin\omega_A\cos\psi_A & \cos\omega_A \end{pmatrix}. \tag{4–57}$$

Since ω_A is always positive, the accumulated planetary precession χ_A is given unambiguously by

$$\chi_A = \mathrm{plg}(-A_{13},\,-A_{23}). \tag{4–58}$$

In practice M_{23} is always negative so that χ_A always falls in the first or fourth quadrant. The angles ω_A and ψ_A can also be determined easily from equation (4–57), but ω_A is one of the variables being integrated and ψ_A is easily found from ϕ via equation (4–24).

The last classical precession angle is the accumulated general precession in longitude, p_A. Following the development of Andoyer (1911) and Lieske *et al.* (1977), write

$$p_A \equiv \Pi_A - \Lambda_A, \tag{4–59}$$

where Λ_A is measured along the ecliptic of date from the vernal equinox of date to the intersection of the ecliptics of date and of epoch; Π_A is measured along the ecliptic of epoch from the vernal equinox of epoch to the same intersection. This intersection, denoted by $\mathbf{N}$, lies in the direction (expressed in E_0 coordinates)

$$\mathbf{N} = \begin{cases} (-c, -s, 0)^{\mathrm{T}}, & \text{if } T < 0; \\ \lim\limits_{T \to 0+} (+c, +s, 0)^{\mathrm{T}}, & \text{if } T = 0; \\ (+c, +s, 0)^{\mathrm{T}}, & \text{if } T > 0; \end{cases} \tag{4–60}$$

where c and s are the values interpolated from Laskar's tables. As $T \to 0$, the direction of $\mathbf{N}$ is well defined even though the magnitude of the vector vanishes. When $\mathbf{N}$ is rotated into the E system, its longitude angle is Λ_A. The orthogonal matrix to perform this rotation is simply

$$\mathsf{E} = \begin{pmatrix} \Upsilon^{\mathrm{T}} \\ (\mathbf{E} \times \Upsilon)^{\mathrm{T}} \\ \mathbf{E}^{\mathrm{T}} \end{pmatrix} \tag{4–61}$$

so that

$$\Lambda_A = \operatorname{plg}\left[(\mathsf{E}\,\mathbf{N})_2,\ (\mathsf{E}\,\mathbf{N})_1\right]. \tag{4–62}$$

The longitude of the node, Π_A, is similarly given as

$$\Pi_A = \operatorname{plg}(N_2,\ N_1). \tag{4–63}$$

If $T = 0$, E is the identity matrix, giving $\Lambda_A = \Pi_A$, or $p_A = 0$.

Finally, the angles relative to the invariable plane must be found. From Chapter 3, the precession matrix P can be written as

$$\mathsf{P} = \mathsf{R}_3(-L)\,\mathsf{R}_1(-I)\,\mathsf{R}_3(-\Delta)\,\mathsf{R}_1(I_0)\,\mathsf{R}_3(L_0). \tag{3–4}$$

Define the matrix Q by

$$\mathbf{Q} \equiv \mathbf{P}\,\mathbf{R}_3(-L_0)\,\mathbf{R}_1(-I_0) \tag{4-64}$$

$$= \mathbf{R}_3(-L)\,\mathbf{R}_1(-I)\,\mathbf{R}_3(-\Delta) \tag{4-65}$$

$$= \begin{pmatrix} \begin{matrix}\cos L\cos\Delta \\ -\sin L\cos I\sin\Delta\end{matrix} & \begin{matrix}-\cos L\sin\Delta \\ -\sin L\cos I\cos\Delta\end{matrix} & \sin L\sin I \\ \\ \begin{matrix}\sin L\cos\Delta \\ +\cos L\cos I\sin\Delta\end{matrix} & \begin{matrix}-\sin L\sin\Delta \\ +\cos L\cos I\cos\Delta\end{matrix} & -\cos L\sin I \\ \\ \sin I\sin\Delta & \sin I\cos\Delta & \cos I \end{pmatrix}. \tag{4-66}$$

Equation (4–66) yields the desired angles by

$$L = \operatorname{plg}(+Q_{13},\, -Q_{23}); \tag{4-67}$$

$$I = \cos^{-1} Q_{33}; \tag{4-68}$$

$$\Delta = \operatorname{plg}(Q_{31},\, Q_{32}). \tag{4-69}$$

These results are unambiguous, even for $T = 0$, because $I > 0$ for all times T. In practice, $\mathbf{Q}$ is found from equation (4–64), using the matrix $\mathbf{P}$ found by equation (4–49).

The equations in this section were evaluated once per century, using the output file from the numerical integrator and the values of L_0 and I_0 determined in Chapter 2. The eleven angles thus found (ε, ζ_A, θ_A, z_A, ψ_A, χ_A, ω_A, p_A, L, I, and Δ) were written to a file for further processing.

Figure 4–2 presents plots of the eleven angles over the entire time span covered by the integration. The plots for ψ_A, p_A, and Δ are nearly straight lines on the scale of the figure; however, these angles are by no means linear functions of time.

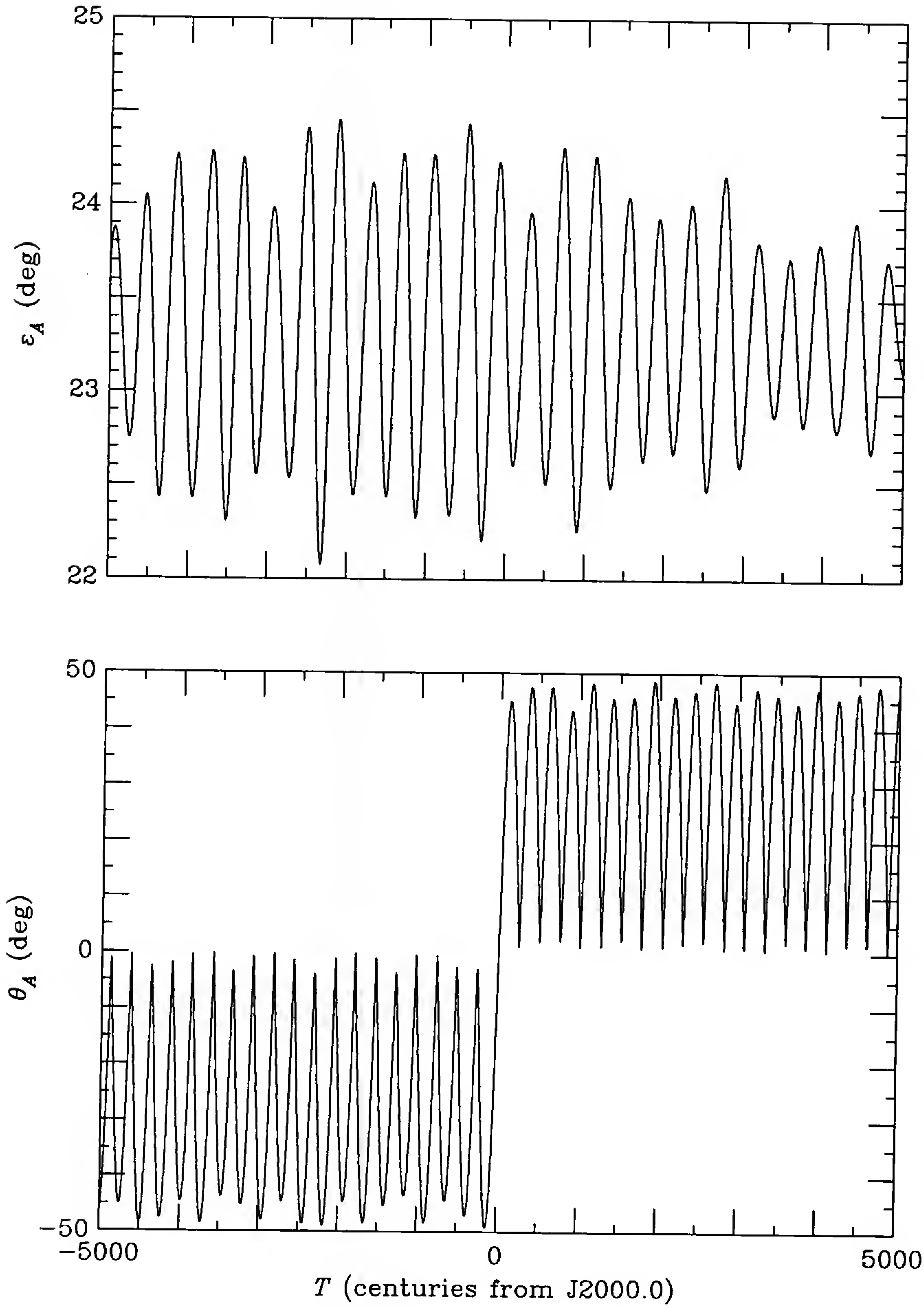

Figure 4–2. The Precession Angles for One Million Years: *a*) Obliquity, ε; *b*) Precession Angle θ_A.

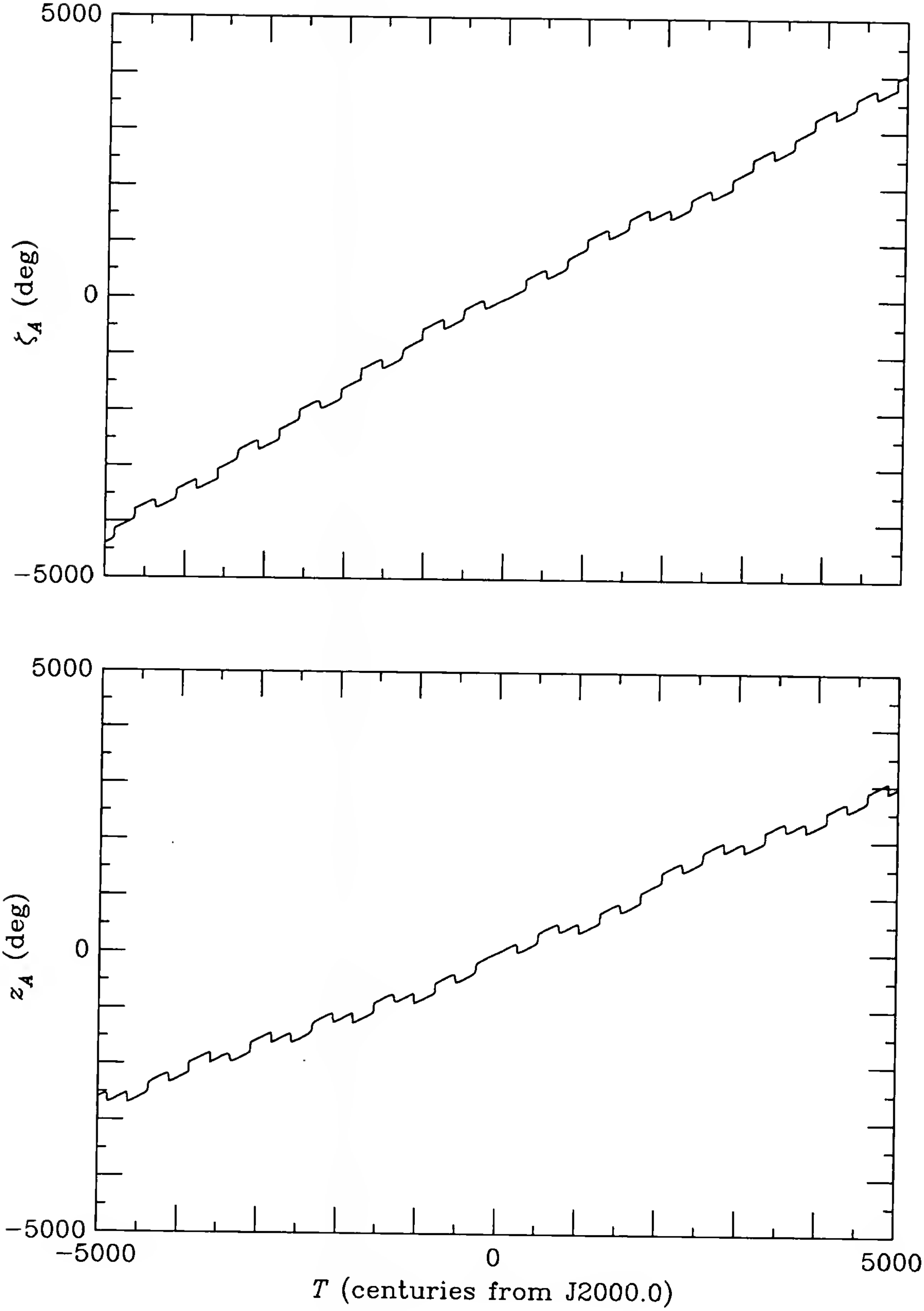

Figure 4–2, continued. *c*) Precession Angle ζ_A; *d*) Precession Angle z_A.

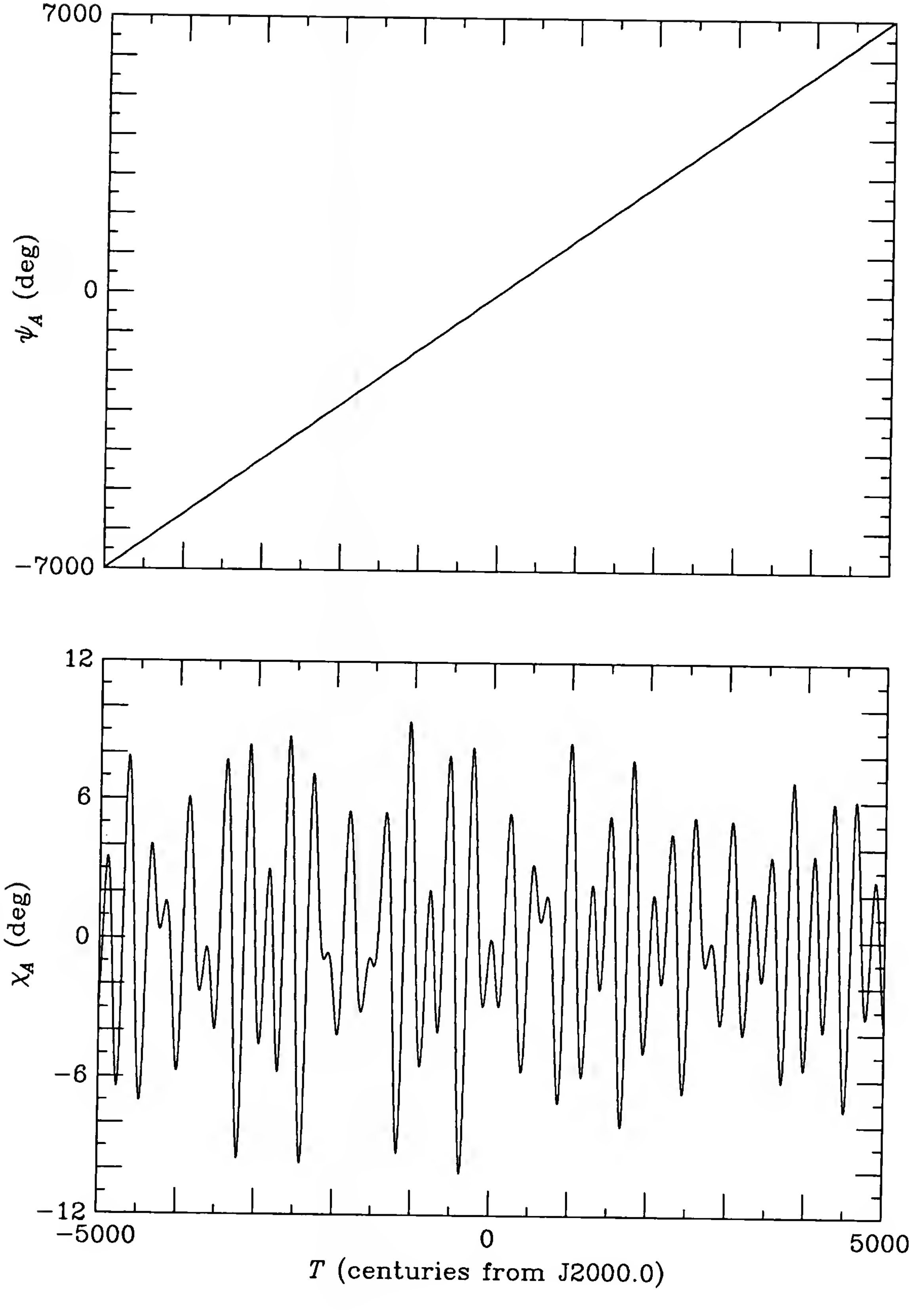

Figure 4–2, continued. *e*) Luni-Solar Precession, ψ_A; *f*) Planetary Precession, χ_A.

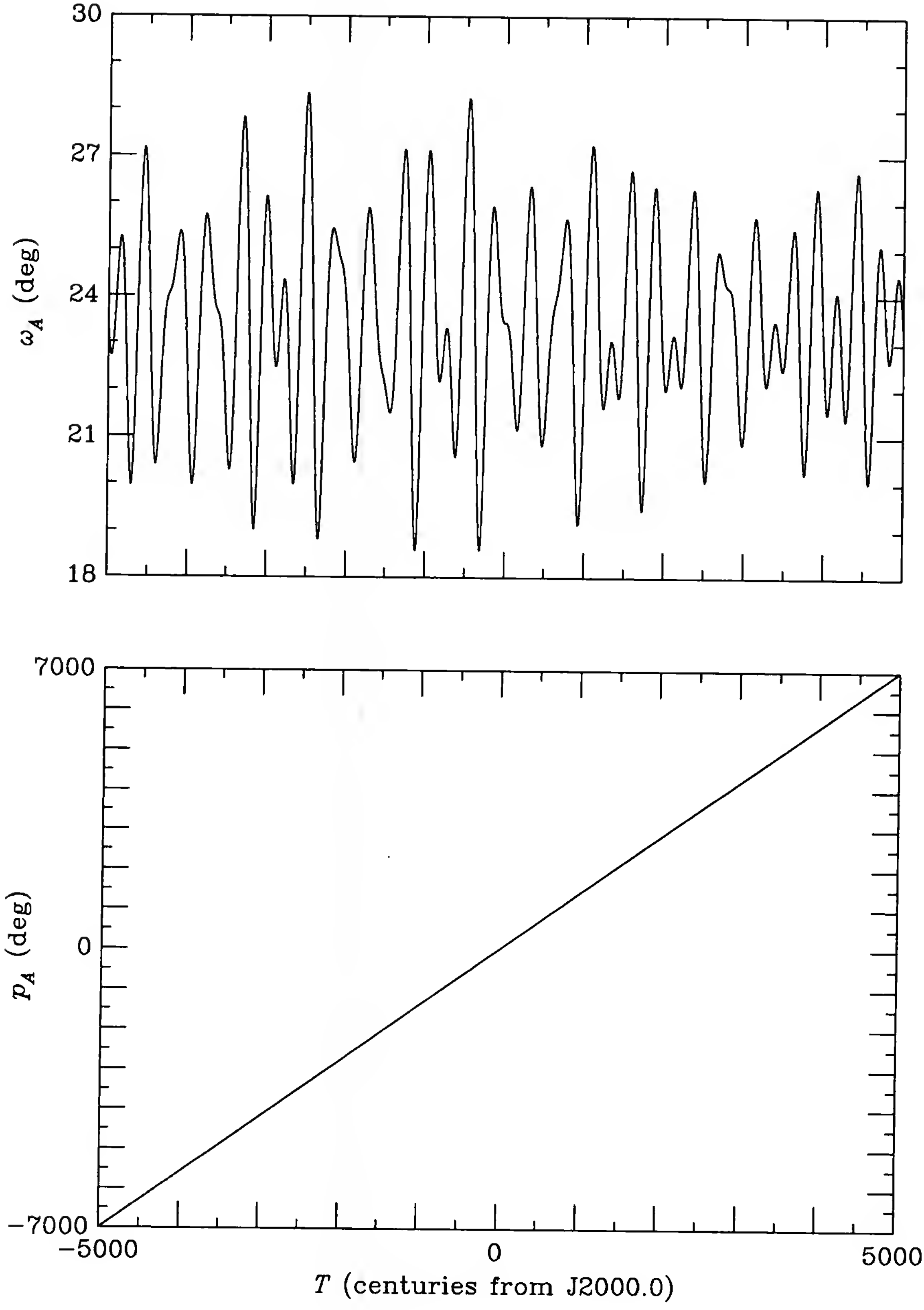

Figure 4-2, continued. *g*) Inclination of Ecliptic of Date to Ecliptic of Epoch, ω_A; *h*) General Precession in Longitude, p_A.

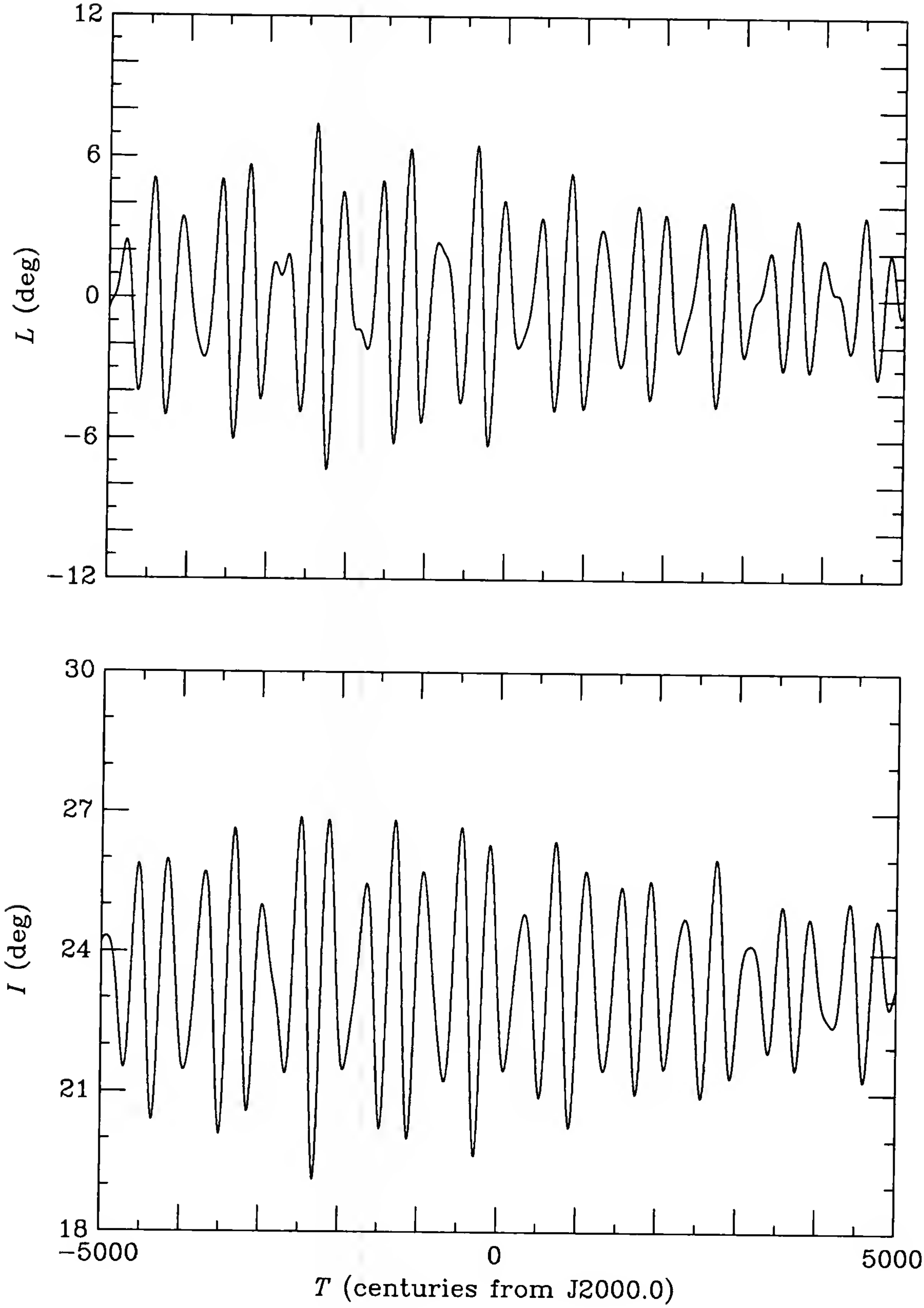

Figure 4–2, continued. *i*) Precession Angle L; *j*) Precession Angle I.

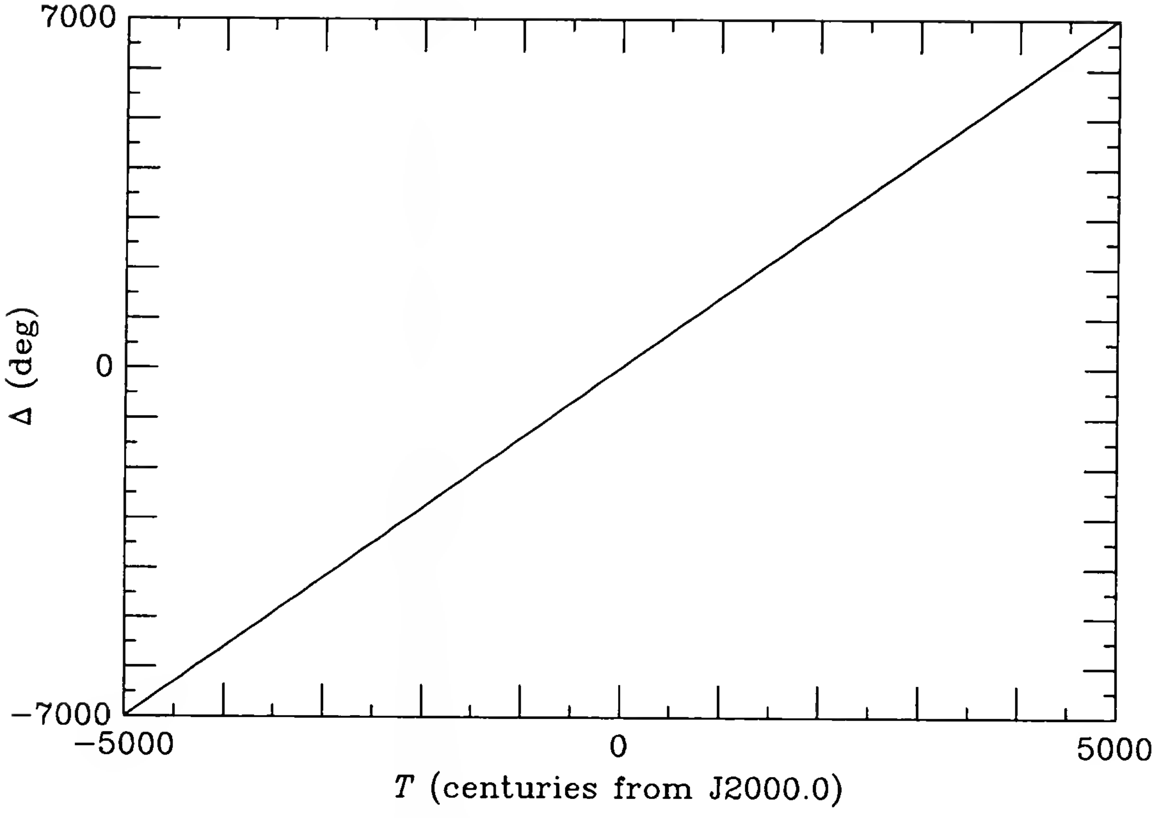

Figure 4–2, continued. k) Precession Angle Δ.

4.6. The Chebyshev Representation of the Precession Angles

The final step in the long-term theory is to provide polynomials through which the precession angles can be interpolated at any desired time. Since the use of Chebyshev polynomials has proved to be worthwhile in the construction of ephemeris files at JPL, this was a natural choice here as well. Both the length of the time intervals to be fit and the degree of the polynomials are arbitrary; a good choice should minimize the total number of coefficients required for the entire million years while retaining acceptable accuracy.

One feature deemed desirable proved to restrict the time intervals to just a few possible values. In order to provide a way of relating the short-term and long-term theories, the time $T = 0$ must occur in the center of an interval rather than at a boundary. This implies an odd number of intervals. Now the integration interval was a million years or 10,000 centuries; therefore the number of fitting intervals must be an odd factor of 10,000, namely, 5, 25, 125, or 625. Since the celestial pole makes approximately 39 revolutions about the ecliptic pole of epoch during the million years, the first two choices are obviously out of the question. The fourth option would not have shortened the interpolating polynomials enough to compensate for the fivefold increase in intervals. Consequently the million-year integration was broken into 125 intervals of 8000 years each.

The classical precession angles ζ_A, θ_A, and z_A proved impossible to fit. Recall that θ_A is the north polar distance of the celestial pole of date as measured in the Q_0 system; ζ_A is the negative of the right ascension of the celestial pole of date, again measured in the Q_0 system; and z_A is 180° plus the right ascension of the celestial pole of epoch with respect to the Q system. Now every 25,000 years or so the Earth completes a precessional cycle, and the celestial pole makes a close approach to the celestial pole of epoch. If the two poles were ever exactly parallel, θ_A would have a cusp at 0, and ζ_A and z_A would each undergo 180° discontinuities. Even though the two poles never do coincide (except at

$T = 0$), the qualitative behavior is similar: θ_A goes through a sharp minimum (although not a cusp), and ζ_A and z_A each move quickly (but not instantaneously) through 180°. Figure 4–3 presents plots of these angles at the close of the next cycle, some 25,100 years in the future. The time scale over which ζ_A and z_A change is about 20 centuries; since this is one fortieth of a fitting interval, one would need a polynomial of at least fortieth degree in order to fit the curve. Consequently these three angles were not fit. This is no great loss, because the precession matrix can be obtained just as easily from the other classical angles.

The remaining eight angles were approximated by ninth-degree polynomials to an average accuracy of about 10^{-9} degree, on the order of a μarcsecond. This accuracy should suffice for any long-term work.

Subroutine DPFIT (JPL Applied Mathematics Group 1987) performed the fitting; this is the same routine that provided the interpolating polynomials for Laskar's tables. The interpolating polynomials were constrained to match the tabular values at the end-points of each fitting interval, assuring continuity of the interpolated values across interval boundaries. In addition, the polynomials in the center interval were constrained to pass through the tabular values at $T = 0$. The constraints were programmed simply by giving these points a standard deviation of 10^{-10} instead of unity; according to the subroutine documentation, this is sufficient to force the polynomials to these points to the computer's internal precision.

DPFIT returns, in addition to the Chebyshev coefficients, a variable giving the *a posteriori* standard deviation of the difference between the tabular values and the polynomial approximations. The largest values found were 3.59×10^{-6} degree or $0\overset{''}{.}013$, for ψ_A and χ_A within the fifty-ninth 8000-year interval ($-360 < T < -280$). The worst fit to the angles

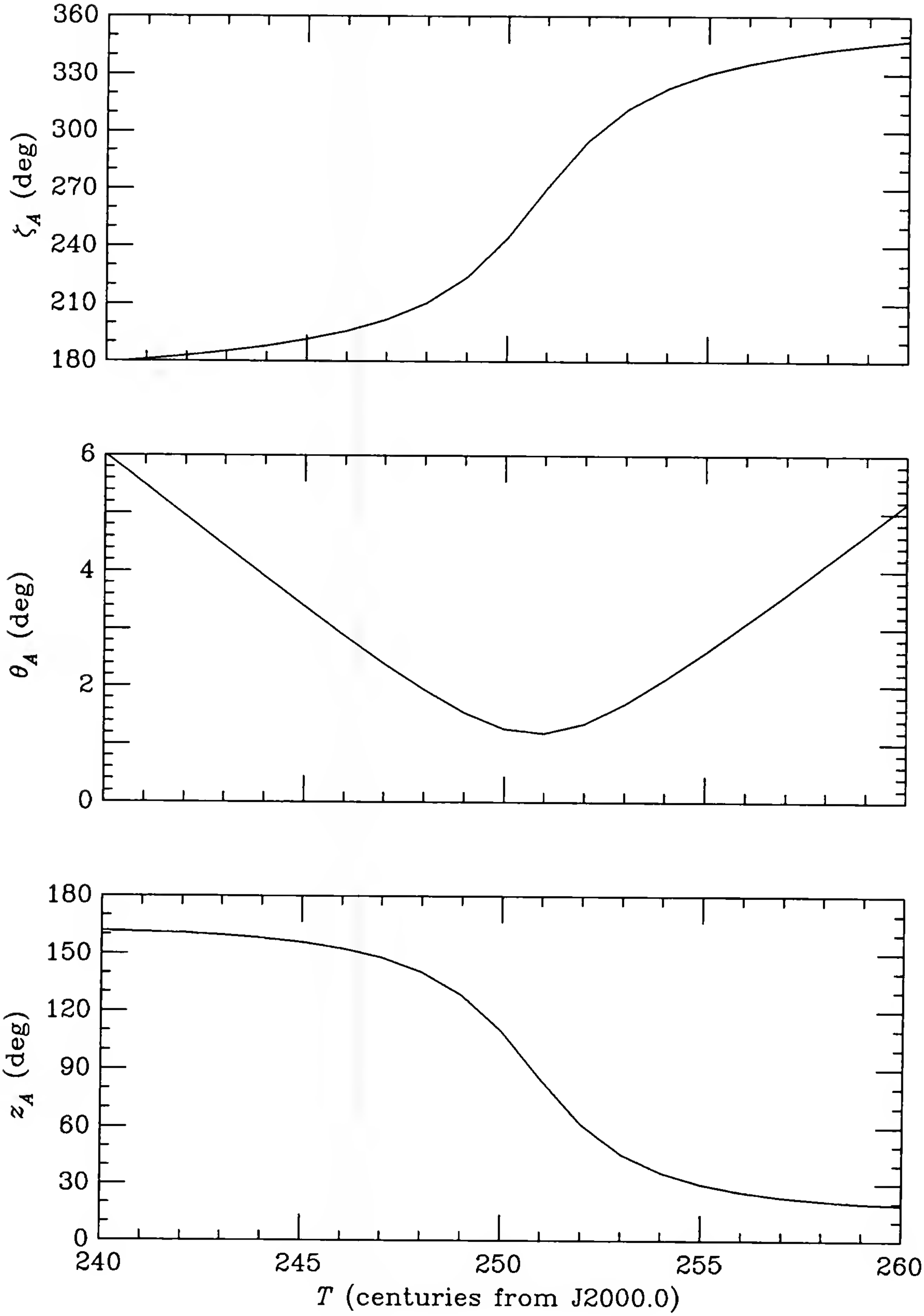

Figure 4–3. The Precession Angles ζ_A, θ_A, and z_A Near $T = +250$

L, I, and Δ was more than an order of magnitude better. More typical results were on the order of 10^{-9} degree.

Table 4–2 presents the principal results of this investigation: Chebyshev coefficients for the remaining eight angles. Each page contains all the coefficients for one time interval. The column of numbers under each angle gives the coefficients α_i for angle α, with α_0 at the top and α_9 at the bottom. (The symbol α here represents any of the eight angles.) All of the α_i are given in degrees.

Given a time T, measured in Julian centuries from J2000, with $|T| \leq 5000$, Table 4–2 is employed as follows: First determine i, the interval number, by

$$i = \lfloor (T + 5080)/80 \rfloor . \tag{4–70}$$

The symbols $\lfloor$ and $\rfloor$ denote that the quotient is to be truncated to an integer. Consequently the value of i will range from 1 to 125. The central time T_c of the interval is given by

$$T_c = 80i - 5040. \tag{4–71}$$

The argument of the Chebyshev polynomials is then (*cf.* equation (2–2))

$$\tau = \frac{T - T_c}{40}. \tag{4–72}$$

Finally, angle α is found by

$$\alpha = \sum_{k=0}^{9} \alpha_k T_k(\tau), \tag{4–73}$$

where $T_k(\tau)$ is the Chebyshev polynomial of the first kind of degree k as in equation (2–1). The result is $\alpha(T)$, expressed in degrees.

Copies of Table 4–2 in computer-readable form may be obtained from the author.

4.7. Sources of Error in the Long-Term Theory

During the verification of the numerical integrator, several tests were run to make sure that numerical problems were not significant. The first check on the integration was made by starting another integration at $T = -5000$ centuries, using the results of the first integration as initial conditions, and proceeding forward one million years to $T = +5000$ centuries. At $T = 0$, the second integration reproduced ε_0 to the last bit, and its value of the precession angle θ_A was 2×10^{-11} arcsec. At $T = +5000$ centuries, the precession angles agreed to better than five nanoarcseconds. This insignificant difference can be ascribed to round-off error during the integration.

A different approach is required to evaluate the difference between the numerically integrated results and the true (unknown) integral. This error arises because the Runge-Kutta procedure assumes a fourth-order polynomial for the derivative function, whereas the true derivative contains higher-order terms. The difference between the true and numerical answers, when integrated over a constant interval, is proportional to the fourth power of the stepsize h. Accordingly, the million-year forward integration above was repeated, with h doubled to $\frac{1}{4}$ century. At the end of the integration ($T = 5000$), the precession angles agreed with the previous results to better than one μarcsec. The difference between the initial integration and the truth should therefore be a factor of 16 smaller. Thus the integration procedure itself is not expected to be the dominant source of error.

The uncertainty in the orientation of the invariable plane was found at the end of Chapter 2 to be on the order of $0\rlap{.}''04$. However, this too is not worrisome, since the subsequent developments require only a fixed plane, not necessarily the invariable plane itself. So the results for I_0 and L_0 can be considered to be conventional constants without error; if the determination of the invariable plane were to change, the angles L, I, and Δ in Table 4–2 could then be interpreted as referring to an arbitrary plane close to, but not

coincident with, the invariable plane. But if I_0 and L_0 were to be changed, only L, I, and Δ would change as a result, and those only by an amount similar to the changes in I_0 or L_0. The numerical integration itself would not be affected.

The dominant uncertainty in the long-term theory is in the initial speed of general precession in longitude. The currently-accepted value of $5029''.0966$/century may be in error by a significant fraction of an arcsecond per century; in other words, by one part in 10^4. And since this uncertainty is in a rate, the error in the accumulated angle p_A will grow roughly linearly with time. After 500,000 years, an error of $0''.36$/century in p_1 would cause an error of half a degree in p_A. An error of this magnitude easily swamps all other effects.

For this reason it is fruitless at the moment to extend the integration much past the million-year interval covered here. There is considerable hope that very-long baseline interferometry (VLBI) and lunar laser ranging (LLR) data will soon provide a much more precise value of p_1. Zhu *et al.* (1990) have already published a new value for the rate of luni-solar precession, based on VLBI data alone, whose standard error is only $0''.047$/century; a forthcoming paper by Williams *et al.* (1990), using LLR data, gives a similar standard error. When more precise values will have been adopted by the IAU, it will be possible to extend the integration over at least ten times the current interval. Results from such a project when compared with the geological record might even give insight into the long-term change in length of the day and the tidal deceleration of the Moon.

Table 4–2. Chebyshev Polynomial Coefficients for the Long-Term Theory

Interval 1: Central time $T_c = -4960$, covering the time span $-5000 \leq T \leq -4920$

	ε (deg)
0	23.818664397623563
1	$6.2699867038191869 \times 10^{-2}$
2	$-4.5701998128459612 \times 10^{-2}$
3	$-6.3995001930043744 \times 10^{-4}$
4	$2.6193018764804876 \times 10^{-4}$
5	$1.0653773760018432 \times 10^{-5}$
6	$-1.1370833649758035 \times 10^{-6}$
7	$-3.0762036624011159 \times 10^{-7}$
8	$3.4849968414360932 \times 10^{-8}$
9	$4.9125847475205783 \times 10^{-9}$

	ψ_A (deg)
0	-6947.1817916751140
1	57.988831745256441
2	$-1.2514648137521231 \times 10^{-2}$
3	$-1.5399728381420450 \times 10^{-1}$
4	$-2.0229105141836574 \times 10^{-4}$
5	$3.4309932438000724 \times 10^{-3}$
6	$6.4884071719344012 \times 10^{-6}$
7	$-7.1150274266214423 \times 10^{-5}$
8	$-2.4715878147915127 \times 10^{-7}$
9	$1.6552941373363334 \times 10^{-6}$

	ω_A (deg)
0	23.060251739442413
1	$6.7481678725657374 \times 10^{-2}$
2	$3.2561467594939639 \times 10^{-1}$
3	$1.0087812131847197 \times 10^{-3}$
4	$-9.1600795131963509 \times 10^{-3}$
5	$-2.5817521574085075 \times 10^{-5}$
6	$1.2847875316548974 \times 10^{-4}$
7	$5.2735777825026838 \times 10^{-7}$
8	$-2.0429279616977182 \times 10^{-6}$
9	$-1.2321701913478007 \times 10^{-8}$

	χ_A (deg)
0	$1.7579257768399799 \times 10^{-1}$
1	3.1293086165127557
2	$1.2182130339216408 \times 10^{-2}$
3	$-1.7980003160735650 \times 10^{-1}$
4	$-4.3499345964194971 \times 10^{-4}$
5	$3.6316984685754724 \times 10^{-3}$
6	$1.2148503584153746 \times 10^{-5}$
7	$-7.2448127012273611 \times 10^{-5}$
8	$-3.6010216691377741 \times 10^{-7}$
9	$1.6699872891592286 \times 10^{-6}$

	p_A (deg)
0	-6947.3423766173318
1	55.119151351552834
2	$-2.2854666780248683 \times 10^{-2}$
3	$1.2456343374957200 \times 10^{-2}$
4	$1.6174745952367560 \times 10^{-4}$
5	$-3.6065919582192644 \times 10^{-5}$
6	$-4.5676134208786222 \times 10^{-6}$
7	$4.5192368450507862 \times 10^{-7}$
8	$9.6971766145124571 \times 10^{-8}$
9	$-1.2385897206242930 \times 10^{-8}$

	Δ (deg)
0	-6943.6556948609772
1	54.864602792740368
2	$-2.0058062039669857 \times 10^{-3}$
3	$-4.6094526591024886 \times 10^{-3}$
4	$-5.2690843404782897 \times 10^{-4}$
5	$2.3535431879999314 \times 10^{-4}$
6	$6.8447509562771559 \times 10^{-6}$
7	$-1.6726348965035335 \times 10^{-6}$
8	$-1.0014551416406377 \times 10^{-7}$
9	$1.5054460132865079 \times 10^{-8}$

	I (deg)
0	24.278883635242686
1	$5.8124784629582700 \times 10^{-2}$
2	$-2.6034029591636675 \times 10^{-2}$
3	$1.5150487807949463 \times 10^{-3}$
4	$-9.2010580028001137 \times 10^{-4}$
5	$-3.1808829795997960 \times 10^{-5}$
6	$1.0308087870064916 \times 10^{-5}$
7	$3.9177373397507516 \times 10^{-7}$
8	$-6.5150507407559364 \times 10^{-8}$
9	$-5.1965942947992761 \times 10^{-9}$

	L (deg)
0	$-1.5911035524283148 \times 10^{-1}$
1	$2.8349725382249824 \times 10^{-1}$
2	$-2.2832895975057642 \times 10^{-2}$
3	$1.8613355542927282 \times 10^{-2}$
4	$7.6287571145027620 \times 10^{-4}$
5	$-2.9995966213654493 \times 10^{-4}$
6	$-1.2635482725393354 \times 10^{-5}$
7	$2.3499262619685270 \times 10^{-6}$
8	$2.1560412343013913 \times 10^{-7}$
9	$-2.9685309397376271 \times 10^{-8}$

Table 4–2, continued.

Interval 2: Central time $T_c = -4880$, covering the time span $-4920 \leq T \leq -4840$

	ε (deg)		p_A (deg)
0	23.606535016764365	0	-6836.8261115604678
1	$-2.5970116821480491 \times 10^{-1}$	1	55.512295531285741
2	$-2.7772307630643748 \times 10^{-2}$	2	$1.1395383413877476 \times 10^{-1}$
3	$3.3609181464471659 \times 10^{-3}$	3	$8.5290291116745100 \times 10^{-3}$
4	$1.8605189421624906 \times 10^{-4}$	4	$-5.3877728738194862 \times 10^{-4}$
5	$-3.5031154300816966 \times 10^{-6}$	5	$-2.0250762498184661 \times 10^{-5}$
6	$9.0817436236039225 \times 10^{-7}$	6	$-1.6648527552111220 \times 10^{-6}$
7	$-1.0371227043267859 \times 10^{-7}$	7	$-5.1330174611322831 \times 10^{-7}$
8	$-2.8523221611967729 \times 10^{-8}$	8	$3.3822057284303545 \times 10^{-8}$
9	$2.0409109010138838 \times 10^{-9}$	9	$7.7678057371945635 \times 10^{-9}$

	ψ_A (deg)		Δ (deg)
0	-6834.9662536509426	0	-6833.9962075706194
1	53.713004439765062	1	54.818528604396031
2	$-6.1671693990336106 \times 10^{-1}$	2	$3.2291392324716688 \times 10^{-2}$
3	$7.1617057067268673 \times 10^{-2}$	3	$1.7912404367114537 \times 10^{-2}$
4	$1.0050519426999416 \times 10^{-2}$	4	$2.3361956264705157 \times 10^{-3}$
5	$-1.1663776862545524 \times 10^{-3}$	5	$-4.4143279209932971 \times 10^{-5}$
6	$8.1890572349062742 \times 10^{-5}$	6	$-1.6542954024190492 \times 10^{-5}$
7	$1.0194552966399281 \times 10^{-5}$	7	$3.9140065104284544 \times 10^{-7}$
8	$-2.7352252096424704 \times 10^{-6}$	8	$-2.3680846027579883 \times 10^{-8}$
9	$1.8357791105666273 \times 10^{-7}$	9	$-1.1997525213281733 \times 10^{-8}$

	ω_A (deg)		I (deg)
0	24.544214965414648	0	24.090891849935034
1	$9.5448004572306359 \times 10^{-1}$	1	$-2.8914739600390418 \times 10^{-1}$
2	$-2.0033307758980940 \times 10^{-1}$	2	$-7.4585302024903922 \times 10^{-2}$
3	$-5.2810313836441126 \times 10^{-2}$	3	$-5.6698137639444905 \times 10^{-3}$
4	$3.6074249899681430 \times 10^{-3}$	4	$5.4773393671183883 \times 10^{-4}$
5	$4.7828583004881589 \times 10^{-4}$	5	$1.2194638385605118 \times 10^{-4}$
6	$-4.1663772032824437 \times 10^{-5}$	6	$-1.9355465990943640 \times 10^{-6}$
7	$1.9696540237390936 \times 10^{-6}$	7	$-5.1168795001412213 \times 10^{-7}$
8	$2.0935720563854280 \times 10^{-7}$	8	$3.1748102977766377 \times 10^{-8}$
9	$-5.8127877522087914 \times 10^{-8}$	9	$-8.2556599325367207 \times 10^{-10}$

	χ_A (deg)		L (deg)
0	2.0346484054914048	0	$7.8483520673705942 \times 10^{-1}$
1	-1.9653232858433521	1	$7.5936173319815703 \times 10^{-1}$
2	$-8.0436598073185256 \times 10^{-1}$	2	$8.7793488472002232 \times 10^{-2}$
3	$6.8504838881229831 \times 10^{-2}$	3	$-1.0515901653955752 \times 10^{-2}$
4	$1.2263458568593089 \times 10^{-2}$	4	$-3.1481954592046039 \times 10^{-3}$
5	$-1.2011922578723845 \times 10^{-3}$	5	$3.2703020635846523 \times 10^{-5}$
6	$6.9297369207512561 \times 10^{-5}$	6	$1.7173028091201902 \times 10^{-5}$
7	$1.0934768379785479 \times 10^{-5}$	7	$-1.0054467936509935 \times 10^{-6}$
8	$-2.7009272579233909 \times 10^{-6}$	8	$5.1894113590652879 \times 10^{-8}$
9	$1.7602246861029177 \times 10^{-7}$	9	$2.0994817646436191 \times 10^{-8}$

Table 4–2, continued.

Interval 3: Central time $T_c = -4800$, covering the time span $-4840 \leq T \leq -4760$

	ε (deg)		p_A (deg)
0	23.024546093662606	0	-6724.7042766355148
1	$-2.7609401298836143 \times 10^{-1}$	1	56.632659665766099
2	$2.7240831349677995 \times 10^{-2}$	2	$1.3980374968206817 \times 10^{-1}$
3	$5.2426266384672827 \times 10^{-3}$	3	$-6.6267617615960967 \times 10^{-3}$
4	$-6.5444517682342942 \times 10^{-5}$	4	$-1.4206025195015459 \times 10^{-3}$
5	$-3.4804349389649285 \times 10^{-5}$	5	$-2.4928186803291643 \times 10^{-5}$
6	$-1.7141554337662453 \times 10^{-6}$	6	$7.8441919827767649 \times 10^{-6}$
7	$1.9132831528864827 \times 10^{-7}$	7	$6.7729550746867505 \times 10^{-7}$
8	$1.9024078404737952 \times 10^{-8}$	8	$-3.8714296306443867 \times 10^{-8}$
9	$-1.0897153065881361 \times 10^{-9}$	9	$-5.4417232724089859 \times 10^{-9}$

	ψ_A (deg)		Δ (deg)
0	-6728.5501503163573	0	-6723.1023581163678
1	53.870277187795890	1	56.367281972725838
2	$7.6840547924135050 \times 10^{-1}$	2	$3.7310105425014816 \times 10^{-1}$
3	$1.3737241969343236 \times 10^{-1}$	3	$2.6390579217846045 \times 10^{-2}$
4	$-9.8589253543130440 \times 10^{-4}$	4	$-2.7544309079580886 \times 10^{-3}$
5	$-1.2128299005059301 \times 10^{-3}$	5	$-5.0710278932929111 \times 10^{-4}$
6	$-2.3279896807209312 \times 10^{-4}$	6	$-2.1078587892539429 \times 10^{-5}$
7	$-2.5238160455851756 \times 10^{-5}$	7	$1.6910997771234073 \times 10^{-6}$
8	$2.4082604557558057 \times 10^{-8}$	8	$4.4540505839224657 \times 10^{-7}$
9	$3.7482959906751830 \times 10^{-7}$	9	$3.7953582417875444 \times 10^{-8}$

	ω_A (deg)		I (deg)
0	23.852524820198505	0	22.905818244614831
1	-1.7311698809243734	1	$-8.5913113721750520 \times 10^{-1}$
2	$-2.9585352628835003 \times 10^{-1}$	2	$-2.8166391480042323 \times 10^{-2}$
3	$4.5425216828311642 \times 10^{-2}$	3	$1.6297063737168860 \times 10^{-2}$
4	$7.1476754469986897 \times 10^{-3}$	4	$1.6240273282561156 \times 10^{-3}$
5	$-6.4106816831847140 \times 10^{-5}$	5	$-6.0622089905822952 \times 10^{-5}$
6	$-3.4219068987473043 \times 10^{-5}$	6	$-1.4620968773203016 \times 10^{-5}$
7	$-4.8552093988726082 \times 10^{-6}$	7	$-6.4241344515251500 \times 10^{-7}$
8	$-6.1588410500559805 \times 10^{-7}$	8	$1.2539307694159822 \times 10^{-8}$
9	$-2.7116443270380051 \times 10^{-8}$	9	$7.8664298237883372 \times 10^{-9}$

	χ_A (deg)		L (deg)
0	-4.1804429556208883	0	2.1154941980236582
1	-2.9801139640561064	1	$2.7902185058114794 \times 10^{-1}$
2	$6.9947005160162694 \times 10^{-1}$	2	$-2.5431307886646264 \times 10^{-1}$
3	$1.5480931224043567 \times 10^{-1}$	3	$-3.5155314349219089 \times 10^{-2}$
4	$-8.4587637361382280 \times 10^{-4}$	4	$1.5416214499452762 \times 10^{-3}$
5	$-1.3234460572683966 \times 10^{-3}$	5	$5.0980507794863407 \times 10^{-4}$
6	$-2.3012434872790400 \times 10^{-4}$	6	$2.8182529521556650 \times 10^{-5}$
7	$-2.5132320943327151 \times 10^{-5}$	7	$-1.1356106337220945 \times 10^{-6}$
8	$3.3229431655192394 \times 10^{-9}$	8	$-4.8585070832187697 \times 10^{-7}$
9	$3.7743427384566012 \times 10^{-7}$	9	$-4.3198212665713461 \times 10^{-8}$

Table 4–2, continued.

Interval 4: Central time $T_c = -4720$, covering the time span $-4760 \leq T \leq -4680$

	ε (deg)
0	22.840135164375138
1	$1.2011118605183176 \times 10^{-1}$
2	$6.0212779394957960 \times 10^{-2}$
3	$-1.2016743746824789 \times 10^{-3}$
4	$-6.0424074439334419 \times 10^{-4}$
5	$3.8354477324632466 \times 10^{-6}$
6	$3.3401848882296466 \times 10^{-6}$
7	$-1.1377753986444457 \times 10^{-7}$
8	$-2.3284540272391351 \times 10^{-8}$
9	$1.6761471191222817 \times 10^{-9}$

	p_A (deg)
0	-6610.8184215687144
1	57.090773792220673
2	$-5.1645934332201964 \times 10^{-2}$
3	$-2.0844264444507571 \times 10^{-2}$
4	$2.7005691283524463 \times 10^{-4}$
5	$1.4852166017188857 \times 10^{-4}$
6	$-2.5117575660635799 \times 10^{-6}$
7	$-9.3121880949740539 \times 10^{-7}$
8	$4.8951911078419546 \times 10^{-8}$
9	$8.0462907849223213 \times 10^{-9}$

	ψ_A (deg)
0	-6612.1320240901592
1	62.507541141954176
2	$6.1814061384224829 \times 10^{-1}$
3	$-2.8224512441903863 \times 10^{-1}$
4	$-3.2098437189695914 \times 10^{-2}$
5	$6.6528972161132308 \times 10^{-3}$
6	$1.2951314870376148 \times 10^{-3}$
7	$-1.3665717876874964 \times 10^{-4}$
8	$-5.0774264941812185 \times 10^{-5}$
9	$1.7764219018014871 \times 10^{-6}$

	Δ (deg)
0	-6607.3987924881333
1	59.137625893000684
2	$1.2598831958515039 \times 10^{-1}$
3	$-7.7890870402873557 \times 10^{-2}$
4	$-5.5182550825404708 \times 10^{-3}$
5	$9.2192727729410407 \times 10^{-4}$
6	$1.0709930851942679 \times 10^{-4}$
7	$-9.3053779665752952 \times 10^{-6}$
8	$-1.9058653269490879 \times 10^{-6}$
9	$7.3315948178890241 \times 10^{-8}$

	ω_A (deg)
0	20.683856989657203
1	$-6.7245004958300708 \times 10^{-1}$
2	$5.8131141644439868 \times 10^{-1}$
3	$4.8032593351361947 \times 10^{-2}$
4	$-1.3136604795685989 \times 10^{-2}$
5	$-1.3567237860777039 \times 10^{-3}$
6	$2.0618953409697026 \times 10^{-4}$
7	$3.9925749992294870 \times 10^{-5}$
8	$-3.0540647617561648 \times 10^{-6}$
9	$-1.2901223441840511 \times 10^{-6}$

	I (deg)
0	21.747437471543140
1	$-8.7396159316614836 \times 10^{-2}$
2	$2.1693283006368155 \times 10^{-1}$
3	$1.2858557422834304 \times 10^{-2}$
4	$-2.7987143495871685 \times 10^{-3}$
5	$-2.3740074065559215 \times 10^{-4}$
6	$2.2515522892957213 \times 10^{-5}$
7	$3.0475001127501817 \times 10^{-6}$
8	$-1.5788010311979041 \times 10^{-7}$
9	$-4.1881973965533186 \times 10^{-8}$

	χ_A (deg)
0	-1.4191205870973486
1	5.8394739243341355
2	$7.1261936585592622 \times 10^{-1}$
3	$-2.7579852569370304 \times 10^{-1}$
4	$-3.3253817215295446 \times 10^{-2}$
5	$6.6590840243643159 \times 10^{-3}$
6	$1.3038777965120428 \times 10^{-3}$
7	$-1.3649666618914944 \times 10^{-4}$
8	$-5.0837777641884433 \times 10^{-5}$
9	$1.7697418133189003 \times 10^{-6}$

	L (deg)
0	$1.3585124399140972 \times 10^{-1}$
1	-2.2174110731703671
2	$-1.9236041117111411 \times 10^{-1}$
3	$6.0570613803702049 \times 10^{-2}$
4	$6.1153756306476221 \times 10^{-3}$
5	$-7.9762129658861718 \times 10^{-4}$
6	$-1.1226893644493565 \times 10^{-4}$
7	$8.4548617754942538 \times 10^{-6}$
8	$1.9692825551574797 \times 10^{-6}$
9	$-6.5188453542120874 \times 10^{-8}$

Table 4–2, continued.

Interval 5: Central time $T_c = -4640$, covering the time span $-4680 \le T \le -4600$

	ε (deg)		p_A (deg)
0	23.417127071250902	0	−6497.6707557206083
1	$4.0583466592017596 \times 10^{-1}$	1	55.929378393607887
2	$-9.0763263032046608 \times 10^{-6}$	2	$-2.0177176139786571 \times 10^{-1}$
3	$-7.5055888044528308 \times 10^{-3}$	3	$-4.0072766029386163 \times 10^{-4}$
4	$-9.3409354847831064 \times 10^{-5}$	4	$1.7940165379468497 \times 10^{-3}$
5	$3.6072464819188211 \times 10^{-5}$	5	$7.4296539942505194 \times 10^{-6}$
6	$8.9929040134509799 \times 10^{-7}$	6	$-4.2041260332181449 \times 10^{-6}$
7	$7.5988404049748119 \times 10^{-8}$	7	$-3.5253065077170602 \times 10^{-8}$
8	$-7.6039691913380088 \times 10^{-9}$	8	$-5.1396009213279347 \times 10^{-8}$
9	$-3.2623036454233008 \times 10^{-9}$	9	$1.3367803501579730 \times 10^{-9}$

	ψ_A (deg)		Δ (deg)
0	−6491.7323240025131	0	−6491.0472058896499
1	56.081089278439220	1	56.546327830696328
2	−1.4474878714939393	2	$-6.4412585815711937 \times 10^{-1}$
3	$7.6439633801051263 \times 10^{-2}$	3	$-1.1223168185592697 \times 10^{-2}$
4	$2.0228958127208187 \times 10^{-2}$	4	$8.6110462608584373 \times 10^{-3}$
5	$-3.9982762869547930 \times 10^{-3}$	5	$-3.3117425957988991 \times 10^{-4}$
6	$2.5629047002286796 \times 10^{-4}$	6	$-6.5242107899225512 \times 10^{-5}$
7	$5.0257595801341078 \times 10^{-5}$	7	$1.0212993302284753 \times 10^{-5}$
8	$-1.4647804124737236 \times 10^{-5}$	8	$-3.9387737661920222 \times 10^{-9}$
9	$1.3555271531755915 \times 10^{-6}$	9	$-1.3646301313799178 \times 10^{-7}$

	ω_A (deg)		I (deg)
0	23.280240563184776	0	23.183004763638829
1	2.7683268207973622	1	1.3996773903951890
2	$2.4024016634889118 \times 10^{-2}$	2	$7.3535567682826250 \times 10^{-2}$
3	$-8.8214547617832957 \times 10^{-2}$	3	$-3.1079667806679730 \times 10^{-2}$
4	$3.2257071351615406 \times 10^{-3}$	4	$-8.4119667903257132 \times 10^{-4}$
5	$7.5536994574118821 \times 10^{-4}$	5	$2.7418509309681261 \times 10^{-4}$
6	$-1.1654341307068695 \times 10^{-4}$	6	$-7.1779250971202812 \times 10^{-6}$
7	$7.8751204437164467 \times 10^{-6}$	7	$-1.6806520806915024 \times 10^{-6}$
8	$8.5248730045197334 \times 10^{-7}$	8	$2.1936669134759034 \times 10^{-7}$
9	$-3.1460183310909927 \times 10^{-7}$	9	$1.1709640775333777 \times 10^{-9}$

	χ_A (deg)		L (deg)
0	6.4687741859971271	0	−3.3644482522272349
1	$2.3475485628823442 \times 10^{-1}$	1	$-6.9943830315431159 \times 10^{-1}$
2	−1.3533511168278861	2	$4.7810816612691630 \times 10^{-1}$
3	$7.3424385735039241 \times 10^{-2}$	3	$1.3767532199577500 \times 10^{-2}$
4	$2.0305939815122274 \times 10^{-2}$	4	$-7.2264923780789703 \times 10^{-3}$
5	$-3.9672718173106631 \times 10^{-3}$	5	$3.1144999171921662 \times 10^{-4}$
6	$2.4663724439027773 \times 10^{-4}$	6	$6.3511487761374682 \times 10^{-5}$
7	$5.0025279593065053 \times 10^{-5}$	7	$-1.0166226825677696 \times 10^{-5}$
8	$-1.4537082646641654 \times 10^{-5}$	8	$-6.4505328526243906 \times 10^{-8}$
9	$1.3563579777435469 \times 10^{-6}$	9	$1.3788675402854099 \times 10^{-7}$

Table 4–2, continued.

Interval 6: Central time $T_c = -4560$, covering the time span $-4600 \leq T \leq -4520$

	ε (deg)
0	23.965712569520440
1	$8.1070137507108527 \times 10^{-2}$
2	$-7.2486970194800267 \times 10^{-2}$
3	$-2.6030484259827079 \times 10^{-3}$
4	$6.5183695941823358 \times 10^{-4}$
5	$1.6066666025280645 \times 10^{-5}$
6	$-3.6162747333110249 \times 10^{-6}$
7	$1.1256693485449113 \times 10^{-8}$
8	$4.6592725930330565 \times 10^{-8}$
9	$3.6380443182125569 \times 10^{-11}$

	p_A (deg)
0	-6387.1168491142016
1	54.751576882820703
2	$-5.1582611303892873 \times 10^{-2}$
3	$2.2483197296702231 \times 10^{-2}$
4	$6.2405503219453443 \times 10^{-4}$
5	$-1.1644004872140250 \times 10^{-4}$
6	$-5.8206654971633952 \times 10^{-7}$
7	$7.6207345024177147 \times 10^{-7}$
8	$-1.6987487456189449 \times 10^{-8}$
9	$-1.2364547212758640 \times 10^{-8}$

	ψ_A (deg)
0	-6386.7969731744093
1	50.074396886118218
2	$-1.6584494409645646 \times 10^{-2}$
3	$1.1633492521364799 \times 10^{-1}$
4	$-1.1885535564436165 \times 10^{-3}$
5	$2.8206558167398318 \times 10^{-4}$
6	$-6.9969038469390292 \times 10^{-6}$
7	$-7.8056051057377058 \times 10^{-6}$
8	$1.9333201857043163 \times 10^{-7}$
9	$-7.3118102673139892 \times 10^{-8}$

	Δ (deg)
0	-6382.3290955608263
1	52.517203364157477
2	$-2.4500829873807289 \times 10^{-1}$
3	$5.8066611085307175 \times 10^{-2}$
4	$1.2402228147936060 \times 10^{-3}$
5	$-1.0991183639930707 \times 10^{-4}$
6	$1.9511578514985864 \times 10^{-5}$
7	$-1.8553924956981989 \times 10^{-6}$
8	$-3.9675715458267454 \times 10^{-8}$
9	$9.9507591119989444 \times 10^{-9}$

	ω_A (deg)
0	26.635717555874233
1	$1.3050024093635318 \times 10^{-1}$
2	$-5.2088021120302472 \times 10^{-1}$
3	$1.9514516345719059 \times 10^{-3}$
4	$5.8006756546692251 \times 10^{-3}$
5	$-6.5472899113674666 \times 10^{-5}$
6	$-2.0013615200473262 \times 10^{-6}$
7	$-1.0492504607595969 \times 10^{-7}$
8	$-1.0289378606328587 \times 10^{-7}$
9	$1.6508680341925306 \times 10^{-9}$

	I (deg)
0	25.444701367942309
1	$5.9162517464842503 \times 10^{-1}$
2	$-2.4545708133287269 \times 10^{-1}$
3	$-1.4889524975958279 \times 10^{-2}$
4	$2.1143200903168092 \times 10^{-3}$
5	$5.7505523979528343 \times 10^{-5}$
6	$-2.6972448473998809 \times 10^{-6}$
7	$2.9672253857570422 \times 10^{-7}$
8	$-5.3093128027390680 \times 10^{-8}$
9	$1.5808633913190472 \times 10^{-10}$

	χ_A (deg)
0	$3.5139351875175307 \times 10^{-1}$
1	-5.1653002708885780
2	$3.5659543086760438 \times 10^{-2}$
3	$1.0993882033998863 \times 10^{-1}$
4	$-1.9683473111405189 \times 10^{-3}$
5	$2.2537140762021708 \times 10^{-4}$
6	$-4.2286314016953239 \times 10^{-6}$
7	$-7.6386531504895290 \times 10^{-6}$
8	$1.9417452931241490 \times 10^{-7}$
9	$-6.3125020432388748 \times 10^{-8}$

	L (deg)
0	-1.3879277566931226
1	2.4481681874126174
2	$2.1797818824463402 \times 10^{-1}$
3	$-4.0268644726973394 \times 10^{-2}$
4	$-9.8004654248740430 \times 10^{-4}$
5	$2.2892669230292406 \times 10^{-5}$
6	$-1.7838993813317142 \times 10^{-5}$
7	$2.6308746248282732 \times 10^{-6}$
8	$1.4790445967263225 \times 10^{-8}$
9	$-2.4624791147965544 \times 10^{-8}$

Table 4–2, continued.

Interval 7: Central time $T_c = -4480$, covering the time span $-4520 \le T \le -4440$

	ε (deg)	p_A (deg)
0	23.575991417001520	-6277.1602608790300
1	$-4.4443190947545271 \times 10^{-1}$	55.424589412075412
2	$-4.1186000917210255 \times 10^{-2}$	$2.0816957500754808 \times 10^{-1}$
3	$7.4078571918914910 \times 10^{-3}$	$1.5881481683356100 \times 10^{-2}$
4	$5.0971845828055419 \times 10^{-4}$	$-1.4178109574625101 \times 10^{-3}$
5	$-1.9927642762038199 \times 10^{-5}$	$-1.2140559330740230 \times 10^{-4}$
6	$-2.4704916733326653 \times 10^{-6}$	$-5.1731291215420605 \times 10^{-6}$
7	$-3.9503841920634605 \times 10^{-7}$	$2.6562882110975096 \times 10^{-7}$
8	$2.3229394014961144 \times 10^{-9}$	$1.6580961300054467 \times 10^{-7}$
9	$7.6953226128109170 \times 10^{-9}$	$3.6560117523203963 \times 10^{-9}$

	ψ_A (deg)	Δ (deg)
0	-6282.5764167716350	-6276.8733139463527
1	55.188612645605684	53.585903454477304
2	1.2101778275784235	$5.2864098410829663 \times 10^{-1}$
3	$5.5601470050332444 \times 10^{-2}$	$6.6437964012072321 \times 10^{-2}$
4	$-1.5526801642309962 \times 10^{-2}$	$-1.0083785015748842 \times 10^{-3}$
5	$-2.3495912766505691 \times 10^{-3}$	$-3.8468546673082717 \times 10^{-4}$
6	$-8.9405258721106330 \times 10^{-5}$	$-5.1459676090475857 \times 10^{-5}$
7	$3.3585461013737947 \times 10^{-5}$	$-3.5857194911869403 \times 10^{-6}$
8	$6.5645068175052427 \times 10^{-6}$	$4.2856878285849280 \times 10^{-8}$
9	$3.3146592015552606 \times 10^{-7}$	$4.2013837811693309 \times 10^{-8}$

	ω_A (deg)	I (deg)
0	23.819401066229035	24.549327652372807
1	-2.5183504552212391	-1.4445791209574703
2	$-1.4108284405186655 \times 10^{-2}$	$-1.9360028452364847 \times 10^{-1}$
3	$7.2555433522037660 \times 10^{-2}$	$2.4737395112041443 \times 10^{-2}$
4	$1.3652404937671124 \times 10^{-3}$	$2.5478807880052707 \times 10^{-3}$
5	$-6.2889516661317191 \times 10^{-4}$	$-4.1061295265188175 \times 10^{-5}$
6	$-6.2401965475016310 \times 10^{-5}$	$-9.4181722493355793 \times 10^{-6}$
7	$-1.9415593147615293 \times 10^{-6}$	$-7.5279168040846323 \times 10^{-7}$
8	$6.9255727765089135 \times 10^{-7}$	$-6.6521724744919380 \times 10^{-8}$
9	$1.3886634278103713 \times 10^{-7}$	$-4.5632838819741200 \times 10^{-9}$

	χ_A (deg)	L (deg)
0	-5.9134651857237934	3.5283306290381441
1	$-1.9689628014686395 \times 10^{-1}$	1.9851876989104539
2	1.0955477359397591	$-3.6035457852484075 \times 10^{-1}$
3	$3.5431793739434752 \times 10^{-2}$	$-5.4366106717630037 \times 10^{-2}$
4	$-1.5667771366048855 \times 10^{-2}$	$7.7650097567567658 \times 10^{-5}$
5	$-2.1574014331612050 \times 10^{-3}$	$2.9396217359625386 \times 10^{-4}$
6	$-7.2973567435653390 \times 10^{-5}$	$4.2991444383768206 \times 10^{-5}$
7	$3.2812597052002370 \times 10^{-5}$	$3.7247478856801817 \times 10^{-6}$
8	$6.3490474629104658 \times 10^{-6}$	$1.5125402107412041 \times 10^{-7}$
9	$3.3042212193166131 \times 10^{-7}$	$-3.8215394555746224 \times 10^{-8}$

Table 4–2, continued.

Interval 8: Central time $T_c = -4400$, covering the time span $-4440 \leq T \leq -4360$

	ε (deg)		p_A (deg)
0	22.697692972132041	0	-6164.4352881315627
1	$-3.4072666630117481 \times 10^{-1}$	1	57.285368948536593
2	$6.8182733019134615 \times 10^{-2}$	2	$1.8742703542962976 \times 10^{-1}$
3	$7.7375798389460035 \times 10^{-3}$	3	$-2.2692323960672745 \times 10^{-2}$
4	$-6.6879798078830967 \times 10^{-4}$	4	$-2.5027673788925501 \times 10^{-3}$
5	$-6.8988003670687156 \times 10^{-5}$	5	$1.4645904151558227 \times 10^{-4}$
6	$3.8541334291483861 \times 10^{-6}$	6	$2.2040433137916666 \times 10^{-5}$
7	$5.0879485868833472 \times 10^{-7}$	7	$-7.2891287358014689 \times 10^{-7}$
8	$-3.0672784219114997 \times 10^{-8}$	8	$-1.8173412136370483 \times 10^{-7}$
9	$-2.8023631224815667 \times 10^{-9}$	9	$5.3315685730141724 \times 10^{-9}$

	ψ_A (deg)		Δ (deg)
0	-6164.6760637941491	0	-6163.8289622327309
1	61.506463348870803	1	59.669682349840066
2	$-1.7575372817223181 \times 10^{-1}$	2	$7.4674920390248098 \times 10^{-1}$
3	$-2.0819976657238286 \times 10^{-1}$	3	$-7.4514788929278267 \times 10^{-2}$
4	$1.5124017599935879 \times 10^{-2}$	4	$-1.6555224810275649 \times 10^{-2}$
5	$4.5816383127191538 \times 10^{-3}$	5	$9.1941903280141270 \times 10^{-5}$
6	$-5.2410283217040262 \times 10^{-4}$	6	$2.7194181408548075 \times 10^{-4}$
7	$-9.7166854448959236 \times 10^{-5}$	7	$2.1185320341288488 \times 10^{-5}$
8	$1.7541163952781903 \times 10^{-5}$	8	$-3.2545274788324971 \times 10^{-6}$
9	$2.0249981429043953 \times 10^{-6}$	9	$-6.8623816986620154 \times 10^{-7}$

	ω_A (deg)		I (deg)
0	20.862417914437064	0	21.419147137081985
1	$-3.0108811360843661 \times 10^{-2}$	1	-1.3044719425873434
2	$4.4970243498463276 \times 10^{-1}$	2	$2.5384135878163799 \times 10^{-1}$
3	$-2.7733044336809580 \times 10^{-2}$	3	$3.6863176740079397 \times 10^{-2}$
4	$-9.1569305176258612 \times 10^{-3}$	4	$-2.7132439918818094 \times 10^{-3}$
5	$7.8553472028722070 \times 10^{-4}$	5	$-5.0406941046704919 \times 10^{-4}$
6	$1.3596244216639089 \times 10^{-4}$	6	$-4.1097104337283639 \times 10^{-7}$
7	$-1.7503304226767023 \times 10^{-5}$	7	$6.0222394004663471 \times 10^{-6}$
8	$-2.1535432489705940 \times 10^{-6}$	8	$5.1181276843407795 \times 10^{-7}$
9	$4.5873394950695581 \times 10^{-7}$	9	$-5.3715734291807095 \times 10^{-8}$

	χ_A (deg)		L (deg)
0	$-2.6242928503981910 \times 10^{-1}$	0	3.1446027805843073
1	4.5494082968497834	1	-2.5887695739416918
2	$-3.9438759341967356 \times 10^{-1}$	2	$-5.9180376833354570 \times 10^{-1}$
3	$-1.9539162657883273 \times 10^{-1}$	3	$5.6525345602905855 \times 10^{-2}$
4	$1.8568038023571860 \times 10^{-2}$	4	$1.4421144984775873 \times 10^{-2}$
5	$4.5331472830609887 \times 10^{-3}$	5	$2.1118351573102366 \times 10^{-5}$
6	$-5.5457341756550372 \times 10^{-4}$	6	$-2.5141508497578959 \times 10^{-4}$
7	$-9.6909281027349898 \times 10^{-5}$	7	$-2.1849951140997618 \times 10^{-5}$
8	$1.7752248830608079 \times 10^{-5}$	8	$3.0746252204117707 \times 10^{-6}$
9	$2.0206138322763796 \times 10^{-6}$	9	$6.9250594902917497 \times 10^{-7}$

Table 4–2, continued.

Interval 9: Central time $T_c = -4320$, covering the time span $-4360 \leq T \leq -4280$

	ε (deg)		p_A (deg)
0	22.687423247955907	0	-6049.4847217800561
1	$3.3597155353933975 \times 10^{-1}$	1	57.356240294930533
2	$7.4811455398655705 \times 10^{-2}$	2	$-1.7353072582387725 \times 10^{-1}$
3	$-6.6333950986405548 \times 10^{-3}$	3	$-2.4974150480497263 \times 10^{-2}$
4	$-6.8892131611232367 \times 10^{-4}$	4	$2.1547955949632744 \times 10^{-3}$
5	$5.6782395875433582 \times 10^{-5}$	5	$1.3754897398655208 \times 10^{-4}$
6	$1.9113887491175400 \times 10^{-6}$	6	$-1.8407232431028744 \times 10^{-5}$
7	$-4.0108083557810620 \times 10^{-7}$	7	$-9.1390267107572022 \times 10^{-9}$
8	$1.1670309198027687 \times 10^{-8}$	8	$1.5000647054282947 \times 10^{-7}$
9	$3.1841415094271436 \times 10^{-9}$	9	$-7.0192712776814479 \times 10^{-9}$

	ψ_A (deg)		Δ (deg)
0	-6046.6199506706105	0	-6043.1357405015794
1	56.255271837143806	1	59.728817681763499
2	$-5.7587195565212657 \times 10^{-1}$	2	$-7.2706404708762197 \times 10^{-1}$
3	$8.7461434860709713 \times 10^{-2}$	3	$-7.2193130296759837 \times 10^{-2}$
4	$2.4644329553179803 \times 10^{-3}$	4	$1.6510606997581533 \times 10^{-2}$
5	$-1.4932136719985579 \times 10^{-3}$	5	$1.3590590840096368 \times 10^{-5}$
6	$1.4270260986629163 \times 10^{-4}$	6	$-2.6911536736645694 \times 10^{-4}$
7	$-7.3486960168757501 \times 10^{-6}$	7	$2.2133733629652750 \times 10^{-5}$
8	$-1.4091113713403295 \times 10^{-6}$	8	$3.0812392013382428 \times 10^{-6}$
9	$3.7818937784047098 \times 10^{-7}$	9	$-6.8713305470352292 \times 10^{-7}$

	ω_A (deg)		I (deg)
0	22.586254476049321	0	21.386697431733631
1	1.2542018256726548	1	1.2694893752295218
2	$-1.1912586287348058 \times 10^{-1}$	2	$2.5137652678177495 \times 10^{-1}$
3	$-2.7662928532105604 \times 10^{-2}$	3	$-3.6544535231411564 \times 10^{-2}$
4	$5.5197079427045825 \times 10^{-3}$	4	$-2.4460983940736166 \times 10^{-3}$
5	$3.5161563475874338 \times 10^{-5}$	5	$5.1796533204979077 \times 10^{-4}$
6	$-4.6610751206908362 \times 10^{-5}$	6	$-3.0866701431563118 \times 10^{-6}$
7	$3.5425551935362345 \times 10^{-6}$	7	$-6.0502360821062469 \times 10^{-6}$
8	$-2.5883737845722400 \times 10^{-7}$	8	$5.1775879244035979 \times 10^{-7}$
9	$-1.1984604058033744 \times 10^{-8}$	9	$5.0136502182117750 \times 10^{-8}$

	χ_A (deg)		L (deg)
0	3.1004590274184833	0	-3.0562669362870719
1	-1.1759905681987503	1	-2.5701612820923938
2	$-4.3930487193703312 \times 10^{-1}$	2	$5.8686880597664387 \times 10^{-1}$
3	$1.2021797742721103 \times 10^{-1}$	3	$5.1433535417997318 \times 10^{-2}$
4	$8.2897874551907062 \times 10^{-4}$	4	$-1.4779772806386016 \times 10^{-2}$
5	$-1.7512254276014630 \times 10^{-3}$	5	$9.3705111359924254 \times 10^{-5}$
6	$1.5770350661770795 \times 10^{-4}$	6	$2.5304195812538503 \times 10^{-4}$
7	$-6.5384206667176167 \times 10^{-6}$	7	$-2.1985224991062792 \times 10^{-5}$
8	$-1.5434791828896781 \times 10^{-6}$	8	$-2.9370143994689238 \times 10^{-6}$
9	$3.8159205051124946 \times 10^{-7}$	9	$6.7927156225642495 \times 10^{-7}$

Table 4-2, continued.

Interval 10: Central time $T_c = -4240$, covering the time span $-4280 \leq T \leq -4200$

	ε (deg)		p_A (deg)
0	23.632363123248477	0	-5936.6419031296072
1	$5.1779162612791918 \times 10^{-1}$	1	55.432176444809571
2	$-3.2491069736212345 \times 10^{-2}$	2	$-2.3981870191974584 \times 10^{-1}$
3	$-8.5143444543083857 \times 10^{-3}$	3	$1.2714124363220905 \times 10^{-2}$
4	$3.8990454377868252 \times 10^{-4}$	4	$1.7993392545660327 \times 10^{-3}$
5	$4.0998211452448053 \times 10^{-5}$	5	$-1.0437764547529339 \times 10^{-4}$
6	$-1.5250172989302688 \times 10^{-6}$	6	$-3.4737666309584845 \times 10^{-6}$
7	$-1.1246243041076886 \times 10^{-7}$	7	$1.2403154920678618 \times 10^{-7}$
8	$-1.6903028898970712 \times 10^{-8}$	8	$-2.0138762163739443 \times 10^{-9}$
9	$6.7569347109976269 \times 10^{-10}$	9	$6.6338119353697764 \times 10^{-9}$

	ψ_A (deg)		Δ (deg)
0	-5935.8931819782846	0	-5929.7907131183119
1	55.066993888595859	1	53.820494139312794
2	$1.1820938292236795 \times 10^{-1}$	2	$-5.1827874874431007 \times 10^{-1}$
3	$1.2019349062047616 \times 10^{-3}$	3	$6.1260103326706782 \times 10^{-2}$
4	$-8.7565678640467094 \times 10^{-3}$	4	$4.1503666371473195 \times 10^{-4}$
5	$1.3762671468337826 \times 10^{-4}$	5	$-3.7383473324449513 \times 10^{-4}$
6	$8.3824223904116970 \times 10^{-5}$	6	$5.3186528858918611 \times 10^{-5}$
7	$4.6010595752450675 \times 10^{-6}$	7	$-3.7587180020770429 \times 10^{-6}$
8	$-5.5783719174982977 \times 10^{-7}$	8	$-5.1252212192268308 \times 10^{-8}$
9	$-1.3613296267512310 \times 10^{-7}$	9	$4.2835280331643713 \times 10^{-8}$

	ω_A (deg)		I (deg)
0	24.004180555423175	0	24.489405405946519
1	$2.4289636645648181 \times 10^{-1}$	1	1.4749353775995546
2	$-3.6195340149488204 \times 10^{-2}$	2	$-1.6943716270229383 \times 10^{-1}$
3	$2.5947035810221554 \times 10^{-2}$	3	$-2.1636559686828944 \times 10^{-2}$
4	$-4.3249199243628933 \times 10^{-4}$	4	$2.4178729455357789 \times 10^{-3}$
5	$-4.6897238520701574 \times 10^{-4}$	5	$-5.6378969222265510 \times 10^{-6}$
6	$3.6628612394348481 \times 10^{-6}$	6	$-9.6491012595617987 \times 10^{-6}$
7	$3.0951836314483639 \times 10^{-6}$	7	$1.1280222756066770 \times 10^{-6}$
8	$1.6438312038053508 \times 10^{-7}$	8	$-6.2326387284237891 \times 10^{-8}$
9	$-1.5344091463667370 \times 10^{-8}$	9	$2.8347231229402212 \times 10^{-10}$

	χ_A (deg)		L (deg)
0	$8.1779379376014537 \times 10^{-1}$	0	-3.6231645500466370
1	$-3.9613840241574566 \times 10^{-1}$	1	1.7462262715121550
2	$3.9053917857500309 \times 10^{-1}$	2	$3.1398118676332339 \times 10^{-1}$
3	$-1.1923679408270016 \times 10^{-2}$	3	$-5.2134504218061958 \times 10^{-2}$
4	$-1.1618028106713097 \times 10^{-2}$	4	$1.0547023714723996 \times 10^{-3}$
5	$2.6029626332642981 \times 10^{-4}$	5	$3.0342500305836451 \times 10^{-4}$
6	$9.7356083350866371 \times 10^{-5}$	6	$-5.5719386661957746 \times 10^{-5}$
7	$4.3044404544768121 \times 10^{-6}$	7	$3.6600496941996057 \times 10^{-6}$
8	$-6.0220701620936336 \times 10^{-7}$	8	$5.2593068104079316 \times 10^{-8}$
9	$-1.4200164220659403 \times 10^{-7}$	9	$-3.4362727215312610 \times 10^{-8}$

Table 4–2, continued.

Interval 11: Central time $T_c = -4160$, covering the time span $-4200 \leq T \leq -4120$

	ε (deg)		p_A (deg)
0	24.188074110017610	0	-5826.9838234818113
1	$-2.2388885312299751 \times 10^{-3}$	1	54.436548806811951
2	$-8.0331221711979867 \times 10^{-2}$	2	$8.4447632660682504 \times 10^{-3}$
3	$9.9312497160090454 \times 10^{-4}$	3	$2.2857095774336630 \times 10^{-2}$
4	$5.6716550851130969 \times 10^{-4}$	4	$-4.3290680499391995 \times 10^{-4}$
5	$-2.4178518500840307 \times 10^{-5}$	5	$-7.1076439661189837 \times 10^{-5}$
6	$-1.0555376208513331 \times 10^{-6}$	6	$6.8633428942651327 \times 10^{-6}$
7	$3.6518342386925326 \times 10^{-7}$	7	$-2.9174291364819168 \times 10^{-8}$
8	$4.4828349072704748 \times 10^{-9}$	8	$-8.9210146002165026 \times 10^{-8}$
9	$-4.5799807717987222 \times 10^{-9}$	9	$-2.3297080957509837 \times 10^{-9}$

	ψ_A (deg)		Δ (deg)
0	-5825.8849591750585	0	-5824.0776102657728
1	54.566929635388225	1	52.449183119321760
2	$-2.9678869900713397 \times 10^{-1}$	2	$1.4727636656411795 \times 10^{-1}$
3	$-2.7238670383973184 \times 10^{-2}$	3	$4.6304727175863810 \times 10^{-2}$
4	$6.7252805074846387 \times 10^{-3}$	4	$-1.3668660437655432 \times 10^{-3}$
5	$5.0080060838101628 \times 10^{-4}$	5	$-5.1240260094497479 \times 10^{-5}$
6	$-9.5214355255432375 \times 10^{-5}$	6	$-1.0351831632616383 \times 10^{-5}$
7	$-1.8604530300568485 \times 10^{-7}$	7	$-6.2425056305669779 \times 10^{-7}$
8	$1.3439648405297290 \times 10^{-6}$	8	$9.8655648063753080 \times 10^{-8}$
9	$-6.1819624394926964 \times 10^{-8}$	9	$2.6769990311074472 \times 10^{-9}$

	ω_A (deg)		I (deg)
0	24.744163262075083	0	25.692030257261059
1	$5.4811452992516069 \times 10^{-1}$	1	$-3.1382864122822942 \times 10^{-1}$
2	$1.9991651875915786 \times 10^{-2}$	2	$-2.2116763213811225 \times 10^{-1}$
3	$-2.3003483867130398 \times 10^{-2}$	3	$1.0566102134193483 \times 10^{-2}$
4	$-2.8035658630782932 \times 10^{-3}$	4	$1.4886151229007740 \times 10^{-3}$
5	$3.3354552732051438 \times 10^{-4}$	5	$-5.8287361717064141 \times 10^{-5}$
6	$2.3884734355817250 \times 10^{-5}$	6	$-1.6697488651113132 \times 10^{-6}$
7	$-3.4876562866598556 \times 10^{-6}$	7	$-3.3039983256881513 \times 10^{-7}$
8	$-3.0574035303092194 \times 10^{-8}$	8	$-1.0183190818670982 \times 10^{-8}$
9	$3.8072530198992210 \times 10^{-8}$	9	$4.0857125231007643 \times 10^{-9}$

	χ_A (deg)		L (deg)
0	1.2074453293622207	0	$7.0963505057338877 \times 10^{-1}$
1	$1.4546889968205213 \times 10^{-1}$	1	2.1959149909136972
2	$-3.3553896788797673 \times 10^{-1}$	2	$-1.5581202733421077 \times 10^{-1}$
3	$-5.5513111227716779 \times 10^{-2}$	3	$-2.7087139824101427 \times 10^{-2}$
4	$7.8284753025566691 \times 10^{-3}$	4	$1.2042667825413408 \times 10^{-3}$
5	$6.5901694407555846 \times 10^{-4}$	5	$4.8168706622589510 \times 10^{-7}$
6	$-1.0816335606868752 \times 10^{-4}$	6	$1.6276039625264375 \times 10^{-5}$
7	$-6.8276307358790051 \times 10^{-7}$	7	$5.4755884147857844 \times 10^{-7}$
8	$1.4664799924276537 \times 10^{-6}$	8	$-2.0074926414527761 \times 10^{-7}$
9	$-5.7717505939828266 \times 10^{-8}$	9	$-5.3021063904519547 \times 10^{-9}$

Table 4–2, continued.

Interval 12: Central time $T_c = -4080$, covering the time span $-4120 \leq T \leq -4040$

	ε (deg)
0	23.665335269640449
1	$-4.7635775873615469 \times 10^{-1}$
2	$-2.7982115256536419 \times 10^{-2}$
3	$6.9073445424447969 \times 10^{-3}$
4	$2.1766500120049183 \times 10^{-4}$
5	$-2.0828380194764327 \times 10^{-5}$
6	$-2.9206920326306598 \times 10^{-6}$
7	$-2.3454348759156991 \times 10^{-7}$
8	$5.1125789245625978 \times 10^{-8}$
9	$5.6515930956443743 \times 10^{-9}$

	p_A (deg)
0	-5717.3046235228462
1	55.413135363376891
2	$2.1153962301359059 \times 10^{-1}$
3	$8.9738637935491020 \times 10^{-3}$
4	$-1.3472892820287205 \times 10^{-3}$
5	$-7.2996160160420141 \times 10^{-5}$
6	$-1.7254402061296726 \times 10^{-6}$
7	$1.0419340914050115 \times 10^{-6}$
8	$1.1896878955889185 \times 10^{-7}$
9	$-1.1955698117337218 \times 10^{-8}$

	ψ_A (deg)
0	-5718.7561271227644
1	52.866987661999135
2	$6.2737197238961619 \times 10^{-2}$
3	$7.9314343435769997 \times 10^{-2}$
4	$4.5304600557535898 \times 10^{-3}$
5	$-2.3298604965192697 \times 10^{-4}$
6	$8.3750127037761767 \times 10^{-8}$
7	$-4.1533344071689039 \times 10^{-6}$
8	$-7.0112625657943057 \times 10^{-7}$
9	$-2.6958170760685720 \times 10^{-9}$

	Δ (deg)
0	-5716.6105789810644
1	55.313243913564842
2	$4.9584833410203825 \times 10^{-1}$
3	$4.2824894653964639 \times 10^{-3}$
4	$-4.1309025360763195 \times 10^{-3}$
5	$-9.4052822793108721 \times 10^{-5}$
6	$2.0838255373860971 \times 10^{-5}$
7	$1.8842874594608660 \times 10^{-6}$
8	$-1.0586175380855125 \times 10^{-7}$
9	$-1.7835138506577112 \times 10^{-8}$

	ω_A (deg)
0	24.965396766748864
1	$-5.8639550210649449 \times 10^{-1}$
2	$-2.7777767667791636 \times 10^{-1}$
3	$-9.0194976708562742 \times 10^{-3}$
4	$4.0101058771914215 \times 10^{-3}$
5	$2.1466411548748902 \times 10^{-4}$
6	$-1.3083310104629373 \times 10^{-5}$
7	$5.1773172260478201 \times 10^{-8}$
8	$-6.5662725398576937 \times 10^{-8}$
9	$-1.3385835761653602 \times 10^{-8}$

	I (deg)
0	23.907478306952636
1	-1.2860665559717100
2	$-3.3933508583659058 \times 10^{-3}$
3	$2.0780201807736654 \times 10^{-2}$
4	$-4.6828847667794971 \times 10^{-4}$
5	$-1.2393810138822825 \times 10^{-4}$
6	$2.1618466858100750 \times 10^{-6}$
7	$7.2118797753681785 \times 10^{-7}$
8	$1.0305806548353579 \times 10^{-9}$
9	$-5.9716767854226117 \times 10^{-9}$

	χ_A (deg)
0	-1.5867358399053381
1	-2.7852072669664335
2	$-1.5572864729451383 \times 10^{-1}$
3	$7.9102896781185615 \times 10^{-2}$
4	$6.3398957590727643 \times 10^{-3}$
5	$-2.6156348263663715 \times 10^{-4}$
6	$-3.0321741644564456 \times 10^{-6}$
7	$-4.7035541415893986 \times 10^{-6}$
8	$-7.9947715699352167 \times 10^{-7}$
9	$9.3113696375314742 \times 10^{-9}$

	L (deg)
0	3.1293133430583314
1	$8.9925367974669585 \times 10^{-2}$
2	$-3.1202117249052663 \times 10^{-1}$
3	$6.5074592565518095 \times 10^{-3}$
4	$3.0297095967315395 \times 10^{-3}$
5	$-6.3055272429550715 \times 10^{-6}$
6	$-2.3939441533141978 \times 10^{-5}$
7	$-5.8509041266799083 \times 10^{-7}$
8	$2.3941180464757204 \times 10^{-7}$
9	$3.1757643773993105 \times 10^{-9}$

Table 4–2, continued.

Interval 13: Central time $T_c = -4000$, covering the time span $-4040 \leq T \leq -3960$

	ε (deg)		p_A (deg)
0	22.758997037666096	0	−5604.7524988708399
1	$-3.6158342279207431 \times 10^{-1}$	1	57.098857405788612
2	$5.2594707071093653 \times 10^{-2}$	2	$1.6384116552205424 \times 10^{-1}$
3	$4.6310040922682836 \times 10^{-3}$	3	$-1.6302368249121253 \times 10^{-2}$
4	$-4.5895570828094842 \times 10^{-4}$	4	$-1.0705074121138985 \times 10^{-3}$
5	$-6.9238082130767509 \times 10^{-6}$	5	$1.1354997896269458 \times 10^{-4}$
6	$4.1351108244444807 \times 10^{-6}$	6	$-2.5239854966365866 \times 10^{-7}$
7	$-2.8483688865673123 \times 10^{-7}$	7	$-1.4499763250921851 \times 10^{-6}$
8	$-6.7031824166247168 \times 10^{-8}$	8	$9.2728418285065761 \times 10^{-8}$
9	$6.0997467081154178 \times 10^{-9}$	9	$2.4655330931751926 \times 10^{-8}$

	ψ_A (deg)		Δ (deg)
0	−5609.0880258026554	0	−5602.6035244144871
1	57.693708370115458	1	58.438954037058593
2	1.0931421583208017	2	$1.9826465534346218 \times 10^{-1}$
3	$3.8266526251380357 \times 10^{-2}$	3	$-4.3309330836919764 \times 10^{-2}$
4	$-1.8562295721237562 \times 10^{-2}$	4	$-1.2239259694556435 \times 10^{-4}$
5	$-2.7041567645348646 \times 10^{-3}$	5	$3.7587539620312959 \times 10^{-4}$
6	$-3.7794820081890900 \times 10^{-5}$	6	$-6.7618758200325464 \times 10^{-6}$
7	$4.8754375445419459 \times 10^{-5}$	7	$-2.8821871935720668 \times 10^{-6}$
8	$8.1726779415775550 \times 10^{-6}$	8	$2.7683890691952949 \times 10^{-8}$
9	$1.6331502429027435 \times 10^{-7}$	9	$1.5081718496248322 \times 10^{-8}$

	ω_A (deg)		I (deg)
0	22.123082290633411	0	21.912220229962857
1	−1.9689740062085110	1	$-5.8566709411102821 \times 10^{-1}$
2	$6.4795755925118305 \times 10^{-2}$	2	$1.4308044139735211 \times 10^{-1}$
3	$6.3424985962367203 \times 10^{-2}$	3	$1.3696084180638732 \times 10^{-3}$
4	$2.3983954318444527 \times 10^{-3}$	4	$-1.3411087936457335 \times 10^{-3}$
5	$-6.7031474528675811 \times 10^{-4}$	5	$5.5362242474251710 \times 10^{-5}$
6	$-8.2625773687586456 \times 10^{-5}$	6	$7.2842098212396369 \times 10^{-6}$
7	$-1.7934497780501445 \times 10^{-6}$	7	$-2.9782315135622503 \times 10^{-7}$
8	$9.8605776840100457 \times 10^{-7}$	8	$-1.5760669500190704 \times 10^{-8}$
9	$1.8101488106213375 \times 10^{-7}$	9	$-1.1564406798118929 \times 10^{-9}$

	χ_A (deg)		L (deg)
0	−4.6931399879646926	0	1.5214561025296204
1	$6.7933361119707421 \times 10^{-1}$	1	−1.4591634155721001
2	$9.9998582100782432 \times 10^{-1}$	2	$-3.4271557152182961 \times 10^{-2}$
3	$5.3949449187702951 \times 10^{-2}$	3	$2.8792569755276198 \times 10^{-2}$
4	$-1.8818746744487038 \times 10^{-2}$	4	$-1.1041197059695190 \times 10^{-3}$
5	$-2.8254325495694644 \times 10^{-3}$	5	$-2.6685782484336021 \times 10^{-4}$
6	$-2.6043943699543542 \times 10^{-5}$	6	$7.2408385135939408 \times 10^{-6}$
7	$5.0339491165859925 \times 10^{-5}$	7	$1.3156938821039271 \times 10^{-6}$
8	$8.0111329180468596 \times 10^{-6}$	8	$7.7423368068690992 \times 10^{-8}$
9	$1.3716424669169754 \times 10^{-7}$	9	$1.1956996879180691 \times 10^{-8}$

Table 4–2, continued.

Interval 14: Central time $T_c = -3920$, covering the time span $-3960 \leq T \leq -3880$

	ε (deg)		p_A (deg)
0	22.547813727833042	0	−5489.9750460890289
1	$1.6190830673331016 \times 10^{-1}$	1	57.477092077379750
2	$6.6558199025034038 \times 10^{-2}$	2	$-7.5226852583642695 \times 10^{-2}$
3	$-2.1241196121248115 \times 10^{-3}$	3	$-1.9904074573796089 \times 10^{-2}$
4	$-4.1414341133274284 \times 10^{-4}$	4	$4.7934214768266299 \times 10^{-4}$
5	$-2.8119292317357847 \times 10^{-6}$	5	$8.3461672152430346 \times 10^{-5}$
6	$1.1759735721882882 \times 10^{-6}$	6	$3.5410521804124448 \times 10^{-6}$
7	$3.3472679409709165 \times 10^{-7}$	7	$-4.3785592456580759 \times 10^{-7}$
8	$-1.8410967508920335 \times 10^{-8}$	8	$-1.1559710653572349 \times 10^{-7}$
9	$-4.7721481331761114 \times 10^{-9}$	9	$9.5704933236703371 \times 10^{-9}$

	ψ_A (deg)		Δ (deg)
0	−5488.1400141665871	0	−5485.5339333406819
1	61.782258014562961	1	58.320239326186223
2	$-5.8343097197559804 \times 10^{-1}$	2	$-1.7050046296427783 \times 10^{-1}$
3	$-2.0404182413402943 \times 10^{-1}$	3	$-1.3349037607317593 \times 10^{-2}$
4	$2.2211791927519208 \times 10^{-2}$	4	$1.7578647096686515 \times 10^{-3}$
5	$4.0525367225744933 \times 10^{-3}$	5	$-2.0893983790734107 \times 10^{-4}$
6	$-7.4416699884824399 \times 10^{-4}$	6	$-1.1056003680298186 \times 10^{-5}$
7	$-6.6962540137867586 \times 10^{-5}$	7	$2.8998323628389556 \times 10^{-6}$
8	$2.3990505060277285 \times 10^{-5}$	8	$5.0403184753528353 \times 10^{-8}$
9	$6.1716495168636546 \times 10^{-7}$	9	$-2.1690156957327074 \times 10^{-8}$

	ω_A (deg)		I (deg)
0	20.652140062514723	0	21.762090456525576
1	$8.4866612790047313 \times 10^{-1}$	1	$3.8539743972189858 \times 10^{-1}$
2	$4.5635602181136721 \times 10^{-1}$	2	$8.6551561861274887 \times 10^{-2}$
3	$-3.4043802216282232 \times 10^{-2}$	3	$-6.1503517765093712 \times 10^{-3}$
4	$-9.2081548380548211 \times 10^{-3}$	4	$3.8436370831379663 \times 10^{-4}$
5	$8.3690155581348572 \times 10^{-4}$	5	$4.5746742754738562 \times 10^{-5}$
6	$1.2609591293174543 \times 10^{-4}$	6	$-9.5999235939922499 \times 10^{-6}$
7	$-2.1151190113027896 \times 10^{-5}$	7	$-3.9344873051678688 \times 10^{-7}$
8	$-1.5289557655772935 \times 10^{-6}$	8	$7.1373057917624544 \times 10^{-8}$
9	$5.6554718335066992 \times 10^{-7}$	9	$3.7195821959635946 \times 10^{-9}$

	χ_A (deg)		L (deg)
0	1.9810118887704784	0	$-9.7118024177335244 \times 10^{-1}$
1	4.6397031988222159	1	$-9.1970981995198577 \times 10^{-1}$
2	$-5.3519288238273040 \times 10^{-1}$	2	$1.0140649868594021 \times 10^{-1}$
3	$-1.9492638892163796 \times 10^{-1}$	3	$-7.1541654672802420 \times 10^{-3}$
4	$2.2226209393646027 \times 10^{-2}$	4	$-1.3470798017179746 \times 10^{-3}$
5	$4.0898360009488905 \times 10^{-3}$	5	$3.1081950153990267 \times 10^{-4}$
6	$-7.5210310588791290 \times 10^{-4}$	6	$1.5774297269866934 \times 10^{-5}$
7	$-6.7166768657127049 \times 10^{-5}$	7	$-3.4674320121763785 \times 10^{-6}$
8	$2.4132952858232196 \times 10^{-5}$	8	$-1.8440477192128804 \times 10^{-7}$
9	$6.0851701120876462 \times 10^{-7}$	9	$3.2410544693793568 \times 10^{-8}$

Table 4–2, continued.

Interval 15: Central time $T_c = -3840$, covering the time span $-3880 \leq T \leq -3800$

	ε (deg)		p_A (deg)
0	23.252168164403464	0	-5376.2052544578061
1	$4.9319914005891942 \times 10^{-1}$	1	56.173008889155154
2	$8.3194192537968069 \times 10^{-3}$	2	$-2.1680426177652113 \times 10^{-1}$
3	$-6.4872808483335196 \times 10^{-3}$	3	$-1.0919901648163385 \times 10^{-3}$
4	$-4.5028808456972132 \times 10^{-6}$	4	$1.4260194667854052 \times 10^{-3}$
5	$2.9119609734562057 \times 10^{-5}$	5	$-3.7326883991273942 \times 10^{-5}$
6	$-1.7229960128749831 \times 10^{-6}$	6	$-6.1813895368227151 \times 10^{-6}$
7	$-2.4634374727177470 \times 10^{-7}$	7	$7.5325945091862501 \times 10^{-7}$
8	$2.3995013077180114 \times 10^{-8}$	8	$6.2374653738944919 \times 10^{-8}$
9	$3.1428951157101989 \times 10^{-9}$	9	$-6.6793224155027121 \times 10^{-9}$

	ψ_A (deg)		Δ (deg)
0	-5372.0683183022473	0	-5370.6203064990798
1	54.207694244703291	1	56.505048573979883
2	$-7.4505110882558290 \times 10^{-1}$	2	$-2.8040346968903457 \times 10^{-1}$
3	$1.0187380368092461 \times 10^{-1}$	3	$-9.8968684005017462 \times 10^{-3}$
4	$1.9067271637980558 \times 10^{-3}$	4	$-2.0032587292737541 \times 10^{-5}$
5	$-1.1123839982196282 \times 10^{-3}$	5	$1.3643737983897666 \times 10^{-4}$
6	$1.5640544650657373 \times 10^{-4}$	6	$1.3533348971241122 \times 10^{-5}$
7	$-1.1847425470160705 \times 10^{-5}$	7	$-2.0047195322908191 \times 10^{-6}$
8	$-7.5487481964285973 \times 10^{-7}$	8	$-9.2027890192599711 \times 10^{-8}$
9	$2.9105451169909522 \times 10^{-7}$	9	$1.9378867874179440 \times 10^{-8}$

	ω_A (deg)		I (deg)
0	23.924341674561594	0	23.060688139431997
1	1.8472344045996831	1	$8.7693762173648369 \times 10^{-1}$
2	$-2.0607658052311381 \times 10^{-1}$	2	$4.0953587884171553 \times 10^{-2}$
3	$-3.8955088158591123 \times 10^{-2}$	3	$-4.0640560472184671 \times 10^{-3}$
4	$4.9983693743467218 \times 10^{-3}$	4	$-5.3200435382974450 \times 10^{-4}$
5	$9.7423937926828782 \times 10^{-5}$	5	$-6.7993270289123260 \times 10^{-5}$
6	$-3.2000597667629211 \times 10^{-5}$	6	$5.8804970612369504 \times 10^{-6}$
7	$3.2866094745194275 \times 10^{-6}$	7	$6.7439110404390921 \times 10^{-7}$
8	$-3.0056721092924595 \times 10^{-7}$	8	$-6.2831664642526352 \times 10^{-8}$
9	$-2.7828212241685378 \times 10^{-9}$	9	$-4.6859685946287246 \times 10^{-9}$

	χ_A (deg)		L (deg)
0	4.5047511479213611	0	-2.2355339193789030
1	-2.1062018714104154	1	$-3.7762444624180579 \times 10^{-1}$
2	$-5.8805491558283252 \times 10^{-1}$	2	$6.8310012690988653 \times 10^{-2}$
3	$1.0957370384917981 \times 10^{-1}$	3	$9.8378039408046716 \times 10^{-3}$
4	$1.4679686525110959 \times 10^{-3}$	4	$1.6157934212589002 \times 10^{-3}$
5	$-1.1587546185671673 \times 10^{-3}$	5	$-1.8430049490489077 \times 10^{-4}$
6	$1.5515072802394319 \times 10^{-4}$	6	$-2.1996237561695501 \times 10^{-5}$
7	$-1.2143402253606865 \times 10^{-5}$	7	$2.8677776587416440 \times 10^{-6}$
8	$-7.8755333229812034 \times 10^{-7}$	8	$1.7362906893552822 \times 10^{-7}$
9	$2.9646954243429672 \times 10^{-7}$	9	$-2.6921075161938768 \times 10^{-8}$

Table 4–2, continued.

Interval 16: Central time $T_c = -3760$, covering the time span $-3800 \le T \le -3720$

	ε (deg)		p_A (deg)
0	24.075897351724677	0	-5265.4078640745818
1	$2.7402691913864281 \times 10^{-1}$	1	54.703955651484701
2	$-5.9169658448544950 \times 10^{-2}$	2	$-1.2260417724100163 \times 10^{-1}$
3	$-4.3156844539686810 \times 10^{-3}$	3	$1.5158164627537252 \times 10^{-2}$
4	$2.1999722334628734 \times 10^{-4}$	4	$8.3045538302499456 \times 10^{-4}$
5	$1.7049333420090973 \times 10^{-5}$	5	$7.2350495515960511 \times 10^{-6}$
6	$2.7978238246163851 \times 10^{-6}$	6	$-6.3000791302275730 \times 10^{-7}$
7	$3.4945771185753901 \times 10^{-8}$	7	$-1.0048584660852780 \times 10^{-6}$
8	$-5.3119890292100926 \times 10^{-8}$	8	$-1.4535894312212395 \times 10^{-8}$
9	$-1.1553365133335358 \times 10^{-9}$	9	$1.3157370196997198 \times 10^{-8}$

	ψ_A (deg)		Δ (deg)
0	-5265.9888879609676	0	-5260.0804103446634
1	52.746605262074289	1	54.012060692296632
2	$2.9225853520134978 \times 10^{-1}$	2	$-3.0318601992966561 \times 10^{-1}$
3	$5.3759385464598557 \times 10^{-2}$	3	$1.1926990723002147 \times 10^{-2}$
4	$-6.2536601021963119 \times 10^{-3}$	4	$2.1594633987640196 \times 10^{-3}$
5	$-4.0279705101552546 \times 10^{-4}$	5	$-1.7288924612178538 \times 10^{-6}$
6	$-1.4646225027926505 \times 10^{-5}$	6	$-6.4802880043820513 \times 10^{-6}$
7	$4.1243187912165425 \times 10^{-6}$	7	$1.0906746921707353 \times 10^{-6}$
8	$7.8542485932368795 \times 10^{-7}$	8	$1.0664158420803538 \times 10^{-8}$
9	$3.8472430770063570 \times 10^{-9}$	9	$-1.3203427536624575 \times 10^{-8}$

	ω_A (deg)		I (deg)
0	25.430874510643995	0	24.857092137616788
1	$-3.4673910045047810 \times 10^{-1}$	1	$8.2432905112354858 \times 10^{-1}$
2	$-2.1840522886260777 \times 10^{-1}$	2	$-7.2463816685978517 \times 10^{-2}$
3	$3.0298698999381762 \times 10^{-2}$	3	$-1.3910559156941199 \times 10^{-2}$
4	$2.3884105056360984 \times 10^{-3}$	4	$-2.1893181827219755 \times 10^{-4}$
5	$-3.3180902165093825 \times 10^{-4}$	5	$7.0181606458059708 \times 10^{-5}$
6	$-1.8775265066924701 \times 10^{-5}$	6	$7.8273487291816245 \times 10^{-7}$
7	$6.7756000753228209 \times 10^{-8}$	7	$-2.3590337629091788 \times 10^{-7}$
8	$1.4321726887497567 \times 10^{-7}$	8	$4.9332572909417223 \times 10^{-8}$
9	$1.6502351386609702 \times 10^{-8}$	9	$7.5844691484527477 \times 10^{-10}$

	χ_A (deg)		L (deg)
0	$-6.3974880333238673 \times 10^{-1}$	0	-1.9775410701998159
1	-2.1539026139668361	1	$7.5241307633026586 \times 10^{-1}$
2	$4.5765922389586301 \times 10^{-1}$	2	$2.0182670485036325 \times 10^{-1}$
3	$4.3570458092243972 \times 10^{-2}$	3	$3.9492570458603579 \times 10^{-3}$
4	$-8.1769030301164138 \times 10^{-3}$	4	$-1.5334560928325652 \times 10^{-3}$
5	$-4.6310489779278414 \times 10^{-4}$	5	$-3.2658624668311807 \times 10^{-6}$
6	$-4.2490090998306602 \times 10^{-6}$	6	$6.7376119574550991 \times 10^{-6}$
7	$5.5001545038348684 \times 10^{-6}$	7	$-2.1729822972982870 \times 10^{-6}$
8	$7.5420319942717326 \times 10^{-7}$	8	$-3.0704484334821426 \times 10^{-8}$
9	$-1.1707876876491186 \times 10^{-8}$	9	$2.8748296524170069 \times 10^{-8}$

Table 4–2, continued.

Interval 17: Central time $T_c = -3680$, covering the time span $-3720 \leq T \leq -3640$

	ε (deg)
0	24.055074749197618
1	$-3.0667483895308663 \times 10^{-1}$
2	$-7.2366569274078548 \times 10^{-2}$
3	$3.3601326141441370 \times 10^{-3}$
4	$6.5253300930054469 \times 10^{-4}$
5	$-7.1736907903639456 \times 10^{-6}$
6	$-3.6887028016596117 \times 10^{-6}$
7	$1.9339664720229003 \times 10^{-7}$
8	$3.8205323255297718 \times 10^{-8}$
9	$-3.9434936358445072 \times 10^{-9}$

	p_A (deg)
0	-5156.2634383744901
1	54.646657038105566
2	$1.2239769822291725 \times 10^{-1}$
3	$2.2537359786383646 \times 10^{-2}$
4	$-4.0424548017308571 \times 10^{-4}$
5	$-1.2108174080841341 \times 10^{-4}$
6	$5.9985729530658260 \times 10^{-7}$
7	$6.8901043081758279 \times 10^{-7}$
8	$-6.3587898580451777 \times 10^{-8}$
9	$-9.1165529324939441 \times 10^{-9}$

	ψ_A (deg)
0	-5157.5931472034811
1	55.563591514321240
2	$1.9550210357568748 \times 10^{-1}$
3	$-6.1470898224740814 \times 10^{-2}$
4	$-1.9513649679911814 \times 10^{-3}$
5	$1.2740788797305728 \times 10^{-3}$
6	$4.4768268318134878 \times 10^{-5}$
7	$-1.6515246521310561 \times 10^{-5}$
8	$-8.3723625838727006 \times 10^{-7}$
9	$2.5444372901746231 \times 10^{-7}$

	Δ (deg)
0	-5153.6249566936453
1	52.729987202107949
2	$4.0610186236516162 \times 10^{-2}$
3	$4.5237296585530544 \times 10^{-2}$
4	$2.1284843909076981 \times 10^{-3}$
5	$1.1244178725137972 \times 10^{-5}$
6	$-3.6339419528003618 \times 10^{-6}$
7	$-1.1424748483366568 \times 10^{-6}$
8	$-8.6063137931184448 \times 10^{-8}$
9	$-4.2006769609109093 \times 10^{-9}$

	ω_A (deg)
0	24.201931010568726
1	$-5.9538320382643545 \times 10^{-1}$
2	$1.1056361711352099 \times 10^{-1}$
3	$5.1839203114768082 \times 10^{-3}$
4	$-4.7273415703648458 \times 10^{-3}$
5	$-4.6940652881071560 \times 10^{-5}$
6	$5.4987441188269108 \times 10^{-5}$
7	$1.0463146083790487 \times 10^{-6}$
8	$-5.3439409056499416 \times 10^{-7}$
9	$-1.7012618546545962 \times 10^{-8}$

	I (deg)
0	25.429291666073592
1	$-3.7016999986539426 \times 10^{-1}$
2	$-2.1097711446615192 \times 10^{-1}$
3	$-5.0750291449219079 \times 10^{-3}$
4	$1.4429187569044348 \times 10^{-3}$
5	$1.0559022910199410 \times 10^{-4}$
6	$1.4682908279314303 \times 10^{-6}$
7	$-3.2185319135724213 \times 10^{-7}$
8	$-3.8132221509367381 \times 10^{-8}$
9	$1.5492376484026766 \times 10^{-9}$

	χ_A (deg)
0	-1.4587747839234495
1	1.0097742016409899
2	$7.8275270484616298 \times 10^{-2}$
3	$-9.2160568746446573 \times 10^{-2}$
4	$-1.4895659834228911 \times 10^{-3}$
5	$1.5171383734319976 \times 10^{-3}$
6	$4.3151631732866323 \times 10^{-5}$
7	$-1.7975979914599792 \times 10^{-5}$
8	$-7.6004234030326403 \times 10^{-7}$
9	$2.6647737421086075 \times 10^{-7}$

	L (deg)
0	$9.8163048615518096 \times 10^{-1}$
1	2.1128618238317396
2	$8.6695097469764620 \times 10^{-2}$
3	$-2.6322863725813514 \times 10^{-2}$
4	$-2.7922799551981461 \times 10^{-3}$
5	$-1.1624240982823118 \times 10^{-4}$
6	$7.1389817600269204 \times 10^{-6}$
7	$1.9202700813001724 \times 10^{-6}$
8	$-2.9887094174960553 \times 10^{-9}$
9	$-7.0482276414645633 \times 10^{-9}$

Table 4–2, continued.

Interval 18: Central time $T_c = -3600$, covering the time span $-3640 \leq T \leq -3560$

	ε (deg)		p_A (deg)
0	23.096173657349682	0	-5045.3229974026891
1	$-5.7710693496178751 \times 10^{-1}$	1	56.430184084151684
2	$1.6694162784419621 \times 10^{-2}$	2	$2.8221863985748370 \times 10^{-1}$
3	$1.0079545155704607 \times 10^{-2}$	3	$-9.6067230562627478 \times 10^{-4}$
4	$9.8135725703059162 \times 10^{-5}$	4	$-2.4697294699192700 \times 10^{-3}$
5	$-4.5019057095199284 \times 10^{-5}$	5	$-1.0034146914341003 \times 10^{-4}$
6	$-3.3120859937768781 \times 10^{-6}$	6	$2.2491430296381862 \times 10^{-6}$
7	$-2.9445999910700714 \times 10^{-7}$	7	$1.2612069648500524 \times 10^{-6}$
8	$3.2737343663361597 \times 10^{-8}$	8	$1.7964320035338595 \times 10^{-7}$
9	$7.9755486138475043 \times 10^{-9}$	9	$-5.6663500592798135 \times 10^{-9}$

	ψ_A (deg)		Δ (deg)
0	-5046.5091365040947	0	-5045.7723040467225
1	55.292826785661690	1	55.705678572127283
2	$-5.5706965955324603 \times 10^{-2}$	2	$7.3100271299943093 \times 10^{-1}$
3	$4.7882376883764818 \times 10^{-2}$	3	$5.7340405781598653 \times 10^{-2}$
4	$9.0050723527520542 \times 10^{-3}$	4	$-3.5103307771715022 \times 10^{-3}$
5	$-4.1984363689149005 \times 10^{-4}$	5	$-9.2868838453285584 \times 10^{-4}$
6	$-4.1602272985925939 \times 10^{-5}$	6	$-8.7392303834004083 \times 10^{-5}$
7	$3.8443623681458046 \times 10^{-6}$	7	$-1.4206353231011521 \times 10^{-6}$
8	$-1.0109140822105547 \times 10^{-6}$	8	$9.4483311901133056 \times 10^{-7}$
9	$-7.4487484495297330 \times 10^{-8}$	9	$1.6596634575860142 \times 10^{-7}$

	ω_A (deg)		I (deg)
0	23.390523635931683	0	23.184934396601141
1	$-4.2240671703155480 \times 10^{-1}$	1	-1.7593605017547559
2	$-1.2107797396877662 \times 10^{-1}$	2	$-7.0533905494068518 \times 10^{-2}$
3	$-2.3432747679761945 \times 10^{-2}$	3	$3.1819771530038821 \times 10^{-2}$
4	$2.7578075964245547 \times 10^{-3}$	4	$2.6213644843666922 \times 10^{-3}$
5	$4.4958801730601557 \times 10^{-4}$	5	$-7.7545301297596407 \times 10^{-5}$
6	$-1.8027007868365178 \times 10^{-5}$	6	$-2.2672638786848002 \times 10^{-5}$
7	$-1.0630965953188470 \times 10^{-6}$	7	$-1.7670447670504373 \times 10^{-6}$
8	$1.3710178662912290 \times 10^{-7}$	8	$-7.6592563500343384 \times 10^{-8}$
9	$-2.4849274323876253 \times 10^{-8}$	9	$7.4930870698549096 \times 10^{-9}$

	χ_A (deg)		L (deg)
0	-1.2885377933581205	0	4.3432677020823182
1	-1.2320708747932061	1	$7.6122725603712110 \times 10^{-1}$
2	$-3.6505657922194153 \times 10^{-1}$	2	$-4.9146988505216405 \times 10^{-1}$
3	$5.4103053920540514 \times 10^{-2}$	3	$-6.0659333255596839 \times 10^{-2}$
4	$1.2468617774972700 \times 10^{-2}$	4	$1.5090828022091125 \times 10^{-3}$
5	$-3.8159191384839866 \times 10^{-4}$	5	$8.5739690537917376 \times 10^{-4}$
6	$-5.3374849436169644 \times 10^{-5}$	6	$8.6616499634554274 \times 10^{-5}$
7	$2.8252041263538908 \times 10^{-6}$	7	$2.5599567023732630 \times 10^{-6}$
8	$-1.1519344076678042 \times 10^{-6}$	8	$-7.3809570869967323 \times 10^{-7}$
9	$-6.7939116869644511 \times 10^{-8}$	9	$-1.7198806635310290 \times 10^{-7}$

Table 4–2, continued.

Interval 19: Central time $T_c = -3520$, covering the time span $-3560 \leq T \leq -3480$

	ε (deg)		p_A (deg)
0	22.415370875901241	0	-4930.7553196069655
1	$-2.7849896678197312 \times 10^{-2}$	1	57.916455798315345
2	$1.0500087758675352 \times 10^{-1}$	2	$2.0413419495108795 \times 10^{-2}$
3	$1.1995965253662042 \times 10^{-3}$	3	$-3.8104620040327579 \times 10^{-2}$
4	$-1.1613907822237268 \times 10^{-3}$	4	$-5.3027230027041141 \times 10^{-4}$
5	$-2.0426449017844937 \times 10^{-5}$	5	$3.4457972532763473 \times 10^{-4}$
6	$9.1656115787701575 \times 10^{-6}$	6	$7.2178851493882585 \times 10^{-6}$
7	$1.7426993592338147 \times 10^{-7}$	7	$-3.3701445066172283 \times 10^{-6}$
8	$-9.9979775999359283 \times 10^{-8}$	8	$-6.5554527264036988 \times 10^{-8}$
9	$-4.9403949965509519 \times 10^{-10}$	9	$4.2301924042989717 \times 10^{-8}$

	ψ_A (deg)		Δ (deg)
0	-4933.5224939686283	0	-4928.1648205828145
1	58.501622862006183	1	61.606036719140825
2	$8.6475724120568206 \times 10^{-1}$	2	$3.4645720772109925 \times 10^{-1}$
3	$4.0841971048399903 \times 10^{-2}$	3	$-1.5630133220926905 \times 10^{-1}$
4	$-1.8103870550209455 \times 10^{-2}$	4	$-1.2639905187904085 \times 10^{-2}$
5	$-2.7953507102698987 \times 10^{-3}$	5	$2.4391248221103169 \times 10^{-3}$
6	$-3.9296887949854439 \times 10^{-5}$	6	$3.4293374678689726 \times 10^{-4}$
7	$4.6667105837147470 \times 10^{-5}$	7	$-3.5090521055712439 \times 10^{-5}$
8	$8.3542893681781727 \times 10^{-6}$	8	$-8.9183890082781836 \times 10^{-6}$
9	$2.3207924573441023 \times 10^{-7}$	9	$3.9095952935797711 \times 10^{-7}$

	ω_A (deg)		I (deg)
0	21.538850296938925	0	20.575533798713399
1	-1.2875485223243870	1	$-4.5761225354345343 \times 10^{-1}$
2	$5.2834019961332805 \times 10^{-2}$	2	$3.8340922197657901 \times 10^{-1}$
3	$5.7029363014515610 \times 10^{-2}$	3	$2.2085385310638257 \times 10^{-2}$
4	$4.0552823482380669 \times 10^{-3}$	4	$-5.5984847720630005 \times 10^{-3}$
5	$-6.2824696643654383 \times 10^{-4}$	5	$-4.5773843713281331 \times 10^{-4}$
6	$-9.6223402498872785 \times 10^{-5}$	6	$5.9216814714046196 \times 10^{-5}$
7	$-3.1455509030062428 \times 10^{-6}$	7	$8.8619336036311061 \times 10^{-6}$
8	$8.8277286125458147 \times 10^{-7}$	8	$-6.1357906402418078 \times 10^{-7}$
9	$1.9543269970547837 \times 10^{-7}$	9	$-1.8739248465687323 \times 10^{-7}$

	χ_A (deg)		L (deg)
0	-2.9850802291083910	0	1.0482518071163564
1	$6.4321224903172797 \times 10^{-1}$	1	-3.9622809711731877
2	$9.0709413905699905 \times 10^{-1}$	2	$-3.4591795140430064 \times 10^{-1}$
3	$8.2629158511515879 \times 10^{-2}$	3	$1.2401758388650354 \times 10^{-1}$
4	$-1.8840354424806427 \times 10^{-2}$	4	$1.2452548902167808 \times 10^{-2}$
5	$-3.2193134097135038 \times 10^{-3}$	5	$-2.1311605157466850 \times 10^{-3}$
6	$-3.5907779767465829 \times 10^{-5}$	6	$-3.3792643292269137 \times 10^{-4}$
7	$5.0663906957848022 \times 10^{-5}$	7	$3.1748134462438307 \times 10^{-5}$
8	$8.3591001970118211 \times 10^{-6}$	8	$8.8612107940190817 \times 10^{-6}$
9	$1.8591418057513129 \times 10^{-7}$	9	$-3.4683176511644462 \times 10^{-7}$

Table 4–2, continued.

Interval 20: Central time $T_c = -3440$, covering the time span $-3480 \le T \le -3400$

	ε (deg)		p_A (deg)
0	23.043897747299093	0	-4816.0048264364452
1	$5.8895930210686813 \times 10^{-1}$	1	56.561894377265631
2	$2.6323455957576270 \times 10^{-2}$	2	$-2.9891015913516726 \times 10^{-1}$
3	$-1.1491343246216989 \times 10^{-2}$	3	$-5.7570979427044122 \times 10^{-3}$
4	$-1.3083672287719360 \times 10^{-4}$	4	$3.1023589289217818 \times 10^{-3}$
5	$7.2027018448510935 \times 10^{-5}$	5	$-4.9065606922797002 \times 10^{-5}$
6	$-1.6613938021009053 \times 10^{-6}$	6	$-1.3222983030250772 \times 10^{-5}$
7	$-1.3623634856652995 \times 10^{-7}$	7	$1.1779656620089914 \times 10^{-6}$
8	$1.7490406198672382 \times 10^{-8}$	8	$-3.0809195687923083 \times 10^{-8}$
9	$-2.5239758953896594 \times 10^{-9}$	9	$-4.8430420915403576 \times 10^{-9}$

	ψ_A (deg)		Δ (deg)
0	-4812.6952188429191	0	-4807.7922541020072
1	60.868949470880088	1	57.567620952115838
2	$-7.9479377600719381 \times 10^{-1}$	2	-1.0314998967848724
3	$-2.0533235316684021 \times 10^{-1}$	3	$3.3619928617774850 \times 10^{-3}$
4	$2.2349724698038838 \times 10^{-2}$	4	$1.4876935875056753 \times 10^{-2}$
5	$4.2203392549089712 \times 10^{-3}$	5	$-1.4174818360873067 \times 10^{-3}$
6	$-7.5135373232525580 \times 10^{-4}$	6	$-5.3661925860813457 \times 10^{-5}$
7	$-7.2784666103634669 \times 10^{-5}$	7	$3.0637210618058290 \times 10^{-5}$
8	$2.4564505738387572 \times 10^{-5}$	8	$-2.8245538809570418 \times 10^{-6}$
9	$8.0328529938577541 \times 10^{-7}$	9	$-1.9701752846792661 \times 10^{-7}$

	ω_A (deg)		I (deg)
0	21.351859958254466	0	22.431551610765381
1	1.4824460791222009	1	2.0809574393237889
2	$4.7510002445228974 \times 10^{-1}$	2	$1.1761480705761728 \times 10^{-1}$
3	$-3.1122288745125948 \times 10^{-2}$	3	$-4.9801207922243447 \times 10^{-2}$
4	$-1.0642722749639732 \times 10^{-2}$	4	$-1.0131102212891914 \times 10^{-4}$
5	$6.6809052545328568 \times 10^{-4}$	5	$4.4824090638300603 \times 10^{-4}$
6	$1.5120040212818503 \times 10^{-4}$	6	$-3.5042091829513771 \times 10^{-5}$
7	$-1.9991513067796350 \times 10^{-5}$	7	$-9.6053900362741592 \times 10^{-7}$
8	$-2.0909381351632479 \times 10^{-6}$	8	$5.9604046104595553 \times 10^{-7}$
9	$5.7780730124454275 \times 10^{-7}$	9	$-5.8044160152325494 \times 10^{-8}$

	χ_A (deg)		L (deg)
0	3.5908861926088674	0	-5.0367300517527882
1	4.6800825452914276	1	-1.1342221663198544
2	$-5.1250376225013135 \times 10^{-1}$	2	$7.8483768182356285 \times 10^{-1}$
3	$-2.1346295060405703 \times 10^{-1}$	3	$-5.4043223677225663 \times 10^{-3}$
4	$1.9117556607036576 \times 10^{-2}$	4	$-1.2381831350197449 \times 10^{-2}$
5	$4.4239655095420667 \times 10^{-3}$	5	$1.3336671978491793 \times 10^{-3}$
6	$-7.3598660162594676 \times 10^{-4}$	6	$4.3695648019298358 \times 10^{-5}$
7	$-7.4722394392559696 \times 10^{-5}$	7	$-2.9290526902785716 \times 10^{-5}$
8	$2.4597184867359731 \times 10^{-5}$	8	$2.7785514849624593 \times 10^{-6}$
9	$8.0894759409624777 \times 10^{-7}$	9	$1.9204435071558957 \times 10^{-7}$

Table 4–2, continued.

Interval 21: Central time $T_c = -3360$, covering the time span $-3400 \leq T \leq -3320$

	ε (deg)		p_A (deg)
0	24.031711766714705	0	-4704.9874265987257
1	$3.0421830107867368 \times 10^{-1}$	1	54.596722649699770
2	$-8.5474857297194409 \times 10^{-2}$	2	$-1.3400470775468885 \times 10^{-1}$
3	$-4.8417241409368853 \times 10^{-3}$	3	$2.7379893756704549 \times 10^{-2}$
4	$7.9062993291587583 \times 10^{-4}$	4	$8.5215188647847301 \times 10^{-4}$
5	$2.0509434914633501 \times 10^{-5}$	5	$-1.2328800386320188 \times 10^{-4}$
6	$-1.8914554046578652 \times 10^{-6}$	6	$8.5311541918298181 \times 10^{-8}$
7	$-2.5631675596444088 \times 10^{-8}$	7	$-2.4217884532387131 \times 10^{-7}$
8	$-1.8079127540388985 \times 10^{-8}$	8	$-1.3655900573574686 \times 10^{-8}$
9	$-6.7425277334899643 \times 10^{-10}$	9	$7.3929328117188791 \times 10^{-9}$

	ψ_A (deg)		Δ (deg)
0	-4700.0835928521726	0	-4699.1584257502441
1	51.695244508936367	1	51.677308578504753
2	$-9.1502015594602729 \times 10^{-1}$	2	$-3.2750618588322216 \times 10^{-1}$
3	$1.1069037083608190 \times 10^{-1}$	3	$7.5723834196468965 \times 10^{-2}$
4	$2.7338160454941257 \times 10^{-3}$	4	$-4.3522923929484100 \times 10^{-4}$
5	$-9.9778218216023873 \times 10^{-4}$	5	$-2.9311566253136914 \times 10^{-5}$
6	$1.8280504017710156 \times 10^{-4}$	6	$2.8829825456370036 \times 10^{-5}$
7	$-1.0333294113992447 \times 10^{-5}$	7	$-3.6041967332767214 \times 10^{-6}$
8	$-7.5728400083462887 \times 10^{-7}$	8	$1.7767806752874018 \times 10^{-7}$
9	$2.9925436216523969 \times 10^{-7}$	9	$-8.7391142517807355 \times 10^{-9}$

	ω_A (deg)		I (deg)
0	25.823915433160135	0	25.906897293600057
1	2.3066040380040514	1	1.0257675342694967
2	$-3.1354099155128181 \times 10^{-1}$	2	$-3.1975056861128646 \times 10^{-1}$
3	$-6.0677091114113224 \times 10^{-2}$	3	$-1.6390269294679061 \times 10^{-2}$
4	$4.2507904759534445 \times 10^{-3}$	4	$2.8707634046088340 \times 10^{-3}$
5	$2.2858470583140523 \times 10^{-4}$	5	$3.4122214726061697 \times 10^{-6}$
6	$-2.6961698889618241 \times 10^{-5}$	6	$-2.1433824632046099 \times 10^{-6}$
7	$3.6450842589914244 \times 10^{-6}$	7	$5.0133101780116389 \times 10^{-7}$
8	$-2.6350923906246044 \times 10^{-7}$	8	$-4.8851474420500561 \times 10^{-8}$
9	$-6.4212211876099145 \times 10^{-9}$	9	$3.1578682986229384 \times 10^{-9}$

	χ_A (deg)		L (deg)
0	5.3907170779462821	0	-2.4824397830551130
1	-3.1448655003683841	1	3.2009857574435465
2	$-8.8465256116739487 \times 10^{-1}$	2	$2.2499844917911361 \times 10^{-1}$
3	$8.7998462039045502 \times 10^{-2}$	3	$-5.4977073332776580 \times 10^{-2}$
4	$3.7541126477440398 \times 10^{-3}$	4	$8.8663164127471134 \times 10^{-4}$
5	$-8.9455815590032283 \times 10^{-4}$	5	$-4.3860294629245630 \times 10^{-5}$
6	$1.6881969875822182 \times 10^{-4}$	6	$-2.6767499437321125 \times 10^{-5}$
7	$-1.0180961177593051 \times 10^{-5}$	7	$3.1408975950871842 \times 10^{-6}$
8	$-6.8890954309184888 \times 10^{-7}$	8	$-1.9510883491048063 \times 10^{-7}$
9	$2.9336313452512637 \times 10^{-7}$	9	$1.7495465648999983 \times 10^{-8}$

Table 4–2, continued.

Interval 22: Central time $T_c = -3280$, covering the time span $-3320 \leq T \leq -3240$

	ε (deg)		p_A (deg)
0	23.932080836340938	0	-4595.7889574358087
1	$-3.8596380057402840 \times 10^{-1}$	1	54.877843323507011
2	$-6.4441122789095834 \times 10^{-2}$	2	$1.9234181557702429 \times 10^{-1}$
3	$7.7843074466973168 \times 10^{-3}$	3	$2.0574995094192837 \times 10^{-2}$
4	$5.5989037846608017 \times 10^{-4}$	4	$-1.6769087241576847 \times 10^{-3}$
5	$-4.5505478639698985 \times 10^{-5}$	5	$-1.0632461290814399 \times 10^{-4}$
6	$-1.6595474563732848 \times 10^{-6}$	6	$4.6511637355456870 \times 10^{-6}$
7	$2.5320038384356668 \times 10^{-7}$	7	$1.3764265969466296 \times 10^{-7}$
8	$-4.2906568631478512 \times 10^{-9}$	8	$-5.1720583369342361 \times 10^{-8}$
9	$-4.3848354990117400 \times 10^{-9}$	9	$1.1280782201160198 \times 10^{-9}$

	ψ_A (deg)		Δ (deg)
0	-4599.6015692765053	0	-4595.5930390833994
1	49.981476023238318	1	52.629890148243135
2	$5.2668043728898052 \times 10^{-1}$	2	$5.6327029302476513 \times 10^{-1}$
3	$1.3898433082637077 \times 10^{-1}$	3	$7.3084060770523467 \times 10^{-2}$
4	$4.9571928324882658 \times 10^{-3}$	4	$-8.9820136321134281 \times 10^{-4}$
5	$3.2010007037510175 \times 10^{-4}$	5	$-3.6240082658943783 \times 10^{-4}$
6	$-7.2582428113052774 \times 10^{-5}$	6	$-6.4367321606551790 \times 10^{-5}$
7	$-1.6266495547654817 \times 10^{-5}$	7	$-4.4889190102350912 \times 10^{-6}$
8	$-1.8509431185772635 \times 10^{-6}$	8	$6.2023977970256563 \times 10^{-8}$
9	$-1.8662828604466020 \times 10^{-7}$	9	$5.3360505893755613 \times 10^{-8}$

	ω_A (deg)		I (deg)
0	26.593572051904143	0	25.328566553998281
1	-1.7266749328132231	1	-1.5385744194266238
2	$-5.3680909575006193 \times 10^{-1}$	2	$-2.4123791549007699 \times 10^{-1}$
3	$2.9313562826904555 \times 10^{-2}$	3	$2.9292874517717636 \times 10^{-2}$
4	$6.7458255734277129 \times 10^{-3}$	4	$2.7533668275086485 \times 10^{-3}$
5	$1.1333333504599793 \times 10^{-4}$	5	$-4.1470409971804532 \times 10^{-5}$
6	$-3.1338476636239489 \times 10^{-7}$	6	$-9.8061877949302694 \times 10^{-6}$
7	$-8.5453976447088793 \times 10^{-7}$	7	$-1.3492428940645584 \times 10^{-6}$
8	$-2.1628680190714901 \times 10^{-7}$	8	$-8.4807730099683201 \times 10^{-8}$
9	$-3.3888507603855338 \times 10^{-8}$	9	$1.0573422333000962 \times 10^{-9}$

	χ_A (deg)		L (deg)
0	-4.1917106362922470	0	3.6914842326516864
1	-5.3717367963159763	1	2.4371540776317967
2	$4.0146835148544059 \times 10^{-1}$	2	$-4.2145686542089088 \times 10^{-1}$
3	$1.3460901962111892 \times 10^{-1}$	3	$-5.6890793895150287 \times 10^{-2}$
4	$5.6299213488262289 \times 10^{-3}$	4	$-1.4830350290080517 \times 10^{-4}$
5	$2.4317658260944107 \times 10^{-4}$	5	$2.9112586236505021 \times 10^{-4}$
6	$-7.0590694708896429 \times 10^{-5}$	6	$6.6175947860410381 \times 10^{-5}$
7	$-1.5261081159984618 \times 10^{-5}$	7	$4.4459130949396565 \times 10^{-6}$
8	$-1.8245104970763254 \times 10^{-6}$	8	$-1.1800275803827036 \times 10^{-7}$
9	$-1.9360234262543067 \times 10^{-7}$	9	$-5.1205313698420883 \times 10^{-8}$

Table 4–2, continued.

Interval 23: Central time $T_c = -3200$, covering the time span $-3240 \le T \le -3160$

	ε (deg)		p_A (deg)
0	22.997901279518913	0	-4484.1154148422271
1	$-4.5165481472776805 \times 10^{-1}$	1	56.827508044477065
2	$4.9375209476331254 \times 10^{-2}$	2	$2.2882957693436296 \times 10^{-1}$
3	$8.5050321256102842 \times 10^{-3}$	3	$-1.6629619051350065 \times 10^{-2}$
4	$-5.1297574971829178 \times 10^{-4}$	4	$-2.4551305384697532 \times 10^{-3}$
5	$-5.9878699655756704 \times 10^{-5}$	5	$7.2856481242916826 \times 10^{-5}$
6	$3.3277022041061354 \times 10^{-7}$	6	$1.6809867562168300 \times 10^{-5}$
7	$3.6618303795290522 \times 10^{-7}$	7	$8.4147748109745577 \times 10^{-7}$
8	$4.6751156690032880 \times 10^{-8}$	8	$-1.0886107104044157 \times 10^{-7}$
9	$-2.4160943761440355 \times 10^{-9}$	9	$-2.1455481858515146 \times 10^{-8}$

	ψ_A (deg)		Δ (deg)
0	-4490.0379737714184	0	-4483.9636443957695
1	60.804103066879257	1	59.253865711466510
2	1.8666387251952286	2	$8.2864217367092699 \times 10^{-1}$
3	$-8.0785184209746968 \times 10^{-2}$	3	$-7.8776821855773650 \times 10^{-2}$
4	$-6.0896684472106799 \times 10^{-2}$	4	$-1.8167279875736650 \times 10^{-2}$
5	$-5.5719242876202947 \times 10^{-3}$	5	$1.6009445643756382 \times 10^{-4}$
6	$1.0551517969539353 \times 10^{-3}$	6	$3.2148208550855577 \times 10^{-4}$
7	$3.4645692322065627 \times 10^{-4}$	7	$2.3445345407564126 \times 10^{-5}$
8	$1.7104855842579085 \times 10^{-5}$	8	$-4.3070262592827010 \times 10^{-6}$
9	$-9.8106036449390051 \times 10^{-6}$	9	$-8.3192093073223071 \times 10^{-7}$

	ω_A (deg)		I (deg)
0	21.262859242263313	0	21.824657369809377
1	-2.7845825934554554	1	-1.5327866948040756
2	$4.3171615502980468 \times 10^{-1}$	2	$2.6606657096673503 \times 10^{-1}$
3	$1.1123326395566033 \times 10^{-1}$	3	$3.9909865450270007 \times 10^{-2}$
4	$-3.6733865312064210 \times 10^{-3}$	4	$-3.4277541118070682 \times 10^{-3}$
5	$-2.2149193532008585 \times 10^{-3}$	5	$-5.7538182298126979 \times 10^{-4}$
6	$-1.9108830560439760 \times 10^{-4}$	6	$6.4963165192841898 \times 10^{-6}$
7	$2.4169464338657823 \times 10^{-5}$	7	$7.7469480369375957 \times 10^{-6}$
8	$8.9625474362338644 \times 10^{-6}$	8	$5.2091826123217721 \times 10^{-7}$
9	$6.3741600372253847 \times 10^{-7}$	9	$-8.2707923420474140 \times 10^{-8}$

	χ_A (deg)		L (deg)
0	-6.4159106319835270	0	3.6798465402579300
1	4.3607595125705859	1	-2.6582543784254285
2	1.7314826547449937	2	$-6.3510124811837607 \times 10^{-1}$
3	$-7.7365181719769298 \times 10^{-2}$	3	$6.8462131613615846 \times 10^{-2}$
4	$-5.9996938575390927 \times 10^{-2}$	4	$1.6103715746941413 \times 10^{-2}$
5	$-5.5036730712091634 \times 10^{-3}$	5	$-1.4360868321098824 \times 10^{-4}$
6	$1.0485637745318850 \times 10^{-3}$	6	$-3.0661321319409224 \times 10^{-4}$
7	$3.4480264025713393 \times 10^{-4}$	7	$-2.2282579064322505 \times 10^{-5}$
8	$1.7193855949151386 \times 10^{-5}$	8	$4.2013264457987572 \times 10^{-6}$
9	$-9.7851443712521234 \times 10^{-6}$	9	$8.0809404512771029 \times 10^{-7}$

Table 4–2, continued.

Interval 24: Central time $T_c = -3120$, covering the time span $-3160 \leq T \leq -3080$

	ε (deg)		p_A (deg)
0	22.669380505610600	0	−4369.5738291607951
1	$1.4563350384094731 \times 10^{-1}$	1	57.438428578094419
2	$7.5923682993353192 \times 10^{-2}$	2	$-9.3145006850777772 \times 10^{-2}$
3	$-4.6412944991094391 \times 10^{-3}$	3	$-2.5825601331293544 \times 10^{-2}$
4	$-7.0374082300298143 \times 10^{-4}$	4	$1.6891357102343930 \times 10^{-3}$
5	$5.7349664162841311 \times 10^{-5}$	5	$1.6445999740998943 \times 10^{-4}$
6	$3.0699806357635496 \times 10^{-6}$	6	$-2.0018893133025568 \times 10^{-5}$
7	$-6.5200403453255674 \times 10^{-7}$	7	$-6.4203854370161304 \times 10^{-7}$
8	$-7.2294270417647345 \times 10^{-9}$	8	$2.5198100388327298 \times 10^{-7}$
9	$8.2981602767850104 \times 10^{-9}$	9	$-6.7414148240981735 \times 10^{-10}$

	ψ_A (deg)		Δ (deg)
0	−4364.6258301073771	0	−4363.6876746073555
1	61.284430155942445	1	59.677303224964871
2	−1.7501289582917392	2	$-6.9714463628302956 \times 10^{-1}$
3	$-8.1940736734942093 \times 10^{-2}$	3	$-6.6750001061901626 \times 10^{-2}$
4	$6.1313003206261372 \times 10^{-2}$	4	$1.7906521812061123 \times 10^{-2}$
5	$-5.4839321030028346 \times 10^{-3}$	5	$-1.9357747404721259 \times 10^{-4}$
6	$-1.0718563015347902 \times 10^{-3}$	6	$-2.8298120258512265 \times 10^{-4}$
7	$3.4700942178701916 \times 10^{-4}$	7	$2.8893128961119003 \times 10^{-5}$
8	$-1.6429027475386637 \times 10^{-5}$	8	$2.6895564030032557 \times 10^{-6}$
9	$-9.9749768237916957 \times 10^{-6}$	9	$-8.2360690634448920 \times 10^{-7}$

	ω_A (deg)		I (deg)
0	20.854002921740290	0	21.405009476938047
1	2.3499863565630838	1	1.0850150644250465
2	$4.0328330357261168 \times 10^{-1}$	2	$2.3560028995265871 \times 10^{-1}$
3	$-1.1290878016555907 \times 10^{-1}$	3	$-4.0891795478022850 \times 10^{-2}$
4	$-2.6172728134060601 \times 10^{-3}$	4	$-2.0188275355967845 \times 10^{-3}$
5	$2.2407060004910184 \times 10^{-3}$	5	$6.0325478985296273 \times 10^{-4}$
6	$-2.0259944707871214 \times 10^{-4}$	6	$-1.2754340165864390 \times 10^{-5}$
7	$-2.2650771969214181 \times 10^{-5}$	7	$-6.3042590706086166 \times 10^{-6}$
8	$9.0343659693509952 \times 10^{-6}$	8	$7.2501271958761208 \times 10^{-7}$
9	$-6.8723874887043153 \times 10^{-7}$	9	$3.3587445588885089 \times 10^{-8}$

	χ_A (deg)		L (deg)
0	5.3434218509415232	0	−2.5440078107672228
1	4.1836977628514244	1	−2.4319939793073792
2	−1.7567277796622787	2	$6.4404382573202219 \times 10^{-1}$
3	$-6.6299793908391349 \times 10^{-2}$	3	$4.4570253126363861 \times 10^{-2}$
4	$6.1448635677224174 \times 10^{-2}$	4	$-1.6829029095589318 \times 10^{-2}$
5	$-5.5394596334299947 \times 10^{-3}$	5	$3.3391652656078528 \times 10^{-4}$
6	$-1.0640248944521084 \times 10^{-3}$	6	$2.6629699643022202 \times 10^{-4}$
7	$3.4696558534603358 \times 10^{-4}$	7	$-2.9432402376280234 \times 10^{-5}$
8	$-1.6661045154265209 \times 10^{-5}$	8	$-2.4363592147222608 \times 10^{-6}$
9	$-9.9709701368386628 \times 10^{-6}$	9	$8.2254355227271582 \times 10^{-7}$

Table 4–2, continued.

Interval 25: Central time $T_c = -3040$, covering the time span $-3080 \leq T \leq -3000$

	ε (deg)		p_A (deg)
0	23.315559388034183	0	-4256.0306054165186
1	$4.2791918644172461 \times 10^{-1}$	1	56.011595774968077
2	$-9.3241859331883723 \times 10^{-3}$	2	$-2.0575315542539576 \times 10^{-1}$
3	$-7.1007702869213155 \times 10^{-3}$	3	$6.2013984951886819 \times 10^{-3}$
4	$2.6416400500248541 \times 10^{-4}$	4	$1.5366415620941334 \times 10^{-3}$
5	$2.7405865146078210 \times 10^{-5}$	5	$-8.2148709816968287 \times 10^{-5}$
6	$-1.0323031001554939 \times 10^{-6}$	6	$-1.0685537810913851 \times 10^{-6}$
7	$5.8494207458700184 \times 10^{-8}$	7	$-6.8850796258695927 \times 10^{-8}$
8	$-2.8508421621736142 \times 10^{-8}$	8	$-3.8411801526490573 \times 10^{-8}$
9	$-1.0519667738920308 \times 10^{-9}$	9	$1.1549838220850948 \times 10^{-8}$

	ψ_A (deg)		Δ (deg)
0	-4253.1001083156344	0	-4249.9099409658651
1	51.499557720757789	1	54.382740806035288
2	$-3.6713421117638643 \times 10^{-1}$	2	$-3.9869121081704615 \times 10^{-1}$
3	$1.4660841737748468 \times 10^{-1}$	3	$6.5507156760921692 \times 10^{-2}$
4	$-4.8883462539735953 \times 10^{-3}$	4	$-3.9825159786740552 \times 10^{-4}$
5	$1.6187515814397041 \times 10^{-4}$	5	$-4.2012587942433078 \times 10^{-4}$
6	$4.6224898689670255 \times 10^{-5}$	6	$4.8653495190350278 \times 10^{-5}$
7	$-1.8935080200160497 \times 10^{-5}$	7	$-4.2964352384554979 \times 10^{-6}$
8	$1.7135166795860077 \times 10^{-6}$	8	$5.6130535400859116 \times 10^{-8}$
9	$-1.8662049227080037 \times 10^{-7}$	9	$3.8385787468438765 \times 10^{-8}$

	ω_A (deg)		I (deg)
0	25.225523099543114	0	23.972118447553355
1	1.2681556585668774	1	1.1342646360594699
2	$-4.8642026557149447 \times 10^{-1}$	2	$-1.7246762470348694 \times 10^{-1}$
3	$-1.5096310637335820 \times 10^{-2}$	3	$-1.4853595528271907 \times 10^{-2}$
4	$7.5909457170781251 \times 10^{-3}$	4	$3.0351306637623153 \times 10^{-3}$
5	$-1.4138784109962681 \times 10^{-4}$	5	$-3.8008546939618437 \times 10^{-5}$
6	$-4.9976681025679012 \times 10^{-6}$	6	$-1.2753561999700900 \times 10^{-5}$
7	$2.1871487962258132 \times 10^{-7}$	7	$9.3691947584476507 \times 10^{-7}$
8	$-2.3991230387780694 \times 10^{-7}$	8	$-6.4208785968672749 \times 10^{-8}$
9	$3.1499285413483427 \times 10^{-8}$	9	$3.7461955243289052 \times 10^{-9}$

	χ_A (deg)		L (deg)
0	3.1978056131373040	0	-2.8274646094447717
1	-4.9234493412420115	1	1.7598620230245704
2	$-1.9858405897228250 \times 10^{-1}$	2	$2.1800996653106218 \times 10^{-1}$
3	$1.5749137615905862 \times 10^{-1}$	3	$-6.4381970468968555 \times 10^{-2}$
4	$-5.9526303954350949 \times 10^{-3}$	4	$1.7343417378406153 \times 10^{-3}$
5	$3.2807796814644409 \times 10^{-5}$	5	$3.9432728835396938 \times 10^{-4}$
6	$4.4054286427504064 \times 10^{-5}$	6	$-4.8963678502029202 \times 10^{-5}$
7	$-1.7409972643399942 \times 10^{-5}$	7	$3.8537471548458904 \times 10^{-6}$
8	$1.7723341385977363 \times 10^{-6}$	8	$-9.8599927003123423 \times 10^{-8}$
9	$-2.0729274033824581 \times 10^{-7}$	9	$-2.4038555647743711 \times 10^{-8}$

Table 4–2, continued.

Interval 26: Central time $T_c = -2960$, covering the time span $-3000 \leq T \leq -2920$

	ε (deg)
0	23.898018129475525
1	$1.1451693668789174 \times 10^{-1}$
2	$-5.7051572463359016 \times 10^{-2}$
3	$-4.7369912534530246 \times 10^{-4}$
4	$4.0356557417429835 \times 10^{-4}$
5	$-1.8921405034860663 \times 10^{-5}$
6	$-1.3802890395973291 \times 10^{-6}$
7	$2.5441913810771760 \times 10^{-7}$
8	$9.8848232684818455 \times 10^{-9}$
9	$-3.1511921130589427 \times 10^{-9}$

	p_A (deg)
0	-4145.2113644030879
1	54.946497723834258
2	$-4.2636262317286525 \times 10^{-2}$
3	$1.6466387831882675 \times 10^{-2}$
4	$-2.0874126154486738 \times 10^{-4}$
5	$-5.9719852325439974 \times 10^{-5}$
6	$5.9756652289111490 \times 10^{-6}$
7	$1.8725306679240507 \times 10^{-7}$
8	$-6.1127165366645080 \times 10^{-8}$
9	$-1.3006223207138999 \times 10^{-9}$

	ψ_A (deg)
0	-4148.3791701956421
1	54.285238453141851
2	$8.9106554859786408 \times 10^{-1}$
3	$3.8431866465817924 \times 10^{-2}$
4	$-1.5597536581898558 \times 10^{-2}$
5	$-1.6337625601631324 \times 10^{-3}$
6	$-6.9510174747286766 \times 10^{-6}$
7	$3.2587103212634525 \times 10^{-5}$
8	$4.2051146201654069 \times 10^{-6}$
9	$-3.0623131107590081 \times 10^{-8}$

	Δ (deg)
0	-4142.1821838196155
1	53.840570307318096
2	$1.9020039417451483 \times 10^{-1}$
3	$2.4921465673068539 \times 10^{-2}$
4	$-3.7623330182579075 \times 10^{-3}$
5	$-7.2470929046877800 \times 10^{-5}$
6	$8.1582473538240104 \times 10^{-6}$
7	$1.3669869702743201 \times 10^{-6}$
8	$1.1847342503159382 \times 10^{-7}$
9	$-1.1000748157688508 \times 10^{-8}$

	ω_A (deg)
0	24.566383451268738
1	-1.5733084094792379
2	$-7.2964763600679694 \times 10^{-2}$
3	$6.8965418276438222 \times 10^{-2}$
4	$1.1880651394104119 \times 10^{-3}$
5	$-7.0784256869176333 \times 10^{-4}$
6	$-5.1637486471453962 \times 10^{-5}$
7	$-1.6584662678024304 \times 10^{-7}$
8	$7.4477808344987685 \times 10^{-7}$
9	$1.0730630032185800 \times 10^{-7}$

	I (deg)
0	24.792694603587447
1	$-2.6676692634260044 \times 10^{-1}$
2	$-1.2110670865960776 \times 10^{-1}$
3	$1.7027162841941048 \times 10^{-2}$
4	$5.5226499114321554 \times 10^{-4}$
5	$-1.6852342705059806 \times 10^{-4}$
6	$-1.0572121606819746 \times 10^{-6}$
7	$3.0260143120823855 \times 10^{-7}$
8	$2.4589453653053433 \times 10^{-8}$
9	$3.2297795286244738 \times 10^{-9}$

	χ_A (deg)
0	-3.4718464262684159
1	$-7.0805837006439913 \times 10^{-1}$
2	1.0273960192265348
3	$2.0448947114017169 \times 10^{-2}$
4	$-1.7291669104434105 \times 10^{-2}$
5	$-1.5313707790235043 \times 10^{-3}$
6	$9.1587706856210275 \times 10^{-7}$
7	$3.1912386714996651 \times 10^{-5}$
8	$4.2295190105490406 \times 10^{-6}$
9	$-2.5246176322084182 \times 10^{-8}$

	L (deg)
0	$5.4633521583159185 \times 10^{-1}$
1	1.2160938886016661
2	$-2.5576548045929735 \times 10^{-1}$
3	$-9.6136152611371209 \times 10^{-3}$
4	$4.0336726824097569 \times 10^{-3}$
5	$9.9837585824925439 \times 10^{-6}$
6	$-5.2010520344407831 \times 10^{-6}$
7	$-1.0829010688573415 \times 10^{-6}$
8	$-1.7787228147210893 \times 10^{-7}$
9	$8.8340886028785339 \times 10^{-9}$

Table 4–2, continued.

Interval 27: Central time $T_c = -2880$, covering the time span $-2920 \le T \le -2840$

	ε (deg)		p_A (deg)
0	23.710590573116297	0	-4035.1045564772750
1	$-2.8424367791827999 \times 10^{-1}$	1	55.294039714923181
2	$-3.6337528085073468 \times 10^{-2}$	2	$1.1765778768178334 \times 10^{-1}$
3	$3.2544101163039144 \times 10^{-3}$	3	$9.9525586544632165 \times 10^{-3}$
4	$1.5446498067418464 \times 10^{-4}$	4	$-4.0847050919019319 \times 10^{-4}$
5	$4.8207934515982577 \times 10^{-6}$	5	$-4.1705651760315353 \times 10^{-6}$
6	$1.2146210975382437 \times 10^{-6}$	6	$-4.1481306279862566 \times 10^{-6}$
7	$-1.6916967638794190 \times 10^{-7}$	7	$-4.8302174680774011 \times 10^{-7}$
8	$-2.1796543971590569 \times 10^{-8}$	8	$1.9367248277844076 \times 10^{-8}$
9	$-5.9292816708772701 \times 10^{-10}$	9	$5.5058606154581557 \times 10^{-9}$

	ψ_A (deg)		Δ (deg)
0	-4034.7795421419084	0	-4032.7400859432162
1	58.276857540576708	1	55.565488013261444
2	$-2.9202935250167890 \times 10^{-1}$	2	$1.5334536245313498 \times 10^{-1}$
3	$-1.5653908359400354 \times 10^{-1}$	3	$-2.2689925391171431 \times 10^{-2}$
4	$1.3798865668500323 \times 10^{-2}$	4	$-3.5146479710734212 \times 10^{-4}$
5	$3.2181812668474222 \times 10^{-3}$	5	$4.3803628253478437 \times 10^{-4}$
6	$-3.9186544434517311 \times 10^{-4}$	6	$8.0595856301568089 \times 10^{-6}$
7	$-5.4730943216951464 \times 10^{-5}$	7	$-3.4520239486611533 \times 10^{-6}$
8	$1.1475838108974375 \times 10^{-5}$	8	$-1.3620718114653137 \times 10^{-7}$
9	$8.9034722987195805 \times 10^{-7}$	9	$2.9379540248285445 \times 10^{-8}$

	ω_A (deg)		I (deg)
0	22.855115161736170	0	23.882796556025578
1	$2.2301906450553768 \times 10^{-1}$	1	$-5.1016903175219556 \times 10^{-1}$
2	$3.3575237450325574 \times 10^{-1}$	2	$3.3576711434517018 \times 10^{-2}$
3	$-3.1168336497869493 \times 10^{-2}$	3	$2.5175671632132676 \times 10^{-3}$
4	$-8.9741355679478399 \times 10^{-3}$	4	$-1.7994883464296500 \times 10^{-3}$
5	$6.6328996310450877 \times 10^{-4}$	5	$1.0402349342024038 \times 10^{-5}$
6	$1.1427703244748700 \times 10^{-4}$	6	$1.6624234644857704 \times 10^{-5}$
7	$-1.3032000271954040 \times 10^{-5}$	7	$2.3462369740140816 \times 10^{-7}$
8	$-1.4152621149782211 \times 10^{-6}$	8	$-8.7795061490475003 \times 10^{-8}$
9	$3.0868404789335074 \times 10^{-7}$	9	$-3.0305846134711073 \times 10^{-9}$

	χ_A (deg)		L (deg)
0	$3.5322881524406014 \times 10^{-1}$	0	1.2800529864710865
1	3.2488074037814959	1	$-2.9486776044346551 \times 10^{-1}$
2	$-4.4606578238155933 \times 10^{-1}$	2	$-3.8441383504407325 \times 10^{-2}$
3	$-1.7945573759647592 \times 10^{-1}$	3	$3.5731125507497957 \times 10^{-2}$
4	$1.5048763429024454 \times 10^{-2}$	4	$-1.2847582376527010 \times 10^{-4}$
5	$3.3785836378030285 \times 10^{-3}$	5	$-4.8240526654483186 \times 10^{-4}$
6	$-3.9294150890624557 \times 10^{-4}$	6	$-1.2007368035780991 \times 10^{-5}$
7	$-5.4987866110968106 \times 10^{-5}$	7	$3.1317127924965380 \times 10^{-6}$
8	$1.1455122957923279 \times 10^{-5}$	8	$1.6032638655691453 \times 10^{-7}$
9	$8.8528796786566948 \times 10^{-7}$	9	$-2.3571535032897158 \times 10^{-8}$

Table 4–2, continued.

Interval 28: Central time $T_c = -2800$, covering the time span $-2840 \leq T \leq -2760$

	ε (deg)		p_A (deg)
0	23.008814941357821	0	−3923.3069634783168
1	$-3.7096998669234238 \times 10^{-1}$	1	56.552391700804002
2	$1.9304725662312256 \times 10^{-2}$	2	$1.7404383283030659 \times 10^{-1}$
3	$5.6932282855681751 \times 10^{-3}$	3	$-3.3508721610766321 \times 10^{-3}$
4	$-6.4339113123000629 \times 10^{-6}$	4	$-1.4010925198390176 \times 10^{-3}$
5	$-3.5512681749199974 \times 10^{-5}$	5	$-2.8257177786074690 \times 10^{-5}$
6	$-9.5397775489458948 \times 10^{-7}$	6	$1.0306385136409087 \times 10^{-5}$
7	$4.9189028383779449 \times 10^{-7}$	7	$5.0523713498067920 \times 10^{-7}$
8	$1.9490686309596650 \times 10^{-8}$	8	$-1.6584887454500197 \times 10^{-7}$
9	$-8.2453313344866552 \times 10^{-9}$	9	$-1.0803006871555457 \times 10^{-8}$

	ψ_A (deg)		Δ (deg)
0	−3922.7987859206233	0	−3920.9272055080334
1	53.673214709066639	1	56.183094121139978
2	$-3.5710775749258457 \times 10^{-1}$	2	$8.4511514318771346 \times 10^{-2}$
3	$1.1367651968032759 \times 10^{-1}$	3	$2.1797789773929267 \times 10^{-2}$
4	$7.1673548839208006 \times 10^{-3}$	4	$3.5307223026537630 \times 10^{-3}$
5	$-8.3509761186233043 \times 10^{-4}$	5	$-2.4500724791174503 \times 10^{-4}$
6	$1.0752485509098362 \times 10^{-4}$	6	$-4.0710213936564664 \times 10^{-5}$
7	$-6.6757837220226247 \times 10^{-6}$	7	$5.7489295542322178 \times 10^{-7}$
8	$-1.7846278596777958 \times 10^{-6}$	8	$3.0363146932206140 \times 10^{-8}$
9	$1.8233807100228443 \times 10^{-7}$	9	$-9.1483552044248457 \times 10^{-9}$

	ω_A (deg)		I (deg)
0	23.973016738852659	0	22.973114383926807
1	$3.2731817352659649 \times 10^{-1}$	1	$-4.6594449749729322 \times 10^{-1}$
2	$-3.1355115166053903 \times 10^{-1}$	2	$-3.9622065701490395 \times 10^{-2}$
3	$-3.6345941979731293 \times 10^{-2}$	3	$-6.3754993136279776 \times 10^{-3}$
4	$6.3821824705721860 \times 10^{-3}$	4	$1.3409013686044223 \times 10^{-3}$
5	$3.3238877494411312 \times 10^{-4}$	5	$1.9614932087643723 \times 10^{-4}$
6	$-3.3012718944930746 \times 10^{-5}$	6	$-8.9640578398964538 \times 10^{-6}$
7	$2.4432900901095330 \times 10^{-6}$	7	$-1.3593292322524189 \times 10^{-6}$
8	$-1.5616449688788684 \times 10^{-7}$	8	$2.6542577396691431 \times 10^{-8}$
9	$-1.9929008143729089 \times 10^{-8}$	9	$3.9912140090280936 \times 10^{-9}$

	χ_A (deg)		L (deg)
0	$5.5469304130040246 \times 10^{-1}$	0	1.2658472840621352
1	−3.1374893939933592	1	$4.0232825018853493 \times 10^{-1}$
2	$-5.7926492751560391 \times 10^{-1}$	2	$9.6326395612079035 \times 10^{-2}$
3	$1.2929872129267310 \times 10^{-1}$	3	$-2.7512578857236917 \times 10^{-2}$
4	$9.8397786605032035 \times 10^{-3}$	4	$-5.3120435832488530 \times 10^{-3}$
5	$-9.5929245983113675 \times 10^{-4}$	5	$2.4612807117373696 \times 10^{-4}$
6	$8.4008523273842108 \times 10^{-5}$	6	$5.5529853112647681 \times 10^{-5}$
7	$-6.2950468018704498 \times 10^{-6}$	7	$-2.1743514192882475 \times 10^{-7}$
8	$-1.5133849406096179 \times 10^{-6}$	8	$-2.3777329996578322 \times 10^{-7}$
9	$1.9064069722378975 \times 10^{-7}$	9	$-1.8365615034545534 \times 10^{-9}$

Table 4–2, continued.

Interval 29: Central time $T_c = -2720$, covering the time span $-2760 \leq T \leq -2680$

	ε (deg)		p_A (deg)
0	22.606482874941229	0	-3809.1881767294080
1	$1.0262204662645923 \times 10^{-2}$	1	57.432324265277711
2	$6.8574920502475262 \times 10^{-2}$	2	$1.5072956741919774 \times 10^{-2}$
3	$1.6154413119049147 \times 10^{-3}$	3	$-2.0756200037230374 \times 10^{-2}$
4	$-3.9175315795491656 \times 10^{-4}$	4	$-5.7826009575240554 \times 10^{-4}$
5	$-1.4331895439585232 \times 10^{-5}$	5	$4.8181509213654995 \times 10^{-5}$
6	$-2.4184947751381816 \times 10^{-6}$	6	$1.9053251829967444 \times 10^{-6}$
7	$-1.2904999776610738 \times 10^{-7}$	7	$1.2470335367658443 \times 10^{-6}$
8	$7.6176653630047610 \times 10^{-8}$	8	$6.2692432441562072 \times 10^{-8}$
9	$3.7603341073447588 \times 10^{-9}$	9	$-2.6956718558174975 \times 10^{-8}$

	ψ_A (deg)		Δ (deg)
0	-3813.2724176416975	0	-3806.8168228592375
1	57.207665736803005	1	58.189643693183402
2	1.2255521484738975	2	$3.7538996398287001 \times 10^{-1}$
3	$9.1324693548470559 \times 10^{-2}$	3	$-4.0937899503505490 \times 10^{-3}$
4	$-2.0429859554890236 \times 10^{-2}$	4	$-8.2126204056117192 \times 10^{-3}$
5	$-4.0339885353164596 \times 10^{-3}$	5	$-5.8964469295239939 \times 10^{-4}$
6	$-2.8927639455981380 \times 10^{-4}$	6	$5.9424661406012559 \times 10^{-5}$
7	$4.3180030847796046 \times 10^{-5}$	7	$1.0590027272538036 \times 10^{-5}$
8	$1.4859260751952828 \times 10^{-5}$	8	$3.6167793817017815 \times 10^{-7}$
9	$1.5535561181406690 \times 10^{-6}$	9	$-9.6036022708289937 \times 10^{-8}$

	ω_A (deg)		I (deg)
0	22.153932101317538	0	21.855676134274106
1	-1.9111095133994498	1	$-5.5493338288463775 \times 10^{-1}$
2	$-3.1380884395083884 \times 10^{-2}$	2	$7.5670479234927250 \times 10^{-2}$
3	$8.2749031675974850 \times 10^{-2}$	3	$2.4764831133332081 \times 10^{-2}$
4	$5.5940062358725636 \times 10^{-3}$	4	$9.1570668549625024 \times 10^{-4}$
5	$-7.2697963581238968 \times 10^{-4}$	5	$-2.8959293702823712 \times 10^{-4}$
6	$-1.2091801799545303 \times 10^{-4}$	6	$-2.0294799815368240 \times 10^{-5}$
7	$-9.7180936478435071 \times 10^{-6}$	7	$1.2725459740324826 \times 10^{-6}$
8	$4.6293130546351714 \times 10^{-7}$	8	$1.9122738933109096 \times 10^{-7}$
9	$3.0306243001566564 \times 10^{-7}$	9	$9.5138681059158801 \times 10^{-9}$

	χ_A (deg)		L (deg)
0	-4.4156845903711812	0	1.2761977543243440
1	$-2.1676760664687935 \times 10^{-1}$	1	$-8.1906466532037937 \times 10^{-1}$
2	1.3085420104746516	2	$-3.8844761014470612 \times 10^{-1}$
3	$1.1541356399619381 \times 10^{-1}$	3	$-1.7742281978350615 \times 10^{-2}$
4	$-2.1836456795498567 \times 10^{-2}$	4	$8.1308112929021062 \times 10^{-3}$
5	$-4.1222472020729068 \times 10^{-3}$	5	$6.5679166385713157 \times 10^{-4}$
6	$-2.7420422874496998 \times 10^{-4}$	6	$-6.1558394384645551 \times 10^{-5}$
7	$4.1845408892062680 \times 10^{-5}$	7	$-9.2931435237764902 \times 10^{-6}$
8	$1.4700318668759850 \times 10^{-5}$	8	$-2.7131797797229560 \times 10^{-7}$
9	$1.5858247488623675 \times 10^{-6}$	9	$6.6237027670290960 \times 10^{-8}$

Table 4–2, continued.

Interval 30: Central time $T_c = -2640$, covering the time span $-2680 \leq T \leq -2600$

	ε (deg)
0	23.141140325456913
1	$4.9871943488097623 \times 10^{-1}$
2	$3.7261258156650025 \times 10^{-2}$
3	$-7.4236537611232484 \times 10^{-3}$
4	$-5.4259606077466481 \times 10^{-4}$
5	$4.1910911342637125 \times 10^{-5}$
6	$5.2640684139899370 \times 10^{-6}$
7	$-3.9285237019311858 \times 10^{-7}$
8	$-5.7696460065052824 \times 10^{-8}$
9	$5.9888406281484848 \times 10^{-9}$

	ψ_A (deg)
0	-3692.0664968874410
1	62.216464953782979
2	$-8.0958181462618024 \times 10^{-1}$
3	$-2.9673665434110253 \times 10^{-1}$
4	$3.1634364886686852 \times 10^{-2}$
5	$7.2844455236891701 \times 10^{-3}$
6	$-1.3251302911605969 \times 10^{-3}$
7	$-1.6281901971340012 \times 10^{-4}$
8	$5.3567896007665972 \times 10^{-5}$
9	$2.7689876920428163 \times 10^{-6}$

	ω_A (deg)
0	21.007681517386012
1	1.3556929691585769
2	$6.2540202407796702 \times 10^{-1}$
3	$-3.7298389701146696 \times 10^{-2}$
4	$-1.4927058809627653 \times 10^{-2}$
5	$1.1149783067079769 \times 10^{-3}$
6	$2.4844968510267213 \times 10^{-4}$
7	$-3.8237649659127273 \times 10^{-5}$
8	$-4.3861963981039177 \times 10^{-6}$
9	$1.3343474453451165 \times 10^{-6}$

	χ_A (deg)
0	3.2048437564312239
1	6.1618783467354874
2	$-5.9971347844307130 \times 10^{-1}$
3	$-3.0067486365425738 \times 10^{-1}$
4	$2.9833524929925060 \times 10^{-2}$
5	$7.3322240947012784 \times 10^{-3}$
6	$-1.3123214389889893 \times 10^{-3}$
7	$-1.6218751598497744 \times 10^{-4}$
8	$5.3393638695294368 \times 10^{-5}$
9	$2.7546790595779104 \times 10^{-6}$

	p_A (deg)
0	-3695.0189481321653
1	56.529345395650927
2	$-2.2886581934645747 \times 10^{-1}$
3	$-1.3322362490086555 \times 10^{-2}$
4	$1.9152614581796881 \times 10^{-3}$
5	$1.2908995528245573 \times 10^{-4}$
6	$-1.3219386070001274 \times 10^{-5}$
7	$-1.2755778181791404 \times 10^{-6}$
8	$1.7185562592815717 \times 10^{-7}$
9	$1.2795385575433862 \times 10^{-8}$

	Δ (deg)
0	-3689.2052877307103
1	58.748620684145343
2	$-4.0421160597441404 \times 10^{-1}$
3	$-8.9825753146369940 \times 10^{-2}$
4	$5.0270454437619239 \times 10^{-3}$
5	$1.2714769255347793 \times 10^{-3}$
6	$-9.3894952413891584 \times 10^{-5}$
7	$-1.5993769152830393 \times 10^{-5}$
8	$1.8299264841956918 \times 10^{-6}$
9	$2.0636710632668400 \times 10^{-7}$

	I (deg)
0	22.135051380992761
1	$9.8429163076638562 \times 10^{-1}$
2	$2.4477720765374395 \times 10^{-1}$
3	$-1.0327545137296335 \times 10^{-2}$
4	$-4.0424177873070675 \times 10^{-3}$
5	$7.6152888554004616 \times 10^{-5}$
6	$3.8187565702009325 \times 10^{-5}$
7	$-1.6273995423288544 \times 10^{-6}$
8	$-3.6356047390836829 \times 10^{-7}$
9	$2.9752713762000313 \times 10^{-8}$

	L (deg)
0	-2.4596164327589777
1	-2.4228445826596819
2	$1.8035980903118888 \times 10^{-1}$
3	$8.2224651360294294 \times 10^{-2}$
4	$-2.9522966626044537 \times 10^{-3}$
5	$-1.1861779842267940 \times 10^{-3}$
6	$7.8766250713963034 \times 10^{-5}$
7	$1.4759348745873229 \times 10^{-5}$
8	$-1.6444368217120092 \times 10^{-6}$
9	$-1.9186044091762947 \times 10^{-7}$

Table 4–2, continued.

Interval 31: Central time $T_c = -2560$, covering the time span $-2600 \leq T \leq -2520$

	ε (deg)		p_A (deg)
0	24.105827632713420	0	-3583.8798808387490
1	$3.7719320397780605 \times 10^{-1}$	1	54.650324492779705
2	$-6.7147188852984480 \times 10^{-2}$	2	$-1.8148806772966394 \times 10^{-1}$
3	$-7.2615241587500206 \times 10^{-3}$	3	$1.9594775316345614 \times 10^{-2}$
4	$4.8291592053084418 \times 10^{-4}$	4	$1.4324190000617175 \times 10^{-3}$
5	$2.7796187005930844 \times 10^{-5}$	5	$-9.2994004087329392 \times 10^{-5}$
6	$-2.1674417592449598 \times 10^{-6}$	6	$2.3363906629292082 \times 10^{-6}$
7	$2.2946481133196552 \times 10^{-7}$	7	$3.5102132796719309 \times 10^{-7}$
8	$8.2541790341673421 \times 10^{-9}$	8	$-9.9131236083894822 \times 10^{-8}$
9	$-3.9698953953555849 \times 10^{-9}$	9	$1.9177621527220207 \times 10^{-9}$

	ψ_A (deg)		Δ (deg)
0	-3578.1433090294848	0	-3576.7667010006500
1	51.595271463154882	1	53.472632373680985
2	-1.0484736662925025	2	$-6.6858200430925327 \times 10^{-1}$
3	$1.3390765935892858 \times 10^{-1}$	3	$4.2244999465858361 \times 10^{-2}$
4	$5.7682313574678716 \times 10^{-4}$	4	$5.0785805864572212 \times 10^{-3}$
5	$-1.2293575991563871 \times 10^{-3}$	5	$-5.4569468388001216 \times 10^{-4}$
6	$2.6857812399243714 \times 10^{-4}$	6	$2.6801151006215310 \times 10^{-5}$
7	$-2.4155230729977072 \times 10^{-5}$	7	$3.7536918646973953 \times 10^{-6}$
8	$1.1881979438872004 \times 10^{-7}$	8	$-6.4473357373925839 \times 10^{-7}$
9	$4.1926031322744013 \times 10^{-7}$	9	$3.5634234862383462 \times 10^{-8}$

	ω_A (deg)		I (deg)
0	25.886463268112073	0	25.100262240201678
1	2.6920259742231880	1	1.6664968288228763
2	$-3.2943762531683389 \times 10^{-1}$	2	$-1.2190412538338520 \times 10^{-1}$
3	$-6.7379081153636300 \times 10^{-2}$	3	$-3.7248752062745651 \times 10^{-2}$
4	$5.6706074434678217 \times 10^{-3}$	4	$9.7333346796655147 \times 10^{-4}$
5	$1.3718526209423160 \times 10^{-4}$	5	$2.0871821554031416 \times 10^{-4}$
6	$-3.2895044856403480 \times 10^{-5}$	6	$-1.1883504782187910 \times 10^{-5}$
7	$5.4420832122676488 \times 10^{-6}$	7	$2.1303469391003338 \times 10^{-7}$
8	$-6.0845076882356113 \times 10^{-7}$	8	$6.9390999985681197 \times 10^{-8}$
9	$2.5723243258967361 \times 10^{-8}$	9	$-9.5731280453257020 \times 10^{-9}$

	χ_A (deg)		L (deg)
0	6.3083230838927996	0	-3.9282776348603912
1	-3.2959278827525282	1	1.2560378151861302
2	$-9.8454408997921121 \times 10^{-1}$	2	$5.4288936886680983 \times 10^{-1}$
3	$1.2027700084270840 \times 10^{-1}$	3	$-2.2368069157796147 \times 10^{-2}$
4	$1.1912114712762349 \times 10^{-3}$	4	$-4.4167919156040667 \times 10^{-3}$
5	$-1.1851775005696676 \times 10^{-3}$	5	$4.3267256497761181 \times 10^{-4}$
6	$2.5056753435082092 \times 10^{-4}$	6	$-1.9641147334846057 \times 10^{-5}$
7	$-2.4438392521600693 \times 10^{-5}$	7	$-3.2210273861956023 \times 10^{-6}$
8	$2.9813476286492068 \times 10^{-7}$	8	$5.2162056963513555 \times 10^{-7}$
9	$4.1940009943339132 \times 10^{-7}$	9	$-3.4630065457117887 \times 10^{-8}$

Table 4-2, continued.

Interval 32: Central time $T_c = -2480$, covering the time span $-2520 \le T \le -2440$

	ε (deg)		p_A (deg)
0	24.161546278445122	0	−3475.0938802473806
1	$-3.4446945295689528 \times 10^{-1}$	1	54.403979825313570
2	$-9.4053906531289222 \times 10^{-2}$	2	$1.3741561684414285 \times 10^{-1}$
3	$3.6719615408405499 \times 10^{-3}$	3	$2.9506948335014462 \times 10^{-2}$
4	$8.4116771112531734 \times 10^{-4}$	4	$-2.7105517723688307 \times 10^{-4}$
5	$8.2879837945026795 \times 10^{-6}$	5	$-1.2140904721400634 \times 10^{-4}$
6	$-2.0712863891851801 \times 10^{-6}$	6	$-6.8359242736111135 \times 10^{-6}$
7	$-2.3594092403430598 \times 10^{-7}$	7	$-2.2764105292964980 \times 10^{-7}$
8	$-4.6717414761178430 \times 10^{-9}$	8	$3.7581624307369208 \times 10^{-8}$
9	$9.4564825184946803 \times 10^{-10}$	9	$2.1715186851936807 \times 10^{-9}$

	ψ_A (deg)		Δ (deg)
0	−3478.4900586324969	0	−3472.9550521917938
1	49.303053485317612	1	50.991778160536541
2	$4.5538511604105969 \times 10^{-1}$	2	$1.0924911412233478 \times 10^{-1}$
3	$1.2809676219686906 \times 10^{-1}$	3	$8.0090878214778866 \times 10^{-2}$
4	$3.2364340480103907 \times 10^{-3}$	4	$1.9425982533207809 \times 10^{-3}$
5	$3.8094927877361129 \times 10^{-4}$	5	$1.2078596952456766 \times 10^{-4}$
6	$-5.2078624829437582 \times 10^{-5}$	6	$4.8024014936306536 \times 10^{-6}$
7	$-1.0160375027990985 \times 10^{-5}$	7	$-2.8801303126899798 \times 10^{-6}$
8	$-1.0739883753706396 \times 10^{-6}$	8	$-2.0390148250541194 \times 10^{-7}$
9	$-1.2468578010426260 \times 10^{-7}$	9	$-2.5342241516917816 \times 10^{-8}$

	ω_A (deg)		I (deg)
0	27.215214204753345	0	26.386611280930243
1	−1.5598025281433817	1	$-5.9650515457976964 \times 10^{-1}$
2	$-5.6501935806048541 \times 10^{-1}$	2	$-3.7736969184044773 \times 10^{-1}$
3	$2.8557818910994223 \times 10^{-2}$	3	$1.5766233159779451 \times 10^{-4}$
4	$6.0116978157127021 \times 10^{-3}$	4	$3.2560984080022575 \times 10^{-3}$
5	$5.8469972430475852 \times 10^{-8}$	5	$6.7905522709038345 \times 10^{-5}$
6	$9.1539185967967325 \times 10^{-7}$	6	$-4.0575489890974972 \times 10^{-7}$
7	$-2.8514110115980934 \times 10^{-7}$	7	$2.0767550767114209 \times 10^{-7}$
8	$-1.2444459297166254 \times 10^{-7}$	8	$-3.1127808479159549 \times 10^{-8}$
9	$-2.1521828460005195 \times 10^{-8}$	9	$-2.9446684187895348 \times 10^{-9}$

	χ_A (deg)		L (deg)
0	−3.7453769088591636	0	1.5188935474004847
1	−5.6225354036175266	1	3.7533033147732612
2	$3.8383926381237015 \times 10^{-1}$	2	$2.1387009036309241 \times 10^{-2}$
3	$1.1458685758581546 \times 10^{-1}$	3	$-5.9404478173368270 \times 10^{-2}$
4	$2.3162110229103910 \times 10^{-3}$	4	$-2.2618230750334592 \times 10^{-3}$
5	$3.3931149058241066 \times 10^{-4}$	5	$-1.6826007680682611 \times 10^{-4}$
6	$-3.4816011218419250 \times 10^{-5}$	6	$-1.1056088010164514 \times 10^{-5}$
7	$-9.0568568811888954 \times 10^{-6}$	7	$2.3295616999225877 \times 10^{-6}$
8	$-1.1786257107530764 \times 10^{-6}$	8	$2.4266517665506323 \times 10^{-7}$
9	$-1.2991396887402534 \times 10^{-7}$	9	$2.8354231410368374 \times 10^{-8}$

Table 4–2, continued.

Interval 33: Central time $T_c = -2400$, covering the time span $-2440 \leq T \leq -2360$

	ε (deg)
0	23.011932375967665
1	$-7.0746726558436811 \times 10^{-1}$
2	$2.1707401391887415 \times 10^{-2}$
3	$1.3719042309115719 \times 10^{-2}$
4	$6.3674598013235704 \times 10^{-5}$
5	$-9.4045785232072009 \times 10^{-5}$
6	$-3.3090615732430368 \times 10^{-6}$
7	$4.2207788886851307 \times 10^{-7}$
8	$3.2980266010097417 \times 10^{-8}$
9	$-2.3804382809916068 \times 10^{-9}$

	p_A (deg)
0	-3364.2734791655063
1	56.607214628426730
2	$3.5932365598431041 \times 10^{-1}$
3	$-1.5692569074522686 \times 10^{-3}$
4	$-3.7201429062867714 \times 10^{-3}$
5	$-1.2794383480954889 \times 10^{-4}$
6	$1.7594592926010867 \times 10^{-5}$
7	$1.8459536029950246 \times 10^{-6}$
8	$-2.8567887099647978 \times 10^{-8}$
9	$-1.5116805452995717 \times 10^{-8}$

	ψ_A (deg)
0	-3371.1604563705486
1	59.266303398850656
2	1.8597479211241230
3	$-4.5679444293298975 \times 10^{-3}$
4	$-4.3212063686365168 \times 10^{-2}$
5	$-5.5890259136950059 \times 10^{-3}$
6	$2.7623080884932967 \times 10^{-4}$
7	$2.0613379264240868 \times 10^{-4}$
8	$2.5298700880621755 \times 10^{-5}$
9	$-2.2226127247991402 \times 10^{-6}$

	Δ (deg)
0	-3366.6770173067579
1	56.225673244026596
2	1.2206340795742389
3	$8.2920641045386028 \times 10^{-2}$
4	$-8.9507170222247324 \times 10^{-3}$
5	$-2.2975319120910768 \times 10^{-3}$
6	$-2.4052619511979670 \times 10^{-4}$
7	$2.3013273153119092 \times 10^{-6}$
8	$4.9894877320437473 \times 10^{-6}$
9	$8.2626938698620292 \times 10^{-7}$

	ω_A (deg)
0	21.763409505448767
1	-3.1616467214452521
2	$3.0092801405027967 \times 10^{-1}$
3	$1.0108015641764240 \times 10^{-1}$
4	$-1.2550681828888405 \times 10^{-3}$
5	$-1.4744197762364182 \times 10^{-3}$
6	$-1.6054724236852419 \times 10^{-4}$
7	$4.5844406184870409 \times 10^{-6}$
8	$4.7122355054733018 \times 10^{-6}$
9	$6.3864202803171300 \times 10^{-7}$

	I (deg)
0	22.867543226125958
1	-2.6299110468213470
2	$-2.5105078171529313 \times 10^{-2}$
3	$5.9730658704062631 \times 10^{-2}$
4	$3.4053109504264038 \times 10^{-3}$
5	$-2.4356535488024189 \times 10^{-4}$
6	$-5.5263926134897597 \times 10^{-5}$
7	$-5.9363360980702951 \times 10^{-6}$
8	$-1.3932222049157860 \times 10^{-7}$
9	$7.6844295542262257 \times 10^{-8}$

	χ_A (deg)
0	-7.4670934791584187
1	2.9711547167919116
2	1.5918752311153920
3	$-1.4848823325136740 \times 10^{-2}$
4	$-4.0765940966079683 \times 10^{-2}$
5	$-5.3246996967679371 \times 10^{-3}$
6	$2.6559741670344029 \times 10^{-4}$
7	$2.0359277182092182 \times 10^{-4}$
8	$2.5321907989505685 \times 10^{-5}$
9	$-2.2062534503639989 \times 10^{-6}$

	L (deg)
0	6.4134632353434875
1	$3.3529698806797188 \times 10^{-1}$
2	$-9.3489037108402288 \times 10^{-1}$
3	$-8.4208714441906281 \times 10^{-2}$
4	$6.1797863465607426 \times 10^{-3}$
5	$2.1728687857501425 \times 10^{-3}$
6	$2.5300907683564366 \times 10^{-4}$
7	$-5.0399258219956596 \times 10^{-7}$
8	$-5.0079313715423361 \times 10^{-6}$
9	$-8.4104568870150128 \times 10^{-7}$

Table 4–2, continued.

Interval 34: Central time $T_c = -2320$, covering the time span $-2360 \leq T \leq -2280$

	ε (deg)		p_A (deg)
0	22.207023876260705	0	−3248.9554715056730
1	$-4.8082478604215699 \times 10^{-3}$	1	58.409274733763858
2	$1.2972294191876400 \times 10^{-1}$	2	$4.6322904727763463 \times 10^{-3}$
3	$2.5836716331448080 \times 10^{-4}$	3	$-4.8326727609517022 \times 10^{-2}$
4	$-1.4538840110182747 \times 10^{-3}$	4	$-1.2210247682944000 \times 10^{-4}$
5	$-6.6051546025043994 \times 10^{-6}$	5	$4.3604889812801234 \times 10^{-4}$
6	$9.1172056863858305 \times 10^{-6}$	6	$3.4588269027580589 \times 10^{-6}$
7	$1.8182229252691308 \times 10^{-7}$	7	$-3.1883625187818569 \times 10^{-6}$
8	$-3.2459046399016901 \times 10^{-8}$	8	$-8.4626882403577756 \times 10^{-8}$
9	$-3.5687160204760207 \times 10^{-9}$	9	$1.7715029829873774 \times 10^{-8}$

	ψ_A (deg)		Δ (deg)
0	−3245.8412629994118	0	−3245.4157463636281
1	63.199477689153007	1	64.006272299453387
2	−1.2809964640258065	2	$-1.9493891315657686 \times 10^{-2}$
3	$-1.7866540348819930 \times 10^{-1}$	3	$-2.8814661361724996 \times 10^{-1}$
4	$5.6929101326384694 \times 10^{-2}$	4	$1.2573952134695127 \times 10^{-3}$
5	$-7.0790047894957097 \times 10^{-4}$	5	$7.3125846458487249 \times 10^{-3}$
6	$-1.6573492376487524 \times 10^{-3}$	6	$-5.3163037560167627 \times 10^{-5}$
7	$2.1838305898606537 \times 10^{-4}$	7	$-2.0864437414763075 \times 10^{-4}$
8	$3.0336613951540770 \times 10^{-5}$	8	$2.1708306872677421 \times 10^{-6}$
9	$-1.1499066256903434 \times 10^{-5}$	9	$6.5221115948386046 \times 10^{-6}$

	ω_A (deg)		I (deg)
0	19.925055282668066	0	19.723147105831873
1	1.4952905978224759	1	$2.0199520010078680 \times 10^{-2}$
2	$4.9200296400104472 \times 10^{-1}$	2	$5.8192074224296902 \times 10^{-1}$
3	$-8.7025530495997797 \times 10^{-2}$	3	$-1.7404172592895168 \times 10^{-3}$
4	$-5.7254281532939365 \times 10^{-3}$	4	$-1.1400170375891618 \times 10^{-2}$
5	$2.1548403124904954 \times 10^{-3}$	5	$5.5860983286561174 \times 10^{-5}$
6	$-7.0070086468981521 \times 10^{-5}$	6	$2.0881665474445191 \times 10^{-4}$
7	$-4.1913901719100146 \times 10^{-5}$	7	$-1.6748853208432869 \times 10^{-6}$
8	$6.5554882211976087 \times 10^{-6}$	8	$-4.9075112488223721 \times 10^{-6}$
9	$4.5559222553827286 \times 10^{-7}$	9	$5.5301158715876350 \times 10^{-8}$

	χ_A (deg)		L (deg)
0	3.3482370239948775	0	$-4.1244982412400890 \times 10^{-2}$
1	5.1512106963366161	1	−5.9972022008782238
2	−1.3577821585237459	2	$2.6528790610113569 \times 10^{-2}$
3	$-1.3846144510732059 \times 10^{-1}$	3	$2.4961217477904874 \times 10^{-1}$
4	$5.8556297344871125 \times 10^{-2}$	4	$-1.4255208821219539 \times 10^{-3}$
5	$-1.0795854404773848 \times 10^{-3}$	5	$-6.9565100979927136 \times 10^{-3}$
6	$-1.6730212288266259 \times 10^{-3}$	6	$5.7230292848985748 \times 10^{-5}$
7	$2.2115257032971891 \times 10^{-4}$	7	$2.0584786143643057 \times 10^{-4}$
8	$3.0474762798626184 \times 10^{-5}$	8	$-2.2636945914532202 \times 10^{-6}$
9	$-1.1512613137219713 \times 10^{-5}$	9	$-6.5068754547808273 \times 10^{-6}$

Table 4–2, continued.

Interval 35: Central time $T_c = -2240$, covering the time span $-2280 \leq T \leq -2200$

	ε (deg)		p_A (deg)
0	23.005373771176995	0	-3133.5899451604252
1	$7.1263541300725730 \times 10^{-1}$	1	56.646010048673418
2	$2.4320697904841869 \times 10^{-2}$	2	$-3.5976693417773764 \times 10^{-1}$
3	$-1.3715321730176735 \times 10^{-2}$	3	$-2.2690954003782021 \times 10^{-3}$
4	$7.1489394128771763 \times 10^{-5}$	4	$3.7508003677929136 \times 10^{-3}$
5	$9.5666808351102509 \times 10^{-5}$	5	$-1.4607059622202007 \times 10^{-4}$
6	$-5.0195972265685636 \times 10^{-6}$	6	$-1.8300622393176320 \times 10^{-5}$
7	$-4.5651950113080085 \times 10^{-7}$	7	$2.6306002033409220 \times 10^{-6}$
8	$7.6272761444216019 \times 10^{-8}$	8	$1.9328688171819238 \times 10^{-8}$
9	$2.2686121165956258 \times 10^{-9}$	9	$-2.9733832005665951 \times 10^{-8}$

	ψ_A (deg)		Δ (deg)
0	-3129.4630868443600	0	-3124.2433247294646
1	53.896872777208862	1	56.201951961052505
2	$-5.5945217250107020 \times 10^{-1}$	2	-1.1898521006635427
3	$1.2159576125216195 \times 10^{-1}$	3	$8.4204312330782696 \times 10^{-2}$
4	$-6.1547711656112948 \times 10^{-3}$	4	$8.1135641938515587 \times 10^{-3}$
5	$-3.8048600129321132 \times 10^{-4}$	5	$-2.1871338192232436 \times 10^{-3}$
6	$1.1224861222480538 \times 10^{-4}$	6	$2.3922485323150939 \times 10^{-4}$
7	$-2.1262067269668043 \times 10^{-5}$	7	$-6.8167955856727903 \times 10^{-7}$
8	$2.1539484889968039 \times 10^{-6}$	8	$-4.4140779892489204 \times 10^{-6}$
9	$-7.0302055456584474 \times 10^{-8}$	9	$7.9067423427570960 \times 10^{-7}$

	ω_A (deg)		I (deg)
0	23.837048451111509	0	22.878371379032167
1	1.7538390345572594	1	2.5937008912958379
2	$-2.8453943234909245 \times 10^{-1}$	2	$-3.3250591040855025 \times 10^{-2}$
3	$-1.6571356685016129 \times 10^{-2}$	3	$-5.7706445287787588 \times 10^{-2}$
4	$6.1441940201984738 \times 10^{-3}$	4	$3.5287370610524178 \times 10^{-3}$
5	$-2.7729206908065321 \times 10^{-4}$	5	$2.1256000097930425 \times 10^{-4}$
6	$-1.2737301279515943 \times 10^{-5}$	6	$-5.1603806718958636 \times 10^{-5}$
7	$1.9255054473437084 \times 10^{-6}$	7	$5.8344155650790365 \times 10^{-6}$
8	$-3.6613267127350694 \times 10^{-7}$	8	$-2.1389800356449813 \times 10^{-7}$
9	$4.4793009546187459 \times 10^{-8}$	9	$-6.4069312989873117 \times 10^{-8}$

	χ_A (deg)		L (deg)
0	4.4835983404003568	0	-6.3341635350966407
1	-2.9538798264616020	1	$4.1847054579515378 \times 10^{-1}$
2	$-2.3449226196956652 \times 10^{-1}$	2	$9.0302935170202725 \times 10^{-1}$
3	$1.3449802757613439 \times 10^{-1}$	3	$-8.6750620529737534 \times 10^{-2}$
4	$-9.7718269930752947 \times 10^{-3}$	4	$-5.2803215799863543 \times 10^{-3}$
5	$-3.7418928354470806 \times 10^{-4}$	5	$2.0487006426148069 \times 10^{-3}$
6	$1.3250333958787669 \times 10^{-4}$	6	$-2.5273843479719305 \times 10^{-4}$
7	$-2.3030997158808129 \times 10^{-5}$	7	$3.3207740483878317 \times 10^{-6}$
8	$2.1046233970392800 \times 10^{-6}$	8	$4.4270363771691643 \times 10^{-6}$
9	$-4.3529114396798259 \times 10^{-8}$	9	$-8.2175776159965139 \times 10^{-7}$

Table 4-2, continued.

Interval 36: Central time $T_c = -2160$, covering the time span $-2200 \le T \le -2120$

	ε (deg)		p_A (deg)
0	24.184026322152706	0	−3022.7244034820246
1	$3.6691071065892885 \times 10^{-1}$	1	54.402513182387916
2	$-9.3305863668522391 \times 10^{-2}$	2	$-1.4690896841936233 \times 10^{-1}$
3	$-4.1588700663253747 \times 10^{-3}$	3	$2.9087095464171815 \times 10^{-2}$
4	$8.1006326803580672 \times 10^{-4}$	4	$4.1470415015051646 \times 10^{-4}$
5	$1.0333387426585951 \times 10^{-6}$	5	$-1.1208303305369775 \times 10^{-4}$
6	$-1.3020665805215973 \times 10^{-6}$	6	$3.3660352151297483 \times 10^{-6}$
7	$3.8586978926079471 \times 10^{-9}$	7	$-4.5778678149261783 \times 10^{-7}$
8	$-2.0319582240500160 \times 10^{-8}$	8	$3.0377891073707804 \times 10^{-8}$
9	$2.5899690541742320 \times 10^{-9}$	9	$5.0727027311482682 \times 10^{-9}$

	ψ_A (deg)		Δ (deg)
0	−3022.8392658150298	0	−3017.8049658960790
1	53.389056767543749	1	51.156785083920696
2	$2.3388234906714588 \times 10^{-1}$	2	$-1.0037036878480632 \times 10^{-1}$
3	$8.9702289708136530 \times 10^{-3}$	3	$7.6444589140423879 \times 10^{-2}$
4	$-6.9840536302842482 \times 10^{-3}$	4	$-2.1891282303869935 \times 10^{-3}$
5	$1.5918161092813936 \times 10^{-4}$	5	$1.0254472442146772 \times 10^{-4}$
6	$4.5490213807955990 \times 10^{-5}$	6	$-4.3059071426774788 \times 10^{-6}$
7	$4.1700257960239122 \times 10^{-6}$	7	$-2.4907356895170593 \times 10^{-6}$
8	$-3.6249948875514469 \times 10^{-7}$	8	$1.9630903427489602 \times 10^{-7}$
9	$-7.4654362179437985 \times 10^{-8}$	9	$-2.3141795681880622 \times 10^{-8}$

	ω_A (deg)		I (deg)
0	25.297566800647229	0	26.341537535032508
1	$-1.3566294240280436 \times 10^{-1}$	1	$5.9758388923322327 \times 10^{-1}$
2	$-1.0907634727119822 \times 10^{-1}$	2	$-3.6110118951797063 \times 10^{-1}$
3	$2.8006628209922483 \times 10^{-2}$	3	$1.1947906936337730 \times 10^{-3}$
4	$-8.8586998079919944 \times 10^{-4}$	4	$3.0660006673200731 \times 10^{-3}$
5	$-3.6431094333201312 \times 10^{-4}$	5	$-8.7676998906640608 \times 10^{-5}$
6	$9.1293946564361393 \times 10^{-6}$	6	$-8.8383138529812905 \times 10^{-7}$
7	$1.9855862762670878 \times 10^{-6}$	7	$-4.2800293551921121 \times 10^{-8}$
8	$1.0178534618672878 \times 10^{-7}$	8	$-1.7537562068839781 \times 10^{-8}$
9	$-1.1325111970399779 \times 10^{-8}$	9	$9.8233021159857522 \times 10^{-10}$

	χ_A (deg)		L (deg)
0	$-1.2717806608709507 \times 10^{-1}$	0	−1.5300175991113410
1	−1.1152594052451119	1	3.5897374701033548
2	$4.1875791941148085 \times 10^{-1}$	2	$-4.1725666803633824 \times 10^{-2}$
3	$-2.1506340488549613 \times 10^{-2}$	3	$-5.5713830657562262 \times 10^{-2}$
4	$-8.3787287892284018 \times 10^{-3}$	4	$2.7258281667504318 \times 10^{-3}$
5	$3.2256855977956627 \times 10^{-4}$	5	$-1.4534279336022393 \times 10^{-4}$
6	$5.0064478278301800 \times 10^{-5}$	6	$5.9170932880843878 \times 10^{-6}$
7	$4.2255829379425426 \times 10^{-6}$	7	$1.6935339298170966 \times 10^{-6}$
8	$-4.2223840072704841 \times 10^{-7}$	8	$-1.5076567919459011 \times 10^{-7}$
9	$-7.8339435393856036 \times 10^{-8}$	9	$2.9841724488410864 \times 10^{-8}$

Table 4–2, continued.

Interval 37: Central time $T_c = -2080$, covering the time span $-2120 \leq T \leq -2040$

	ε (deg)		p_A (deg)
0	24.161981062441868	0	-2914.0114962758825
1	$-3.6992365593185191 \times 10^{-1}$	1	54.582384039365445
2	$-7.1150572272184794 \times 10^{-2}$	2	$1.7594526270525754 \times 10^{-1}$
3	$6.9624840067120627 \times 10^{-3}$	3	$2.0122727369731263 \times 10^{-2}$
4	$4.5592785043888713 \times 10^{-4}$	4	$-1.4233793257848416 \times 10^{-3}$
5	$-3.5548061589387947 \times 10^{-5}$	5	$-7.2438228320549097 \times 10^{-5}$
6	$-9.8722607899940287 \times 10^{-7}$	6	$2.1767671955464591 \times 10^{-6}$
7	$6.7022054230610302 \times 10^{-8}$	7	$6.6884555183676357 \times 10^{-8}$
8	$-4.3638226402410763 \times 10^{-9}$	8	$5.7094864593671563 \times 10^{-9}$
9	$-3.5117391760109412 \times 10^{-10}$	9	$2.3579787897095305 \times 10^{-9}$

	ψ_A (deg)		Δ (deg)
0	-2914.7644560721438	0	-2913.7823727147420
1	54.452608141714142	1	53.461836101628901
2	$-2.0275876670625132 \times 10^{-2}$	2	$6.1047541893805553 \times 10^{-1}$
3	$-1.8746894821577393 \times 10^{-2}$	3	$3.6214088463441385 \times 10^{-2}$
4	$5.0358587535629251 \times 10^{-3}$	4	$-4.6489051701490893 \times 10^{-3}$
5	$5.1711669933516706 \times 10^{-4}$	5	$-4.1620913040573538 \times 10^{-4}$
6	$-6.0976654061580367 \times 10^{-5}$	6	$-1.6692834217597107 \times 10^{-5}$
7	$-2.5298901578974652 \times 10^{-6}$	7	$3.3867572399328507 \times 10^{-6}$
8	$7.7372009428857880 \times 10^{-7}$	8	$4.8082948403248003 \times 10^{-7}$
9	$-8.3148776708580337 \times 10^{-9}$	9	$1.8086169052792608 \times 10^{-8}$

	ω_A (deg)		I (deg)
0	24.789508721531933	0	25.185371560958333
1	$-2.8661789965750144 \times 10^{-1}$	1	-1.5474087252578791
2	$-8.7519899086951228 \times 10^{-3}$	2	$-1.1793649888258163 \times 10^{-1}$
3	$-1.4586648130976685 \times 10^{-2}$	3	$3.3499136205723217 \times 10^{-2}$
4	$-2.0814828266361710 \times 10^{-3}$	4	$6.7069715050498835 \times 10^{-4}$
5	$3.0817150602713496 \times 10^{-4}$	5	$-1.8551418412718362 \times 10^{-4}$
6	$2.1107780564532381 \times 10^{-5}$	6	$-8.6717027237318811 \times 10^{-6}$
7	$-2.5223618354293545 \times 10^{-6}$	7	$-1.5460111339205842 \times 10^{-7}$
8	$-6.8526678983284917 \times 10^{-8}$	8	$6.9610145485549862 \times 10^{-8}$
9	$2.2994589122805660 \times 10^{-8}$	9	$9.0481775789827263 \times 10^{-9}$

	χ_A (deg)		L (deg)
0	$-8.2707851863543590 \times 10^{-1}$	0	3.6459648101949835
1	$-1.4011577818186290 \times 10^{-1}$	1	1.2145613828897842
2	$-2.1509393444222844 \times 10^{-1}$	2	$-4.8581896938086187 \times 10^{-1}$
3	$-4.2390393516023600 \times 10^{-2}$	3	$-1.5694692364524507 \times 10^{-2}$
4	$7.1920776749491709 \times 10^{-3}$	4	$3.8946959422991626 \times 10^{-3}$
5	$6.4756379668114264 \times 10^{-4}$	5	$3.2363268981441298 \times 10^{-4}$
6	$-7.0256353411476488 \times 10^{-5}$	6	$1.5393442210880780 \times 10^{-5}$
7	$-2.9553899288065521 \times 10^{-6}$	7	$-3.1946933428058348 \times 10^{-6}$
8	$7.9574233020929025 \times 10^{-7}$	8	$-4.6882001643942888 \times 10^{-7}$
9	$-9.5673178028147003 \times 10^{-9}$	9	$-1.5750587177669582 \times 10^{-8}$

Table 4–2, continued.

Interval 38: Central time $T_c = -2000$, covering the time span $-2040 \leq T \leq -1960$

	ε (deg)		p_A (deg)
0	23.165686332930421	0	-2803.0044390744073
1	$-5.4079851084290183 \times 10^{-1}$	1	56.484214272947516
2	$2.9928569824660758 \times 10^{-2}$	2	$2.4632025597997528 \times 10^{-1}$
3	$7.8020149156166213 \times 10^{-3}$	3	$-9.7480139492292584 \times 10^{-3}$
4	$-3.6080538094809520 \times 10^{-4}$	4	$-1.9062396965112377 \times 10^{-3}$
5	$-3.6718774250996762 \times 10^{-5}$	5	$5.5384520827290119 \times 10^{-5}$
6	$1.6419207545811579 \times 10^{-6}$	6	$8.9225464221514137 \times 10^{-6}$
7	$1.7966878603753131 \times 10^{-7}$	7	$-2.7794128712463866 \times 10^{-8}$
8	$-3.1410129205882367 \times 10^{-9}$	8	$-6.2883906239459133 \times 10^{-8}$
9	$-1.9277463602162034 \times 10^{-9}$	9	$-1.9093115165472528 \times 10^{-9}$

	ψ_A (deg)		Δ (deg)
0	-2805.5646874563344	0	-2801.8366919902282
1	55.046959037758113	1	58.310355352029393
2	$3.3410829927358090 \times 10^{-1}$	2	$4.0132187831872800 \times 10^{-1}$
3	$7.0986374048890162 \times 10^{-2}$	3	$-7.1576249880399182 \times 10^{-2}$
4	$2.5154272505216695 \times 10^{-3}$	4	$-3.8753863561504142 \times 10^{-3}$
5	$-6.8615968708743874 \times 10^{-4}$	5	$9.7400147154599047 \times 10^{-4}$
6	$-6.2474018238248978 \times 10^{-5}$	6	$6.3160392988462171 \times 10^{-5}$
7	$-5.3681551322160887 \times 10^{-6}$	7	$-1.1722400555855305 \times 10^{-5}$
8	$-3.0426659526141036 \times 10^{-7}$	8	$-1.1634390362093405 \times 10^{-6}$
9	$7.5548107092467943 \times 10^{-8}$	9	$1.4464183224654063 \times 10^{-7}$

	ω_A (deg)		I (deg)
0	23.544518554861634	0	22.329621085522068
1	-1.0888348080664923	1	-1.0338600950861056
2	$-1.5256904207852375 \times 10^{-1}$	2	$2.0296135116625238 \times 10^{-1}$
3	$6.9725108742675221 \times 10^{-3}$	3	$9.1889068405016934 \times 10^{-3}$
4	$4.1458240445858769 \times 10^{-3}$	4	$-3.3177926317178385 \times 10^{-3}$
5	$1.3503685281752386 \times 10^{-4}$	5	$-3.5969596422982942 \times 10^{-5}$
6	$-2.6394045306721089 \times 10^{-5}$	6	$3.1366710506153031 \times 10^{-5}$
7	$-1.2874982514116217 \times 10^{-6}$	7	$9.8184215608755686 \times 10^{-7}$
8	$-8.8097209413847411 \times 10^{-8}$	8	$-2.9726974502739227 \times 10^{-7}$
9	$-9.8779514854500664 \times 10^{-9}$	9	$-1.8847224268675630 \times 10^{-8}$

	χ_A (deg)		L (deg)
0	-2.7840092662454768	0	2.5944418326365082
1	-1.5484083786315953	1	-1.9939339402386025
2	$1.0122059178841622 \times 10^{-1}$	2	$-1.6107236917213641 \times 10^{-1}$
3	$8.7752306640793051 \times 10^{-2}$	3	$6.6749898740906245 \times 10^{-2}$
4	$4.4426226171844558 \times 10^{-3}$	4	$1.7907132808322566 \times 10^{-3}$
5	$-8.4672470592889065 \times 10^{-4}$	5	$-9.6131084663023209 \times 10^{-4}$
6	$-7.2457033174798819 \times 10^{-5}$	6	$-5.2323883831997619 \times 10^{-5}$
7	$-4.6080937649058011 \times 10^{-6}$	7	$1.1870549209683904 \times 10^{-5}$
8	$-2.3082989989711403 \times 10^{-7}$	8	$1.0925264077870646 \times 10^{-6}$
9	$7.3959793645696283 \times 10^{-8}$	9	$-1.4696686275613742 \times 10^{-7}$

Table 4–2, continued.

Interval 39: Central time $T_c = -1920$, covering the time span $-1960 \le T \le -1880$

	ε (deg)	p_A (deg)
0	22.526130256703072	-2688.7081677633994
1	$-6.1361065589005518 \times 10^{-2}$	57.612475520013628
2	$7.4512278455132433 \times 10^{-2}$	$1.1539051357358141 \times 10^{-2}$
3	$-7.5594104479529762 \times 10^{-4}$	$-2.3326561624729684 \times 10^{-2}$
4	$-5.1989834299669492 \times 10^{-4}$	$4.0171939833668512 \times 10^{-4}$
5	$1.8439037837902234 \times 10^{-5}$	$1.1408164707467517 \times 10^{-4}$
6	$1.4006751032630643 \times 10^{-6}$	$-4.9108757318457287 \times 10^{-6}$
7	$-8.9218066804151347 \times 10^{-8}$	$-3.4140537813882965 \times 10^{-7}$
8	$4.1751722540764434 \times 10^{-9}$	$1.5240157244076475 \times 10^{-8}$
9	$-7.1386544743692204 \times 10^{-10}$	$-2.2643101592441084 \times 10^{-9}$

	ψ_A (deg)	Δ (deg)
0	-2690.7127422244288	-2684.5231586916695
1	60.125664358180693	58.448315083860962
2	$6.6403140839613292 \times 10^{-1}$	$-2.4333138580605496 \times 10^{-1}$
3	$-8.5016163122238272 \times 10^{-2}$	$-8.9203751571364340 \times 10^{-3}$
4	$-2.1725030555812234 \times 10^{-2}$	$6.0479943473661052 \times 10^{-3}$
5	$1.9040367511047946 \times 10^{-4}$	$-4.5094643691234902 \times 10^{-4}$
6	$4.4014147212109644 \times 10^{-4}$	$-4.8191102358409842 \times 10^{-5}$
7	$3.3094592733243639 \times 10^{-5}$	$7.8519962156535603 \times 10^{-6}$
8	$-6.4803831794301197 \times 10^{-6}$	$-1.5988782724567420 \times 10^{-7}$
9	$-1.3302436324252111 \times 10^{-6}$	$-7.2931634004828174 \times 10^{-8}$

	ω_A (deg)	I (deg)
0	21.152598764637209	21.711169022252863
1	$-9.4597946598919869 \times 10^{-1}$	$3.0863600268733409 \times 10^{-1}$
2	$2.6250684252103117 \times 10^{-1}$	$8.3939061460770199 \times 10^{-2}$
3	$4.4609767194098415 \times 10^{-2}$	$-1.7515580093268469 \times 10^{-2}$
4	$-2.9765130641340458 \times 10^{-3}$	$8.3207512948952296 \times 10^{-4}$
5	$-8.5444314191100560 \times 10^{-4}$	$2.1260539464427299 \times 10^{-4}$
6	$-2.4337136191223944 \times 10^{-6}$	$-1.8698046599190519 \times 10^{-5}$
7	$1.1576603839217811 \times 10^{-5}$	$-9.0322223420673769 \times 10^{-7}$
8	$9.5267420408602097 \times 10^{-7}$	$1.7564826481962959 \times 10^{-7}$
9	$-1.1858170621740433 \times 10^{-7}$	$-6.9719564491651839 \times 10^{-9}$

	χ_A (deg)	L (deg)
0	-2.1638112849743555	$-6.8314796809196843 \times 10^{-1}$
1	2.7153918036077174	$-9.0728557138924260 \times 10^{-1}$
2	$6.9570707427840314 \times 10^{-1}$	$2.7376796265710373 \times 10^{-1}$
3	$-6.6358809042936798 \times 10^{-2}$	$-1.5604078158936019 \times 10^{-2}$
4	$-2.3081034487519038 \times 10^{-2}$	$-5.9956014564796857 \times 10^{-3}$
5	$1.2277234850916410 \times 10^{-4}$	$5.9288926555596083 \times 10^{-4}$
6	$4.5326349907411681 \times 10^{-4}$	$4.5717139313444819 \times 10^{-5}$
7	$3.3119832957205764 \times 10^{-5}$	$-8.3545962721463565 \times 10^{-6}$
8	$-6.5279778752929577 \times 10^{-6}$	$1.6232948863815774 \times 10^{-7}$
9	$-1.3258748663379257 \times 10^{-6}$	$7.0830552222715234 \times 10^{-8}$

Table 4–2, continued.

Interval 40: Central time $T_c = -1840$, covering the time span $-1880 \leq T \leq -1800$

	ε (deg)		p_A (deg)
0	22.894027074016675	0	-2574.1192481194815
1	$3.9216999190774073 \times 10^{-1}$	1	56.814588747304222
2	$3.0934380766956746 \times 10^{-2}$	2	$-1.8133284952862348 \times 10^{-1}$
3	$-5.3212341913739165 \times 10^{-3}$	3	$-7.0367311457396868 \times 10^{-3}$
4	$-7.4666342314106529 \times 10^{-5}$	4	$1.1404956429522861 \times 10^{-3}$
5	$1.0724437208462140 \times 10^{-5}$	5	$-2.4170241332358374 \times 10^{-5}$
6	$-1.8737832979472818 \times 10^{-6}$	6	$-4.7001340665460992 \times 10^{-8}$
7	$8.7963678039244386 \times 10^{-8}$	7	$8.1199234044163707 \times 10^{-7}$
8	$3.9665697867628381 \times 10^{-8}$	8	$-2.7239613924402281 \times 10^{-8}$
9	$6.8876071046698569 \times 10^{-11}$	9	$-1.3604557686771650 \times 10^{-8}$

	ψ_A (deg)		Δ (deg)
0	-2570.4390290914009	0	-2569.2850571160350
1	58.721029548906108	1	56.909425755283083
2	$-9.4003796711849073 \times 10^{-1}$	2	$-1.1823892246953024 \times 10^{-1}$
3	$-4.9070319449966245 \times 10^{-2}$	3	$6.0911643694383543 \times 10^{-3}$
4	$2.1556169834666942 \times 10^{-2}$	4	$-2.9018586459453117 \times 10^{-3}$
5	$-7.0782557487159939 \times 10^{-4}$	5	$-1.3919151143206431 \times 10^{-4}$
6	$-3.2562014412769652 \times 10^{-4}$	6	$3.7079435277850910 \times 10^{-5}$
7	$4.7031260748545978 \times 10^{-5}$	7	$2.8194791045092982 \times 10^{-7}$
8	$1.9114985328310586 \times 10^{-6}$	8	$-7.0027597682990755 \times 10^{-8}$
9	$-1.2100554947265734 \times 10^{-6}$	9	$-5.0449815919987516 \times 10^{-9}$

	ω_A (deg)		I (deg)
0	21.930392817668441	0	22.601845071628815
1	1.6775993192419867	1	$5.2201672792366788 \times 10^{-1}$
2	$2.0873004772524543 \times 10^{-1}$	2	$1.1365146399428901 \times 10^{-2}$
3	$-5.1651635743506169 \times 10^{-2}$	3	$4.7976100545366041 \times 10^{-3}$
4	$-2.4656855094605152 \times 10^{-3}$	4	$7.0081456391001199 \times 10^{-4}$
5	$7.8655780655065055 \times 10^{-4}$	5	$-1.6306843367078275 \times 10^{-4}$
6	$-1.7655597748232052 \times 10^{-5}$	6	$-5.0517365369459665 \times 10^{-6}$
7	$-8.5601597302327058 \times 10^{-6}$	7	$9.5988159403579736 \times 10^{-7}$
8	$1.1155842426322345 \times 10^{-6}$	8	$2.4224247919240762 \times 10^{-9}$
9	$2.9585606873156332 \times 10^{-8}$	9	$-3.8739406251164194 \times 10^{-10}$

	χ_A (deg)		L (deg)
0	3.9881539803637540	0	-1.4017337616930466
1	2.0899781904664245	1	$-1.1452645821738985 \times 10^{-1}$
2	$-8.0945001713348942 \times 10^{-1}$	2	$-6.8912674594125155 \times 10^{-2}$
3	$-4.8212635345884567 \times 10^{-2}$	3	$-1.4306302554955421 \times 10^{-2}$
4	$2.1275806537584660 \times 10^{-2}$	4	$4.3503214887288605 \times 10^{-3}$
5	$-6.0981119182723132 \times 10^{-4}$	5	$1.3129095675028831 \times 10^{-4}$
6	$-3.3241313900183179 \times 10^{-4}$	6	$-3.9671558807922555 \times 10^{-5}$
7	$4.5719994675160372 \times 10^{-5}$	7	$5.1824495331742380 \times 10^{-7}$
8	$1.9651931046583146 \times 10^{-6}$	8	$5.3244315492423173 \times 10^{-8}$
9	$-1.1933105438562751 \times 10^{-6}$	9	$-1.0071408839453152 \times 10^{-8}$

Table 4-2, continued.

Interval 41: Central time $T_c = -1760$, covering the time span $-1800 \leq T \leq -1720$

	ε (deg)		p_A (deg)
0	23.716328633624752	0	-2462.0021824037440
1	$3.7468114090249414 \times 10^{-1}$	1	55.317436575414151
2	$-3.5195711772523653 \times 10^{-2}$	2	$-1.6341261006476099 \times 10^{-1}$
3	$-5.1464917742584100 \times 10^{-3}$	3	$9.9203058750737920 \times 10^{-3}$
4	$1.7192540967644034 \times 10^{-4}$	4	$1.0194512630102631 \times 10^{-3}$
5	$2.5949887495129492 \times 10^{-5}$	5	$-2.6227988034384405 \times 10^{-5}$
6	$-7.1156090982803904 \times 10^{-8}$	6	$-5.7050079437159282 \times 10^{-6}$
7	$-3.4997302203626353 \times 10^{-7}$	7	$1.0082581942360692 \times 10^{-7}$
8	$4.0307345405995474 \times 10^{-9}$	8	$1.1494868476440201 \times 10^{-7}$
9	$6.5839929076454956 \times 10^{-9}$	9	$-3.4287925636310164 \times 10^{-9}$

	ψ_A (deg)		Δ (deg)
0	-2459.5860669342333	0	-2456.6890664196814
1	52.667325745043209	1	55.552414723471180
2	$-3.4575038837719450 \times 10^{-1}$	2	$-2.6980315228292641 \times 10^{-1}$
3	$8.6776456621070013 \times 10^{-2}$	3	$-1.9370069703618591 \times 10^{-2}$
4	$-9.9988476289652811 \times 10^{-4}$	4	$1.5146842483695950 \times 10^{-3}$
5	$-3.2701315028112096 \times 10^{-4}$	5	$3.7232117536449886 \times 10^{-4}$
6	$4.7944002953798022 \times 10^{-5}$	6	$-1.4122711917485513 \times 10^{-5}$
7	$-6.9110101462559737 \times 10^{-6}$	7	$-2.9688542191741504 \times 10^{-6}$
8	$1.9889007534503830 \times 10^{-7}$	8	$1.6043811746045428 \times 10^{-7}$
9	$3.0259648518651303 \times 10^{-8}$	9	$2.6620471348638038 \times 10^{-8}$

	ω_A (deg)		I (deg)
0	25.108343598460245	0	23.890848228610169
1	1.0756213210517480	1	$7.8830742418282305 \times 10^{-1}$
2	$-2.8871296011460851 \times 10^{-1}$	2	$3.2009541506409053 \times 10^{-2}$
3	$-1.5145429239321550 \times 10^{-2}$	3	$-7.6725473952284393 \times 10^{-3}$
4	$4.2016964450812517 \times 10^{-3}$	4	$-1.6674774477417333 \times 10^{-3}$
5	$-2.6139953246129650 \times 10^{-5}$	5	$1.2677408184431379 \times 10^{-5}$
6	$-1.5098944914612164 \times 10^{-5}$	6	$1.5494648047844146 \times 10^{-5}$
7	$1.0172001512505387 \times 10^{-6}$	7	$-7.1385180791737210 \times 10^{-8}$
8	$-1.1319108043103913 \times 10^{-7}$	8	$-9.2769785791236135 \times 10^{-8}$
9	$2.9937619634519372 \times 10^{-9}$	9	$-5.8027522604374424 \times 10^{-10}$

	χ_A (deg)		L (deg)
0	2.6449526999579431	0	-1.9492998464726881
1	-2.8937781192727222	1	$-2.7048861916890648 \times 10^{-1}$
2	$-2.1138662703761025 \times 10^{-1}$	2	$1.1618552030295336 \times 10^{-1}$
3	$8.5295677409950761 \times 10^{-2}$	3	$3.2401771078916977 \times 10^{-2}$
4	$-1.6592342309038460 \times 10^{-3}$	4	$-4.5156895418968613 \times 10^{-4}$
5	$-4.0193528198054045 \times 10^{-4}$	5	$-4.3871113778168300 \times 10^{-4}$
6	$5.1954050946466482 \times 10^{-5}$	6	$6.9088451144555482 \times 10^{-6}$
7	$-6.3777694953623470 \times 10^{-6}$	7	$3.2744640660064090 \times 10^{-6}$
8	$7.5748347242872325 \times 10^{-8}$	8	$-2.3846730104289834 \times 10^{-8}$
9	$3.0968174107329143 \times 10^{-8}$	9	$-3.0606848964184287 \times 10^{-8}$

Table 4–2, continued.

Interval 42: Central time $T_c = -1680$, covering the time span $-1720 \leq T \leq -1640$

	ε (deg)		p_A (deg)
0	24.041240108226881	0	-2352.1456454953133
1	$-7.7860738469939574 \times 10^{-2}$	1	54.696803718287947
2	$-6.8510821904729177 \times 10^{-2}$	2	$2.4017531947318445 \times 10^{-2}$
3	$1.2333653018068166 \times 10^{-4}$	3	$1.9003875211445256 \times 10^{-2}$
4	$4.0401386225924742 \times 10^{-4}$	4	$1.5561428794516021 \times 10^{-4}$
5	$6.6612376171362020 \times 10^{-6}$	5	$-2.9367357318693472 \times 10^{-5}$
6	$1.0700913759895984 \times 10^{-6}$	6	$-4.2847393486732261 \times 10^{-7}$
7	$4.4254014379820280 \times 10^{-8}$	7	$-6.2506045218677575 \times 10^{-7}$
8	$-3.0881518578240446 \times 10^{-8}$	8	$-1.5795398388090899 \times 10^{-8}$
9	$7.9145441689103878 \times 10^{-11}$	9	$7.4777775627094360 \times 10^{-9}$

	ψ_A (deg)		Δ (deg)
0	-2354.1395079533215	0	-2347.9379826953430
1	53.450243032644281	1	53.245756827311830
2	$4.5033076626382528 \times 10^{-1}$	2	$-2.1014832718165581 \times 10^{-1}$
3	$3.3762630533159894 \times 10^{-2}$	3	$3.3532230367632403 \times 10^{-2}$
4	$-6.1924426830661735 \times 10^{-3}$	4	$3.3558375297527770 \times 10^{-3}$
5	$-3.7250542705995605 \times 10^{-4}$	5	$-1.1292991932413841 \times 10^{-4}$
6	$7.5776718399061099 \times 10^{-6}$	6	$-1.8508785241217508 \times 10^{-6}$
7	$6.1097739465256338 \times 10^{-6}$	7	$1.5259463874339639 \times 10^{-6}$
8	$5.3543879393336297 \times 10^{-7}$	8	$-1.3781986014030407 \times 10^{-7}$
9	$-2.7437705180704594 \times 10^{-8}$	9	$-1.0119293510375523 \times 10^{-8}$

	ω_A (deg)		I (deg)
0	25.089071203936638	0	25.193156321358805
1	$-9.6212345518035483 \times 10^{-1}$	1	$3.4969177250736837 \times 10^{-1}$
2	$-1.3585582428210768 \times 10^{-1}$	2	$-1.5977630378642374 \times 10^{-1}$
3	$3.2190638932415539 \times 10^{-2}$	3	$-1.7403214454889133 \times 10^{-2}$
4	$8.1944887688371419 \times 10^{-4}$	4	$9.2428919898618125 \times 10^{-4}$
5	$-3.1209940954190514 \times 10^{-4}$	5	$1.5890029309870551 \times 10^{-4}$
6	$-1.1303642143601351 \times 10^{-5}$	6	$-4.0492574545594256 \times 10^{-6}$
7	$6.9819912219704612 \times 10^{-7}$	7	$-3.2031494914042669 \times 10^{-7}$
8	$1.6375165834056936 \times 10^{-7}$	8	$5.5426229906287956 \times 10^{-8}$
9	$1.1391472508214170 \times 10^{-8}$	9	$-1.8679079298576692 \times 10^{-9}$

	χ_A (deg)		L (deg)
0	-2.1895554554628571	0	$-7.5034565438750643 \times 10^{-1}$
1	-1.3619676447858675	1	1.5941681859352994
2	$4.7312790994248373 \times 10^{-1}$	2	$2.5971995809126007 \times 10^{-1}$
3	$1.5517158783628750 \times 10^{-2}$	3	$-1.6506376156317323 \times 10^{-2}$
4	$-7.3007940397419676 \times 10^{-3}$	4	$-3.7088531717657863 \times 10^{-3}$
5	$-3.3652232927148038 \times 10^{-4}$	5	$9.2616012611803685 \times 10^{-5}$
6	$1.5621478496065185 \times 10^{-5}$	6	$5.3303850966302561 \times 10^{-6}$
7	$6.7038409691295117 \times 10^{-6}$	7	$-2.2337319725097355 \times 10^{-6}$
8	$5.2272928123880704 \times 10^{-7}$	8	$1.0449315856650633 \times 10^{-7}$
9	$-3.5404446416130404 \times 10^{-8}$	9	$1.8846715880826189 \times 10^{-8}$

Table 4-2, continued.

Interval 43: Central time $T_c = -1600$, covering the time span $-1640 \leq T \leq -1560$

	ε (deg)		p_A (deg)
0	23.429436308072304	0	−2241.8555495287528
1	$-4.9539126969606662 \times 10^{-1}$	1	55.769188829413217
2	$-2.1980220180004857 \times 10^{-2}$	2	$2.3184115004670974 \times 10^{-1}$
3	$7.9462998315933038 \times 10^{-3}$	3	$1.1376066446931409 \times 10^{-2}$
4	$4.8298834855077114 \times 10^{-4}$	4	$-1.5766767574129684 \times 10^{-3}$
5	$-2.4141294105078021 \times 10^{-5}$	5	$-1.5851057531072726 \times 10^{-4}$
6	$-4.7599421945261396 \times 10^{-6}$	6	$-2.8479208538035659 \times 10^{-6}$
7	$-1.9631408593703674 \times 10^{-7}$	7	$9.9832054394455399 \times 10^{-7}$
8	$2.2170649818644732 \times 10^{-8}$	8	$1.0338733834801019 \times 10^{-7}$
9	$2.9166823138600620 \times 10^{-9}$	9	$1.1856339298071721 \times 10^{-9}$

	ψ_A (deg)		Δ (deg)
0	−2243.6767300888510	0	−2241.3151229636446
1	56.810873074016617	1	53.926263593608377
2	$2.2182505681828234 \times 10^{-1}$	2	$4.4457611882951247 \times 10^{-1}$
3	$-5.4152818828330070 \times 10^{-2}$	3	$6.9806929025571554 \times 10^{-2}$
4	$1.0283117609668009 \times 10^{-3}$	4	$7.3696558741676088 \times 10^{-4}$
5	$1.1292342310128849 \times 10^{-3}$	5	$-3.2453764746706117 \times 10^{-4}$
6	$-1.1276063957539098 \times 10^{-5}$	6	$-4.9043757671109775 \times 10^{-5}$
7	$-1.5335219436006608 \times 10^{-5}$	7	$-5.6296003552638634 \times 10^{-6}$
8	$8.9379123904198462 \times 10^{-8}$	8	$-2.2615438410187881 \times 10^{-7}$
9	$2.1538712336214650 \times 10^{-7}$	9	$2.5580109184666985 \times 10^{-8}$

	ω_A (deg)		I (deg)
0	23.133836859610876	0	24.264935731120213
1	$-7.7086007685719475 \times 10^{-1}$	1	−1.3102582854195577
2	$1.2114144720153479 \times 10^{-1}$	2	$-1.9463497278656033 \times 10^{-1}$
3	$-1.6026330017764494 \times 10^{-3}$	3	$1.6745369483450484 \times 10^{-2}$
4	$-3.5629428451163230 \times 10^{-3}$	4	$2.9831001152466331 \times 10^{-3}$
5	$1.4463037923634673 \times 10^{-4}$	5	$4.2525456704801585 \times 10^{-5}$
6	$4.5576181110155974 \times 10^{-5}$	6	$-7.4988573189152509 \times 10^{-6}$
7	$-9.1577590647919187 \times 10^{-7}$	7	$-8.0646443445302079 \times 10^{-7}$
8	$-4.4532075723872216 \times 10^{-7}$	8	$-1.1480947740376048 \times 10^{-7}$
9	$7.5092460150546870 \times 10^{-9}$	9	$-7.3769236638513313 \times 10^{-9}$

	χ_A (deg)		L (deg)
0	−1.9853497578145133	0	3.2604921006469830
1	1.1437021042558300	1	1.9971592429153118
2	$-1.4233660604262932 \times 10^{-2}$	2	$-2.4113665467601317 \times 10^{-1}$
3	$-7.1127315849338409 \times 10^{-2}$	3	$-6.3468776425924323 \times 10^{-2}$
4	$3.0336509410492569 \times 10^{-3}$	4	$-2.0693929600146573 \times 10^{-3}$
5	$1.3723662354919010 \times 10^{-3}$	5	$2.1147235622301322 \times 10^{-4}$
6	$-1.2322480165978976 \times 10^{-5}$	6	$4.5100101860457920 \times 10^{-5}$
7	$-1.6933227465459657 \times 10^{-5}$	7	$6.4787802080225715 \times 10^{-6}$
8	$-7.4149927011791949 \times 10^{-9}$	8	$3.3648396759782911 \times 10^{-7}$
9	$2.1645992176579216 \times 10^{-7}$	9	$-2.4345615657189207 \times 10^{-8}$

Table 4–2, continued.

Interval 44: Central time $T_c = -1520$, covering the time span $-1560 \le T \le -1480$

	ε (deg)		p_A (deg)
0	22.607955330586165	0	-2128.4661663068273
1	$-2.3512014565430396 \times 10^{-1}$	1	57.543068310603174
2	$8.4372013518144691 \times 10^{-2}$	2	$1.3753220928757505 \times 10^{-1}$
3	$6.1831991209802247 \times 10^{-3}$	3	$-2.9196389625805633 \times 10^{-2}$
4	$-8.6890661973581595 \times 10^{-4}$	4	$-2.1729965091572489 \times 10^{-3}$
5	$-6.4141383069842838 \times 10^{-5}$	5	$2.2483903451644104 \times 10^{-4}$
6	$6.0955619411381263 \times 10^{-6}$	6	$2.1952598873753843 \times 10^{-5}$
7	$5.3612980842338468 \times 10^{-7}$	7	$-1.7316652829205472 \times 10^{-6}$
8	$-5.6452570577871303 \times 10^{-8}$	8	$-2.2927563247688556 \times 10^{-7}$
9	$-5.5327933170702304 \times 10^{-9}$	9	$1.6133670816008048 \times 10^{-8}$

	ψ_A (deg)		Δ (deg)
0	-2129.3807036638153	0	-2127.8251837150563
1	57.383015500952553	1	59.922960754562279
2	$1.0311912071030351 \times 10^{-1}$	2	$8.3003789619270732 \times 10^{-1}$
3	$3.9385520213077384 \times 10^{-2}$	3	$-6.3705211005820321 \times 10^{-2}$
4	$2.9861397179630001 \times 10^{-3}$	4	$-2.0625554559515067 \times 10^{-2}$
5	$-1.0102999703594522 \times 10^{-3}$	5	$-6.4524569353787313 \times 10^{-4}$
6	$-6.0343355689218968 \times 10^{-5}$	6	$3.0757368621416826 \times 10^{-4}$
7	$3.9160671465885637 \times 10^{-6}$	7	$4.4059345832216351 \times 10^{-5}$
8	$-8.4325071404690017 \times 10^{-8}$	8	$-1.7288389101511299 \times 10^{-6}$
9	$6.9039414794602107 \times 10^{-8}$	9	$-1.1176109268119606 \times 10^{-6}$

	ω_A (deg)		I (deg)
0	22.129355825156241	0	21.256017812547236
1	$-3.6600675572831567 \times 10^{-1}$	1	-1.3221876414699696
2	$-2.2367019208114164 \times 10^{-2}$	2	$2.4893626243193380 \times 10^{-1}$
3	$-2.8334149406550718 \times 10^{-3}$	3	$4.6250343736313698 \times 10^{-2}$
4	$3.5470621987486204 \times 10^{-3}$	4	$-1.7246354725703566 \times 10^{-3}$
5	$1.9691185952913122 \times 10^{-4}$	5	$-6.7214382578267726 \times 10^{-4}$
6	$-3.9446872534092245 \times 10^{-5}$	6	$-2.8374809276357281 \times 10^{-5}$
7	$-1.7176935782606061 \times 10^{-6}$	7	$6.5586573157276321 \times 10^{-6}$
8	$1.0239783713669285 \times 10^{-7}$	8	$1.0868414198941435 \times 10^{-6}$
9	$-6.9060541431071342 \times 10^{-9}$	9	$-6.0204759411277681 \times 10^{-9}$

	χ_A (deg)		L (deg)
0	$-9.8876439544039228 \times 10^{-1}$	0	3.1328957002635053
1	$-1.7090285095045117 \times 10^{-1}$	1	-2.5740805222310130
2	$-3.7331056608118599 \times 10^{-2}$	2	$-7.3560192288252840 \times 10^{-1}$
3	$7.4166037580292684 \times 10^{-2}$	3	$3.8128400203280223 \times 10^{-2}$
4	$5.4853933193864664 \times 10^{-3}$	4	$1.9090864694043847 \times 10^{-2}$
5	$-1.3345563803426827 \times 10^{-3}$	5	$8.5365799683521729 \times 10^{-4}$
6	$-8.6661847904839359 \times 10^{-5}$	6	$-2.8944606460732170 \times 10^{-4}$
7	$6.4380197072632061 \times 10^{-6}$	7	$-4.5847724739727383 \times 10^{-5}$
8	$1.7736039143756032 \times 10^{-7}$	8	$1.5103784541855262 \times 10^{-6}$
9	$4.8012601990575933 \times 10^{-8}$	9	$1.1352675189221534 \times 10^{-6}$

Table 4–2, continued.

Interval 45: Central time $T_c = -1440$, covering the time span $-1480 \leq T \leq -1400$

	ε (deg)
0	22.854240233087038
1	$4.6254052324024652 \times 10^{-1}$
2	$6.2371335478888123 \times 10^{-2}$
3	$-9.0673881507871671 \times 10^{-3}$
4	$-6.1622039028134667 \times 10^{-4}$
5	$5.9234108545220470 \times 10^{-5}$
6	$7.6560938239129899 \times 10^{-7}$
7	$-1.3510058708372241 \times 10^{-7}$
8	$3.7174675706351490 \times 10^{-8}$
9	$-2.4122799414973897 \times 10^{-9}$

	ψ_A (deg)
0	-2012.8710090830325
1	59.169066976511870
2	$1.1761878557051948 \times 10^{-1}$
3	$-8.2188318796812676 \times 10^{-2}$
4	$-1.2267379240726858 \times 10^{-2}$
5	$9.9385678869321200 \times 10^{-4}$
6	$2.6613700782153607 \times 10^{-4}$
7	$-4.5500789435698398 \times 10^{-6}$
8	$-4.8307665623339962 \times 10^{-6}$
9	$-2.1012031599983262 \times 10^{-7}$

	ω_A (deg)
0	21.749025315123901
1	$2.0167902930779438 \times 10^{-1}$
2	$2.2366549082646448 \times 10^{-1}$
3	$2.6923820993179423 \times 10^{-2}$
4	$-2.8892218510616985 \times 10^{-3}$
5	$-6.4024878315281502 \times 10^{-4}$
6	$2.0630980208214760 \times 10^{-5}$
7	$8.2434657797728377 \times 10^{-6}$
8	$6.2917059802668921 \times 10^{-8}$
9	$-1.0725013327306881 \times 10^{-7}$

	χ_A (deg)
0	$7.1459871772196343 \times 10^{-1}$
1	2.2707029405843900
2	$3.8913882004119508 \times 10^{-1}$
3	$-6.4784530386568173 \times 10^{-2}$
4	$-1.5951587538057136 \times 10^{-2}$
5	$9.2044984786924948 \times 10^{-4}$
6	$2.9074555175578776 \times 10^{-4}$
7	$-5.4348938020096636 \times 10^{-6}$
8	$-4.8882807623784731 \times 10^{-6}$
9	$-1.9072096454447028 \times 10^{-7}$

	p_A (deg)
0	-2013.5295368292434
1	57.071153606808544
2	$-2.3928258924997486 \times 10^{-1}$
3	$-2.0718069236773857 \times 10^{-2}$
4	$2.6757581924097409 \times 10^{-3}$
5	$9.4738266486648198 \times 10^{-5}$
6	$-1.5752948039233174 \times 10^{-5}$
7	$6.8093501985270191 \times 10^{-7}$
8	$2.7279055643316159 \times 10^{-8}$
9	$-1.6497788878735608 \times 10^{-8}$

	Δ (deg)
0	-2006.4468900279047
1	59.930096229865390
2	$-8.9566291273097964 \times 10^{-1}$
3	$-9.5113265539092086 \times 10^{-2}$
4	$2.1438707168479277 \times 10^{-2}$
5	$2.6962624460196144 \times 10^{-4}$
6	$-4.2294315036954058 \times 10^{-4}$
7	$3.0210578362151276 \times 10^{-5}$
8	$6.4628322221854667 \times 10^{-6}$
9	$-1.2259567569772356 \times 10^{-6}$

	I (deg)
0	21.467844420666571
1	1.5886359557132506
2	$3.0889742816643889 \times 10^{-1}$
3	$-4.3151289482093561 \times 10^{-2}$
4	$-3.9990000400889965 \times 10^{-3}$
5	$6.7855282569472428 \times 10^{-4}$
6	$8.7253441066032929 \times 10^{-6}$
7	$-1.0333537524706693 \times 10^{-5}$
8	$7.0156032825254650 \times 10^{-7}$
9	$1.2756188482300235 \times 10^{-7}$

	L (deg)
0	-3.8176004453145053
1	-3.1060068057539973
2	$6.9227439616329037 \times 10^{-1}$
3	$8.1234897388863346 \times 10^{-2}$
4	$-1.9159237697746281 \times 10^{-2}$
5	$-2.3575712924017044 \times 10^{-4}$
6	$4.0962133261172692 \times 10^{-4}$
7	$-2.9189842772934986 \times 10^{-5}$
8	$-6.4526398080393327 \times 10^{-6}$
9	$1.2072802403943550 \times 10^{-6}$

Table 4–2, continued.

Interval 46: Central time $T_c = -1360$, covering the time span $-1400 \le T \le -1320$

	ε (deg)		p_A (deg)
0	23.877567083075478	0	-1901.5478153017817
1	$4.5307784118053634 \times 10^{-1}$	1	54.927377261746106
2	$-6.4433409098314931 \times 10^{-2}$	2	$-2.2076190709397304 \times 10^{-1}$
3	$-8.9000822288160233 \times 10^{-3}$	3	$2.1696243357833023 \times 10^{-2}$
4	$6.2627636954186773 \times 10^{-4}$	4	$1.8899478021782344 \times 10^{-3}$
5	$5.0892129688111391 \times 10^{-5}$	5	$-1.3745808477998325 \times 10^{-4}$
6	$-3.1949826977764583 \times 10^{-6}$	6	$-4.5055181246653702 \times 10^{-6}$
7	$-2.5776424970660294 \times 10^{-7}$	7	$6.7303087598505214 \times 10^{-7}$
8	$2.4483093844593952 \times 10^{-8}$	8	$3.4677232001437515 \times 10^{-8}$
9	$3.8851020567259510 \times 10^{-9}$	9	$-6.2700541018713050 \times 10^{-9}$

	ψ_A (deg)		Δ (deg)
0	-1897.3374676219279	0	-1894.1544078062264
1	55.463973295651162	1	52.625204620907480
2	$-9.0196421465014467 \times 10^{-1}$	2	$-6.2888532350222585 \times 10^{-1}$
3	$-1.1578260184394801 \times 10^{-2}$	3	$7.8819999628301935 \times 10^{-2}$
4	$1.3915761175228886 \times 10^{-2}$	4	$5.9219256743088915 \times 10^{-4}$
5	$-5.4329206133082958 \times 10^{-4}$	5	$-3.9365437214324322 \times 10^{-4}$
6	$-1.3822326357439517 \times 10^{-4}$	6	$7.6688254861991974 \times 10^{-5}$
7	$2.3100091209049921 \times 10^{-5}$	7	$-6.1032895458743302 \times 10^{-6}$
8	$1.3274602806996298 \times 10^{-7}$	8	$3.6916770738219811 \times 10^{-8}$
9	$-3.9326007263267737 \times 10^{-7}$	9	$6.3729202813785749 \times 10^{-8}$

	ω_A (deg)		I (deg)
0	24.043611364102583	0	25.270772573721297
1	1.9651178921520175	1	1.7411787493833205
2	$7.2302447605422626 \times 10^{-2}$	2	$-2.4581817860159639 \times 10^{-1}$
3	$-4.9856567380113992 \times 10^{-2}$	3	$-3.2256275728166128 \times 10^{-2}$
4	$-2.3384755142198125 \times 10^{-3}$	4	$2.9164500735683054 \times 10^{-3}$
5	$5.2191385028703845 \times 10^{-4}$	5	$3.2016578017744518 \times 10^{-5}$
6	$-3.6711989554771508 \times 10^{-6}$	6	$-9.3625180237731662 \times 10^{-6}$
7	$-4.0863571217016091 \times 10^{-6}$	7	$1.5716939941360804 \times 10^{-6}$
8	$5.0727711704148303 \times 10^{-7}$	8	$-1.3036407589038459 \times 10^{-7}$
9	$4.2768402041879040 \times 10^{-9}$	9	$1.6054353375789202 \times 10^{-9}$

	χ_A (deg)		L (deg)
0	4.6101543709453653	0	-4.1933677170629271
1	$6.2682300994709089 \times 10^{-1}$	1	2.5008145516017266
2	$-7.4219997009509990 \times 10^{-1}$	2	$4.6336540801199270 \times 10^{-1}$
3	$-4.0836660320116647 \times 10^{-2}$	3	$-6.1461241568889480 \times 10^{-2}$
4	$1.2799428282095717 \times 10^{-2}$	4	$6.1041717009604434 \times 10^{-4}$
5	$-2.9579033182429662 \times 10^{-4}$	5	$2.8777172771177024 \times 10^{-4}$
6	$-1.3691043538669093 \times 10^{-4}$	6	$-7.8188631765232574 \times 10^{-5}$
7	$2.1788908937987769 \times 10^{-5}$	7	$6.6777129956784378 \times 10^{-6}$
8	$8.6213480964276558 \times 10^{-8}$	8	$3.4032680628343459 \times 10^{-9}$
9	$-3.8522065191872219 \times 10^{-7}$	9	$-7.0369815662043145 \times 10^{-8}$

Table 4–2, continued.

Interval 47: Central time $T_c = -1280$, covering the time span $-1320 \leq T \leq -1240$

	ε (deg)		p_A (deg)
0	24.081869910942161	0	-1792.4155110121172
1	$-2.7266391359301094 \times 10^{-1}$	1	54.499087992370520
2	$-9.2740827097642418 \times 10^{-2}$	2	$1.2701134169184787 \times 10^{-1}$
3	$4.6711981543485081 \times 10^{-3}$	3	$2.9480060900409736 \times 10^{-2}$
4	$8.4882617189298569 \times 10^{-4}$	4	$-8.0975925699983019 \times 10^{-4}$
5	$-1.6006972888661980 \times 10^{-5}$	5	$-1.2434048811063542 \times 10^{-4}$
6	$-1.5549272922092043 \times 10^{-6}$	6	$-2.4063258681531714 \times 10^{-6}$
7	$-1.3332336771367886 \times 10^{-7}$	7	$-4.8354463632828123 \times 10^{-7}$
8	$-3.0817836507813495 \times 10^{-8}$	8	$5.8254753967738662 \times 10^{-8}$
9	$2.9568390068794419 \times 10^{-9}$	9	$1.1142847643274835 \times 10^{-8}$

	ψ_A (deg)		Δ (deg)
0	-1792.1561641798912	0	-1790.9596824738860
1	50.324445188323564	1	51.348038586067167
2	$-1.9689596410284536 \times 10^{-1}$	2	$3.0701808885321670 \times 10^{-1}$
3	$9.9306964866499885 \times 10^{-2}$	3	$7.8885981588192624 \times 10^{-2}$
4	$3.0891668310135442 \times 10^{-3}$	4	$6.3756436495136674 \times 10^{-4}$
5	$1.1310913905799158 \times 10^{-4}$	5	$2.5171794341982548 \times 10^{-5}$
6	$5.3251862450155761 \times 10^{-5}$	6	$-2.6095790474582472 \times 10^{-5}$
7	$-3.4383099811400713 \times 10^{-6}$	7	$-3.5790570763107957 \times 10^{-6}$
8	$-1.8044352297867335 \times 10^{-7}$	8	$-2.2598705583181903 \times 10^{-7}$
9	$-1.4079443710975835 \times 10^{-8}$	9	$-1.7451707415658361 \times 10^{-8}$

	ω_A (deg)		I (deg)
0	26.658049948490973	0	26.131106744800320
1	$1.8978988855876341 \times 10^{-1}$	1	$-9.6538920160515482 \times 10^{-1}$
2	$-4.6608432258435723 \times 10^{-1}$	2	$-3.4717221821612869 \times 10^{-1}$
3	$-2.2817201435142893 \times 10^{-2}$	3	$1.5520773194490062 \times 10^{-2}$
4	$4.5628382841803070 \times 10^{-3}$	4	$3.0181994203195489 \times 10^{-3}$
5	$1.9879375504446960 \times 10^{-4}$	5	$2.8320841317802441 \times 10^{-6}$
6	$-4.9027769144503242 \times 10^{-6}$	6	$-1.2442941660420275 \times 10^{-6}$
7	$7.2432498367984491 \times 10^{-7}$	7	$-3.7341239050268968 \times 10^{-7}$
8	$-2.6728791164297065 \times 10^{-8}$	8	$-4.0551912019287242 \times 10^{-8}$
9	$6.6758508173049185 \times 10^{-10}$	9	$-4.9464143484308176 \times 10^{-9}$

	χ_A (deg)		L (deg)
0	$2.8831707582233967 \times 10^{-1}$	0	2.3314293357405834
1	-4.6136639705715278	1	3.4651635486713503
2	$-3.5792867820768260 \times 10^{-1}$	2	$-2.1195164996858860 \times 10^{-1}$
3	$8.2526017445990146 \times 10^{-2}$	3	$-5.6803593827580437 \times 10^{-2}$
4	$4.8713640678510991 \times 10^{-3}$	4	$-1.0520869798534119 \times 10^{-3}$
5	$1.1688318094892166 \times 10^{-4}$	5	$-9.3038718756256680 \times 10^{-5}$
6	$4.5954656899368971 \times 10^{-5}$	6	$2.1490677917261477 \times 10^{-5}$
7	$-2.3384014976292457 \times 10^{-6}$	7	$2.8270892435361264 \times 10^{-6}$
8	$-1.8475532499247289 \times 10^{-7}$	8	$2.9627987094979758 \times 10^{-7}$
9	$-2.7963722508441903 \times 10^{-8}$	9	$3.0541279242596150 \times 10^{-8}$

Table 4–2, continued.

Interval 48: Central time $T_c = -1200$, covering the time span $-1240 \leq T \leq -1160$

	ε (deg)		p_A (deg)
0	23.105651410233283	0	-1681.5738202913563
1	$-6.0531687471772565 \times 10^{-1}$	1	56.501317251364602
2	$2.3204295802931538 \times 10^{-2}$	2	$3.1224029244440636 \times 10^{-1}$
3	$1.2127794034862019 \times 10^{-2}$	3	$-5.2907710900624998 \times 10^{-3}$
4	$-1.5937983440630459 \times 10^{-4}$	4	$-3.3154211957330663 \times 10^{-3}$
5	$-8.3075473325105779 \times 10^{-5}$	5	$-3.6804165445626649 \times 10^{-5}$
6	$-6.3653657274782608 \times 10^{-7}$	6	$1.7563139940279184 \times 10^{-5}$
7	$3.7943295044462735 \times 10^{-7}$	7	$9.4780224113052144 \times 10^{-7}$
8	$3.5974858712305693 \times 10^{-9}$	8	$-6.0438116165035748 \times 10^{-8}$
9	$-1.5072430545364931 \times 10^{-9}$	9	$-3.9447352335339180 \times 10^{-9}$

	ψ_A (deg)		Δ (deg)
0	-1688.4938421162309	0	-1682.9363333278433
1	54.685490498681549	1	57.359678456817417
2	1.4187366557409605	2	1.0990429023934280
3	$1.6523556066703544 \times 10^{-1}$	3	$1.4706355519350935 \times 10^{-2}$
4	$-2.6315643951772690 \times 10^{-3}$	4	$-1.5152164284244121 \times 10^{-2}$
5	$-2.9253554642893920 \times 10^{-3}$	5	$-1.7371734575672078 \times 10^{-3}$
6	$-5.7041380579327579 \times 10^{-4}$	6	$1.1535385103300096 \times 10^{-5}$
7	$-5.1493084002061352 \times 10^{-5}$	7	$3.2473491469099166 \times 10^{-5}$
8	$2.2637973958645296 \times 10^{-6}$	8	$4.1150916397965029 \times 10^{-6}$
9	$1.7101105823774624 \times 10^{-6}$	9	$-5.2123598756767661 \times 10^{-8}$

	ω_A (deg)		I (deg)
0	23.504205200551918	0	22.565933345806673
1	-3.1113378153735012	1	-2.2205827368302340
2	$-1.8093979901270186 \times 10^{-1}$	2	$1.0368931254134528 \times 10^{-1}$
3	$7.8480175974545914 \times 10^{-2}$	3	$5.3918344612330192 \times 10^{-2}$
4	$7.5546411016878170 \times 10^{-3}$	4	$3.8604952108152803 \times 10^{-4}$
5	$-7.6648224337893987 \times 10^{-5}$	5	$-4.5626587225427060 \times 10^{-4}$
6	$-7.0176882773379831 \times 10^{-5}$	6	$-4.4705618394947853 \times 10^{-5}$
7	$-1.3024484976781586 \times 10^{-5}$	7	$-2.6703502560340898 \times 10^{-7}$
8	$-1.4724716337836437 \times 10^{-6}$	8	$6.2502028387246260 \times 10^{-7}$
9	$-3.5161557886985936 \times 10^{-8}$	9	$8.5923077417342281 \times 10^{-8}$

	χ_A (deg)		L (deg)
0	-7.5211549633707079	0	5.3569319791261387
1	-1.8798763612623418	1	$-9.8849705083496495 \times 10^{-1}$
2	1.2190916873369540	2	$-8.4598759626074924 \times 10^{-1}$
3	$1.7481358629058054 \times 10^{-1}$	3	$-1.6388481635940838 \times 10^{-2}$
4	$-1.1468909649605061 \times 10^{-3}$	4	$1.2527544728491110 \times 10^{-2}$
5	$-2.9514926352573052 \times 10^{-3}$	5	$1.6701616336438471 \times 10^{-3}$
6	$-5.7461414807947281 \times 10^{-4}$	6	$2.6380304820971131 \times 10^{-6}$
7	$-5.2060315120636776 \times 10^{-5}$	7	$-3.1440315755353858 \times 10^{-5}$
8	$2.2611140371865546 \times 10^{-6}$	8	$-4.1682571742427244 \times 10^{-6}$
9	$1.7126151086609948 \times 10^{-6}$	9	$4.9014739838479492 \times 10^{-8}$

Table 4–2, continued.

Interval 49: Central time $T_c = -1120$, covering the time span $-1160 \leq T \leq -1080$

	ε (deg)
0	22.435261062706909
1	$5.4506007175135916 \times 10^{-3}$
2	$1.0580911391818705 \times 10^{-1}$
3	$-9.4910228839753060 \times 10^{-4}$
4	$-1.1325008034334297 \times 10^{-3}$
5	$1.9414634317012132 \times 10^{-5}$
6	$7.0312365324526882 \times 10^{-6}$
7	$-1.7422120416386672 \times 10^{-7}$
8	$-5.2246597869178707 \times 10^{-8}$
9	$9.3712804731745557 \times 10^{-10}$

	p_A (deg)
0	-1566.8406663306232
1	57.957545122036635
2	$-9.2508129069564392 \times 10^{-3}$
3	$-3.8442810280223999 \times 10^{-2}$
4	$4.5513015437297126 \times 10^{-4}$
5	$3.1879500740017446 \times 10^{-4}$
6	$-6.6330365108016204 \times 10^{-6}$
7	$-2.4156778003696724 \times 10^{-6}$
8	$6.4260865738673186 \times 10^{-8}$
9	$2.4201697269504086 \times 10^{-8}$

	ψ_A (deg)
0	-1566.9408297790004
1	65.804919762879855
2	$5.3615018807343909 \times 10^{-2}$
3	$-4.8204377425904187 \times 10^{-1}$
4	$-4.0885371715805432 \times 10^{-3}$
5	$1.7264241416704881 \times 10^{-2}$
6	$2.3833300561281932 \times 10^{-4}$
7	$-6.9511997029056134 \times 10^{-4}$
8	$-1.3532636616876354 \times 10^{-5}$
9	$3.0646919833109741 \times 10^{-5}$

	Δ (deg)
0	-1562.3747400481051
1	61.947178040859419
2	$-3.4511332621672657 \times 10^{-1}$
3	$-1.7712685637510607 \times 10^{-1}$
4	$1.3587630838269001 \times 10^{-2}$
5	$3.0932439180193804 \times 10^{-3}$
6	$-3.9997343244227598 \times 10^{-4}$
7	$-5.2631239268958197 \times 10^{-5}$
8	$1.1441226261481257 \times 10^{-5}$
9	$8.0615754185184403 \times 10^{-7}$

	ω_A (deg)
0	19.438622344519786
1	$-5.2574851673264982 \times 10^{-2}$
2	$8.3237145607426606 \times 10^{-1}$
3	$4.4832515716497390 \times 10^{-3}$
4	$-2.2014831922531456 \times 10^{-2}$
5	$-1.9524961066637087 \times 10^{-4}$
6	$5.6149336908843368 \times 10^{-4}$
7	$8.0011146839110283 \times 10^{-6}$
8	$-1.8639181305985944 \times 10^{-5}$
9	$-3.7455976809631917 \times 10^{-7}$

	I (deg)
0	20.503487883803626
1	$4.3413751098405495 \times 10^{-1}$
2	$4.1761682201529356 \times 10^{-1}$
3	$-2.2908461087773841 \times 10^{-2}$
4	$-6.6185730006312625 \times 10^{-3}$
5	$5.0329008272004770 \times 10^{-4}$
6	$8.0651515491804246 \times 10^{-5}$
7	$-1.0643910792335928 \times 10^{-5}$
8	$-1.0506258313720842 \times 10^{-6}$
9	$2.4957085672858475 \times 10^{-7}$

	χ_A (deg)
0	$-1.0779101948181661 \times 10^{-1}$
1	8.4110469842488853
2	$6.6312824909572441 \times 10^{-2}$
3	$-4.6157800483847395 \times 10^{-1}$
4	$-4.6676467840365089 \times 10^{-3}$
5	$1.7128026842551069 \times 10^{-2}$
6	$2.4639622183152596 \times 10^{-4}$
7	$-6.9354117814798160 \times 10^{-4}$
8	$-1.3601778252170397 \times 10^{-5}$
9	$3.0623352271454860 \times 10^{-5}$

	L (deg)
0	$-9.7988210672889230 \times 10^{-1}$
1	-4.3001962202989770
2	$3.5576497246781480 \times 10^{-1}$
3	$1.4552387423752096 \times 10^{-1}$
4	$-1.3491856005147524 \times 10^{-2}$
5	$-2.8256578401516151 \times 10^{-3}$
6	$3.9564429281047726 \times 10^{-4}$
7	$5.0420485491147894 \times 10^{-5}$
8	$-1.1384295292257687 \times 10^{-5}$
9	$-7.8208150135707529 \times 10^{-7}$

Table 4–2, continued.

Interval 50: Central time $T_c = -1040$, covering the time span $-1080 \leq T \leq -1000$

	ε (deg)		p_A (deg)
0	23.085725964130325	0	-1452.1353288905160
1	$5.7028472479989535 \times 10^{-1}$	1	56.515556946427718
2	$1.8630458427385521 \times 10^{-2}$	2	$-2.8374455485228679 \times 10^{-1}$
3	$-1.0387294266045408 \times 10^{-2}$	3	$-1.8285198869789771 \times 10^{-3}$
4	$6.7923139929367769 \times 10^{-5}$	4	$2.6730529749905981 \times 10^{-3}$
5	$5.9285910246924318 \times 10^{-5}$	5	$-8.8747261075432497 \times 10^{-5}$
6	$-2.3091727210252737 \times 10^{-6}$	6	$-8.7823341320441617 \times 10^{-6}$
7	$-6.2354723331330352 \times 10^{-8}$	7	$9.6210818838182191 \times 10^{-7}$
8	$9.6062968007534619 \times 10^{-9}$	8	$-5.2472342390819283 \times 10^{-8}$
9	$-2.5491122745308035 \times 10^{-9}$	9	$-1.7238256308793968 \times 10^{-9}$

	ψ_A (deg)		Δ (deg)
0	-1445.1306769411426	0	-1444.4428202360400
1	54.784575014145583	1	55.656823264981897
2	-1.4772818721943687	2	$-7.7318263655531905 \times 10^{-1}$
3	$1.6753036664700749 \times 10^{-1}$	3	$6.5487209693477240 \times 10^{-2}$
4	$4.5351216891598067 \times 10^{-3}$	4	$3.5769904499418113 \times 10^{-3}$
5	$-3.2769408093722770 \times 10^{-3}$	5	$-1.0617747288905322 \times 10^{-3}$
6	$6.0709167035372884 \times 10^{-4}$	6	$1.1152105502768022 \times 10^{-4}$
7	$-4.8400947138585631 \times 10^{-5}$	7	$-2.8041637862272107 \times 10^{-6}$
8	$-4.0510322450471763 \times 10^{-6}$	8	$-1.1901603154910742 \times 10^{-6}$
9	$2.0499627234695054 \times 10^{-6}$	9	$2.2533571135119193 \times 10^{-7}$

	ω_A (deg)		I (deg)
0	23.443887284447547	0	23.204868810983638
1	3.1619540049211515	1	1.8330168277194613
2	$-1.7235150403341691 \times 10^{-1}$	2	$-8.3273421542886554 \times 10^{-2}$
3	$-8.4452791606231144 \times 10^{-2}$	3	$-3.4767492344315501 \times 10^{-2}$
4	$7.4725555200453912 \times 10^{-3}$	4	$3.0477868441239281 \times 10^{-3}$
5	$1.7067116490551246 \times 10^{-4}$	5	$7.6885097221807336 \times 10^{-5}$
6	$-7.8317506564879088 \times 10^{-5}$	6	$-2.6685040126491909 \times 10^{-5}$
7	$1.4055106544312834 \times 10^{-5}$	7	$2.4972882395757728 \times 10^{-6}$
8	$-1.4761742563503259 \times 10^{-6}$	8	$-1.0708342084796394 \times 10^{-7}$
9	$2.9650473815142023 \times 10^{-9}$	9	$-1.2946293267645174 \times 10^{-8}$

	χ_A (deg)		L (deg)
0	7.6115566619697396	0	-4.5119037926359029
1	-1.7860922570888781	1	$8.8706682199844163 \times 10^{-1}$
2	-1.3127546867213412	2	$5.3682748256951265 \times 10^{-1}$
3	$1.7265269191842678 \times 10^{-1}$	3	$-7.0043225412116394 \times 10^{-2}$
4	$3.9102144928163231 \times 10^{-3}$	4	$-1.4440605205859009 \times 10^{-3}$
5	$-3.2258534652235456 \times 10^{-3}$	5	$1.0095872305712565 \times 10^{-3}$
6	$6.0015168611339118 \times 10^{-4}$	6	$-1.1735597272755674 \times 10^{-4}$
7	$-4.9254577605463554 \times 10^{-5}$	7	$3.5344667424037051 \times 10^{-6}$
8	$-3.9203955210196602 \times 10^{-6}$	8	$1.1232746704629589 \times 10^{-6}$
9	$2.0527307028544640 \times 10^{-6}$	9	$-2.2580234748440097 \times 10^{-7}$

Table 4–2, continued.

Interval 51: Central time $T_c = -960$, covering the time span $-1000 \le T \le -920$

	ε (deg)
0	24.039318925397702
1	$3.0743068418455728 \times 10^{-1}$
2	$-7.1392687877800063 \times 10^{-2}$
3	$-3.2785945619239647 \times 10^{-3}$
4	$5.9949901599687155 \times 10^{-4}$
5	$-7.4311983963967675 \times 10^{-6}$
6	$-2.2042101246887403 \times 10^{-6}$
7	$1.9177849413613603 \times 10^{-7}$
8	$1.3899313263570418 \times 10^{-8}$
9	$-1.6491194975552574 \times 10^{-9}$

	p_A (deg)
0	-1341.0452232083614
1	54.709841417643470
2	$-1.2623169135595095 \times 10^{-1}$
3	$2.1849595335693400 \times 10^{-2}$
4	$2.7464728553556861 \times 10^{-4}$
5	$-9.7504161400791055 \times 10^{-5}$
6	$5.3996489049650597 \times 10^{-6}$
7	$1.8611267761592522 \times 10^{-7}$
8	$-4.2540076066023476 \times 10^{-8}$
9	$-2.7177818730908710 \times 10^{-9}$

	ψ_A (deg)
0	-1341.4678400511106
1	50.315981705815539
2	$2.4474978447775431 \times 10^{-1}$
3	$1.1215003039894302 \times 10^{-1}$
4	$-2.6893654022710652 \times 10^{-3}$
5	$5.7274643982889124 \times 10^{-5}$
6	$-7.6833324321006577 \times 10^{-5}$
7	$-6.8747350444159812 \times 10^{-6}$
8	$3.5503227821733918 \times 10^{-8}$
9	$-3.9686967154802834 \times 10^{-9}$

	Δ (deg)
0	-1336.7566026609385
1	52.674688072972328
2	$-8.4098976582759218 \times 10^{-3}$
3	$4.8860801312562714 \times 10^{-2}$
4	$-2.5310107135775401 \times 10^{-3}$
5	$-2.3265181879192580 \times 10^{-5}$
6	$-1.2447818311005611 \times 10^{-6}$
7	$-8.1803442099277348 \times 10^{-7}$
8	$1.6925556718574322 \times 10^{-7}$
9	$-5.7729237402594848 \times 10^{-9}$

	ω_A (deg)
0	26.596383688031172
1	$-2.7204208632338853 \times 10^{-1}$
2	$-4.9163659571491091 \times 10^{-1}$
3	$2.5987718857300628 \times 10^{-2}$
4	$5.6604301840718018 \times 10^{-3}$
5	$-1.5996917491088826 \times 10^{-4}$
6	$-8.6595514812789444 \times 10^{-6}$
7	$-1.3895323975017346 \times 10^{-6}$
8	$-1.1063522724981279 \times 10^{-7}$
9	$-6.3177511192916801 \times 10^{-9}$

	I (deg)
0	25.456488676172512
1	$3.0419562914652717 \times 10^{-1}$
2	$-2.2353228576097684 \times 10^{-1}$
3	$7.6382194938156155 \times 10^{-3}$
4	$1.7041877446595737 \times 10^{-3}$
5	$-1.1952890731995596 \times 10^{-4}$
6	$-1.1242581279631807 \times 10^{-6}$
7	$2.7680922421244908 \times 10^{-8}$
8	$-1.4506925588594928 \times 10^{-8}$
9	$3.0015540519374724 \times 10^{-9}$

	χ_A (deg)
0	$-4.6842046411319092 \times 10^{-1}$
1	-4.8538266718429276
2	$4.1058586599648906 \times 10^{-1}$
3	$1.0537310251665166 \times 10^{-1}$
4	$-3.9623659736664684 \times 10^{-3}$
5	$-5.9904322136759565 \times 10^{-6}$
6	$-7.3032298830432038 \times 10^{-5}$
7	$-6.2101393437887645 \times 10^{-6}$
8	$2.3360696303582765 \times 10^{-8}$
9	$-3.6798985178834886 \times 10^{-9}$

	L (deg)
0	$-8.3482406228910493 \times 10^{-1}$
1	2.2328456183405945
2	$-1.2528009017069655 \times 10^{-1}$
3	$-3.1174980231051392 \times 10^{-2}$
4	$3.1239544477080961 \times 10^{-3}$
5	$-4.7818154118596941 \times 10^{-5}$
6	$4.0836865149328739 \times 10^{-6}$
7	$9.6914210934907245 \times 10^{-7}$
8	$-2.0520518444940111 \times 10^{-7}$
9	$2.1754450956997527 \times 10^{-9}$

Table 4–2, continued.

Interval 52: Central time $T_c = -880$, covering the time span $-920 \leq T \leq -840$

	ε (deg)
0	24.060207422640327
1	$-2.7842275345452358 \times 10^{-1}$
2	$-6.2219379552814827 \times 10^{-2}$
3	$4.0768007260041271 \times 10^{-3}$
4	$3.2256588023442577 \times 10^{-4}$
5	$-1.2043999046632496 \times 10^{-5}$
6	$-3.5230623577527585 \times 10^{-7}$
7	$-1.3757024528667812 \times 10^{-7}$
8	$6.8854941128986600 \times 10^{-9}$
9	$3.0661167602215681 \times 10^{-9}$

	p_A (deg)
0	-1231.8229618070689
1	54.717981566164517
2	$1.1870118866812715 \times 10^{-1}$
3	$1.6654208455083128 \times 10^{-2}$
4	$-7.2003306610619721 \times 10^{-4}$
5	$-3.6546725938803831 \times 10^{-5}$
6	$-1.3246449998459565 \times 10^{-6}$
7	$5.7796540238480647 \times 10^{-8}$
8	$5.7096232218193211 \times 10^{-8}$
9	$-1.5954870091550938 \times 10^{-9}$

	ψ_A (deg)
0	-1235.6137341680210
1	56.157448664000674
2	$9.4545183778426396 \times 10^{-1}$
3	$-4.8993403521787342 \times 10^{-2}$
4	$-2.1763160150905064 \times 10^{-2}$
5	$-5.3119462210741713 \times 10^{-4}$
6	$3.4319458563954227 \times 10^{-4}$
7	$4.6536935346283139 \times 10^{-5}$
8	$-2.6653527225569075 \times 10^{-6}$
9	$-1.2662453980134784 \times 10^{-6}$

	Δ (deg)
0	-1230.1350783593869
1	54.216873170266355
2	$3.1477419628943430 \times 10^{-1}$
3	$4.0199071103930134 \times 10^{-3}$
4	$-2.7435696356710430 \times 10^{-3}$
5	$9.6805948388330668 \times 10^{-5}$
6	$1.8354861176480868 \times 10^{-5}$
7	$4.4453859193299734 \times 10^{-7}$
8	$-1.6428967393554904 \times 10^{-7}$
9	$-1.0484431838357548 \times 10^{-8}$

	ω_A (deg)
0	23.930683332034000
1	-1.8273473098091937
2	$1.7002735649181237 \times 10^{-1}$
3	$6.0763699966094474 \times 10^{-2}$
4	$-4.0023264256376959 \times 10^{-3}$
5	$-9.0951719387917473 \times 10^{-4}$
6	$-9.3556747414204083 \times 10^{-6}$
7	$1.0411934246376413 \times 10^{-5}$
8	$1.2404087425044155 \times 10^{-6}$
9	$-5.7885569148164668 \times 10^{-8}$

	I (deg)
0	24.769441580915608
1	$-8.4315296281813316 \times 10^{-1}$
2	$-5.0671516295107162 \times 10^{-2}$
3	$1.4897098495388090 \times 10^{-2}$
4	$-7.5596553704341286 \times 10^{-4}$
5	$-9.7865332625520337 \times 10^{-5}$
6	$6.6006013214483348 \times 10^{-6}$
7	$6.3945198179576624 \times 10^{-7}$
8	$-5.5552302987815097 \times 10^{-9}$
9	$-5.4737031833581021 \times 10^{-9}$

	χ_A (deg)
0	-4.1555130269297376
1	1.6062028665529135
2	$8.9687534049837500 \times 10^{-1}$
3	$-7.6071044676202560 \times 10^{-2}$
4	$-2.2232110489502775 \times 10^{-2}$
5	$-3.6072845629536996 \times 10^{-4}$
6	$3.5238728106806594 \times 10^{-4}$
7	$4.5763885191852454 \times 10^{-5}$
8	$-2.7424297673148692 \times 10^{-6}$
9	$-1.2627689374832202 \times 10^{-6}$

	L (deg)
0	2.0169551662655307
1	$5.4372280303174375 \times 10^{-1}$
2	$-2.1660324152012049 \times 10^{-1}$
3	$1.4377552220726847 \times 10^{-2}$
4	$2.2517874208708235 \times 10^{-3}$
5	$-1.6466822016318287 \times 10^{-4}$
6	$-2.0977690371408203 \times 10^{-5}$
7	$-2.0064371927804939 \times 10^{-7}$
8	$2.3171635885490667 \times 10^{-7}$
9	$8.0614642952244484 \times 10^{-9}$

Table 4–2, continued.

Interval 53: Central time $T_c = -800$, covering the time span $-840 \le T \le -760$

	ε (deg)		p_A (deg)
0	23.207116231044139	0	-1120.9799904385453
1	$-5.1621826430950736 \times 10^{-1}$	1	56.215015615768023
2	$7.2290992624360991 \times 10^{-3}$	2	$2.2559245608529787 \times 10^{-1}$
3	$6.6116232836106813 \times 10^{-3}$	3	$-3.0971183112073054 \times 10^{-4}$
4	$-1.3767674787247447 \times 10^{-5}$	4	$-1.2918961708004533 \times 10^{-3}$
5	$-1.3117488175567786 \times 10^{-5}$	5	$-1.3512528084997478 \times 10^{-5}$
6	$7.0036578362914115 \times 10^{-7}$	6	$-7.6405289518186297 \times 10^{-7}$
7	$-1.4529513438561300 \times 10^{-7}$	7	$-3.6729646495685355 \times 10^{-7}$
8	$-3.2576521987658365 \times 10^{-8}$	8	$4.4574105530304746 \times 10^{-8}$
9	$1.9149460378787606 \times 10^{-9}$	9	$1.2699036893234776 \times 10^{-8}$

	ψ_A (deg)		Δ (deg)
0	-1120.2803862902205	0	-1119.3771817439346
1	57.821563888906183	1	56.458845213242390
2	$-5.6844397551261909 \times 10^{-1}$	2	$2.2525106347754051 \times 10^{-1}$
3	$-6.6268901198750426 \times 10^{-2}$	3	$-7.3713268348406077 \times 10^{-3}$
4	$2.2985232271425834 \times 10^{-2}$	4	$1.5275798731082257 \times 10^{-3}$
5	$1.5275630593677397 \times 10^{-4}$	5	$1.3937228105301824 \times 10^{-4}$
6	$-4.1290862389079181 \times 10^{-4}$	6	$-2.4412947164273546 \times 10^{-5}$
7	$3.6911556070007998 \times 10^{-5}$	7	$-1.4339689740344809 \times 10^{-6}$
8	$5.6156812645061355 \times 10^{-6}$	8	$1.1219845355654138 \times 10^{-7}$
9	$-1.3573657895767358 \times 10^{-6}$	9	$-1.2698859661256144 \times 10^{-9}$

	ω_A (deg)		I (deg)
0	22.557300900958182	0	23.049233105145246
1	$4.5579467228188323 \times 10^{-1}$	1	$-8.1030392071550529 \times 10^{-1}$
2	$1.7379117105064080 \times 10^{-1}$	2	$3.0164482805726577 \times 10^{-2}$
3	$-5.8015183522405302 \times 10^{-2}$	3	$-4.8220957296657182 \times 10^{-4}$
4	$-3.0559750636852707 \times 10^{-3}$	4	$-3.9732526490309839 \times 10^{-4}$
5	$1.0462886225828556 \times 10^{-3}$	5	$1.2377751317824079 \times 10^{-4}$
6	$-6.0348963098773962 \times 10^{-6}$	6	$4.2248945297987195 \times 10^{-6}$
7	$-1.2085492383835196 \times 10^{-5}$	7	$-7.7094127078561358 \times 10^{-7}$
8	$1.2084845958541389 \times 10^{-6}$	8	$-1.0629806474972066 \times 10^{-8}$
9	$1.0479787074759497 \times 10^{-7}$	9	$2.2149004282524347 \times 10^{-9}$

	χ_A (deg)		L (deg)
0	$7.5888908779680395 \times 10^{-1}$	0	2.1006770421080121
1	1.7442683269693516	1	$-2.6863607930274017 \times 10^{-1}$
2	$-8.6163261104189136 \times 10^{-1}$	2	$1.7735214831322256 \times 10^{-3}$
3	$-7.1094160040682215 \times 10^{-2}$	3	$7.6572554475762174 \times 10^{-3}$
4	$2.5899361008851495 \times 10^{-2}$	4	$-3.0924376987026376 \times 10^{-3}$
5	$2.3186858743655453 \times 10^{-4}$	5	$-1.5674496214713522 \times 10^{-4}$
6	$-4.2452125383787567 \times 10^{-4}$	6	$2.5885875121099806 \times 10^{-5}$
7	$3.6810884375038687 \times 10^{-5}$	7	$1.0340342156042322 \times 10^{-6}$
8	$5.6043355825646128 \times 10^{-6}$	8	$-7.0632588352913459 \times 10^{-8}$
9	$-1.3680220773937192 \times 10^{-6}$	9	$1.5275748971179376 \times 10^{-8}$

Table 4–2, continued.

Interval 54: Central time $T_c = -720$, covering the time span $-760 \le T \le -680$

	ε (deg)
0	22.465854783070128
1	$-1.6831698530690911 \times 10^{-1}$
2	$7.4410792400590295 \times 10^{-2}$
3	$3.4265576312717111 \times 10^{-3}$
4	$-4.7695789229628874 \times 10^{-4}$
5	$-3.2162264326250330 \times 10^{-5}$
6	$1.4935973960673726 \times 10^{-6}$
7	$3.5615841320740868 \times 10^{-7}$
8	$-2.1501059931094247 \times 10^{-8}$
9	$-5.0704814576426937 \times 10^{-9}$

	ψ_A (deg)
0	-1008.2093780957345
1	54.921287410145865
2	$1.9040089162160678 \times 10^{-1}$
3	$1.3280506811009339 \times 10^{-1}$
4	$1.3968221457313377 \times 10^{-3}$
5	$-9.4106303285937219 \times 10^{-4}$
6	$-5.7140008564297447 \times 10^{-5}$
7	$-2.0577925524050228 \times 10^{-5}$
8	$-5.2669562413830234 \times 10^{-7}$
9	$1.4223181873316491 \times 10^{-7}$

	ω_A (deg)
0	22.915093265549495
1	$-5.0221141871360996 \times 10^{-1}$
2	$-2.9076978215678090 \times 10^{-1}$
3	$7.3956367918008708 \times 10^{-3}$
4	$7.8040934620238725 \times 10^{-3}$
5	$6.7305137582394371 \times 10^{-5}$
6	$-3.1696650011919821 \times 10^{-5}$
7	$-1.1224571931468685 \times 10^{-6}$
8	$-4.2756557276879352 \times 10^{-7}$
9	$-1.5340397410526366 \times 10^{-8}$

	χ_A (deg)
0	-1.2850518759685488
1	-2.9324005023778625
2	$1.1440190760370061 \times 10^{-1}$
3	$1.6914098560120169 \times 10^{-1}$
4	$2.3877295199993411 \times 10^{-3}$
5	$-1.2309310266881378 \times 10^{-3}$
6	$-6.8060851115413886 \times 10^{-5}$
7	$-1.8755424615649189 \times 10^{-5}$
8	$-3.7917288140184855 \times 10^{-7}$
9	$1.2513520666073055 \times 10^{-7}$

	p_A (deg)
0	-1007.0204659281842
1	57.630585007936240
2	$8.8858203185327290 \times 10^{-2}$
3	$-2.2408206834098348 \times 10^{-2}$
4	$-1.1279497105853108 \times 10^{-3}$
5	$9.7675277764031598 \times 10^{-5}$
6	$1.1270758759211604 \times 10^{-5}$
7	$-4.6820982068290636 \times 10^{-7}$
8	$-1.4046570300723191 \times 10^{-7}$
9	$8.7117393836228400 \times 10^{-9}$

	Δ (deg)
0	-1004.6599188544707
1	58.287231636098261
2	$2.5135683490549419 \times 10^{-1}$
3	$7.5902957903221298 \times 10^{-4}$
4	$-2.7610060865916547 \times 10^{-3}$
5	$-5.0528387957425477 \times 10^{-4}$
6	$-2.6086953292139949 \times 10^{-6}$
7	$4.9860557316228530 \times 10^{-6}$
8	$4.1975702838765481 \times 10^{-7}$
9	$-4.5607126020165297 \times 10^{-9}$

	I (deg)
0	21.698540943009792
1	$-5.1463282176476191 \times 10^{-1}$
2	$6.4582717899284428 \times 10^{-2}$
3	$1.0666193608005322 \times 10^{-2}$
4	$1.2130091084428102 \times 10^{-3}$
5	$-5.6289177278638895 \times 10^{-5}$
6	$-1.8744012115149991 \times 10^{-5}$
7	$-4.1912943790191086 \times 10^{-7}$
8	$1.0371078064057119 \times 10^{-7}$
9	$1.0730529597056200 \times 10^{-8}$

	L (deg)
0	1.2821418482638470
1	$-7.0410363292941719 \times 10^{-1}$
2	$-1.7407531956724452 \times 10^{-1}$
3	$-2.4961210160791891 \times 10^{-2}$
4	$1.7362394893905792 \times 10^{-3}$
5	$6.3715361072967708 \times 10^{-4}$
6	$1.3851333582701236 \times 10^{-5}$
7	$-5.6970201567462405 \times 10^{-6}$
8	$-5.6922347256919958 \times 10^{-7}$
9	$1.5168111433747019 \times 10^{-8}$

Table 4–2, continued.

Interval 55: Central time $T_c = -640$, covering the time span $-680 \leq T \leq -600$

	ε (deg)		p_A (deg)
0	22.744310561541511	0	−891.98300120358755
1	$4.3420045989473412 \times 10^{-1}$	1	57.162697574033141
2	$5.9316204095902917 \times 10^{-2}$	2	$-1.9859594333931795 \times 10^{-1}$
3	$-5.9591395618072807 \times 10^{-3}$	3	$-1.8366997096703943 \times 10^{-2}$
4	$-4.9151631170658065 \times 10^{-4}$	4	$1.5756999053096775 \times 10^{-3}$
5	$2.7840940362483687 \times 10^{-5}$	5	$9.0600843123846575 \times 10^{-5}$
6	$1.6899790214789416 \times 10^{-6}$	6	$-8.1384970393169608 \times 10^{-6}$
7	$-7.8673624516262754 \times 10^{-8}$	7	$-2.1803904599163965 \times 10^{-7}$
8	$-7.6876692452382440 \times 10^{-9}$	8	$2.0436011097458063 \times 10^{-8}$
9	$-1.8053304069991831 \times 10^{-9}$	9	$2.7114069717513852 \times 10^{-9}$

	ψ_A (deg)		Δ (deg)
0	−893.16788964348552	0	−886.92029507609429
1	60.664170047413316	1	59.081271704121194
2	$7.5377448096065304 \times 10^{-1}$	2	$-1.9181995631267028 \times 10^{-1}$
3	$-1.5007800597347041 \times 10^{-1}$	3	$-7.1387117738976090 \times 10^{-2}$
4	$-3.5901830942180981 \times 10^{-2}$	4	$-9.4484037853610975 \times 10^{-4}$
5	$1.3114770745664066 \times 10^{-3}$	5	$9.3098376286096088 \times 10^{-4}$
6	$9.8765709450984626 \times 10^{-4}$	6	$2.8973400363891535 \times 10^{-5}$
7	$5.5182899766267316 \times 10^{-5}$	7	$-1.2245076723679683 \times 10^{-5}$
8	$-2.2776592665409828 \times 10^{-5}$	8	$-4.6142705230058666 \times 10^{-7}$
9	$-3.7452374937918741 \times 10^{-6}$	9	$1.6541685461754798 \times 10^{-7}$

	ω_A (deg)		I (deg)
0	21.167951398154167	0	21.678415937402728
1	$-6.7287688907606678 \times 10^{-1}$	1	$6.1811357445491295 \times 10^{-1}$
2	$3.6991897563174279 \times 10^{-1}$	2	$2.0463648796124360 \times 10^{-1}$
3	$6.8819590922041460 \times 10^{-2}$	3	$2.1744177070535398 \times 10^{-3}$
4	$-6.1303832883014705 \times 10^{-3}$	4	$-2.6267174447995471 \times 10^{-3}$
5	$-1.5575991858368848 \times 10^{-3}$	5	$-1.3356612388117867 \times 10^{-4}$
6	$1.6829495095834211 \times 10^{-5}$	6	$2.5056404900376633 \times 10^{-5}$
7	$2.8502119506198946 \times 10^{-5}$	7	$1.3939198383623706 \times 10^{-6}$
8	$2.1440922210206890 \times 10^{-6}$	8	$-2.5485392325222428 \times 10^{-7}$
9	$-4.7875205420186368 \times 10^{-7}$	9	$-1.4471016672934084 \times 10^{-8}$

	χ_A (deg)		L (deg)
0	−1.2783788248695038	0	−1.6284820548368483
1	3.7783187524889770	1	−2.0758682660679872
2	1.0234330662735664	2	$-1.3426218972244288 \times 10^{-2}$
3	$-1.3992203114723734 \times 10^{-1}$	3	$5.6232226244775661 \times 10^{-2}$
4	$-3.9120331944688193 \times 10^{-2}$	4	$2.8423911292944080 \times 10^{-3}$
5	$1.3114745990713352 \times 10^{-3}$	5	$-8.6387487063284159 \times 10^{-4}$
6	$1.0083860402237577 \times 10^{-3}$	6	$-3.9867472568035755 \times 10^{-5}$
7	$5.4769481145512130 \times 10^{-5}$	7	$1.2137898789087324 \times 10^{-5}$
8	$-2.2831478577331694 \times 10^{-5}$	8	$4.8938546619091656 \times 10^{-7}$
9	$-3.7444393318896444 \times 10^{-6}$	9	$-1.6293642319445033 \times 10^{-7}$

Table 4–2, continued.

Interval 56: Central time $T_c = -560$, covering the time span $-600 \leq T \leq -520$

	ε (deg)		p_A (deg)
0	23.799824442605268	0	−779.59485129572548
1	$5.3985393072628900 \times 10^{-1}$	1	55.189252712915944
2	$-3.7073046860345918 \times 10^{-2}$	2	$-2.4206443848096764 \times 10^{-1}$
3	$-8.3463076652863838 \times 10^{-3}$	3	$1.1140360620390110 \times 10^{-2}$
4	$1.9377082478842863 \times 10^{-4}$	4	$1.6562460648028255 \times 10^{-3}$
5	$3.1310195819334419 \times 10^{-5}$	5	$-4.4633181872942152 \times 10^{-5}$
6	$-1.0526180679759474 \times 10^{-7}$	6	$-5.3633863702666341 \times 10^{-7}$
7	$1.1662955141084704 \times 10^{-7}$	7	$6.9153112816837938 \times 10^{-8}$
8	$-2.5106413310637263 \times 10^{-9}$	8	$-6.7269711566195425 \times 10^{-8}$
9	$-3.5005056622132233 \times 10^{-9}$	9	$1.3731147630807152 \times 10^{-9}$

	ψ_A (deg)		Δ (deg)
0	−773.81074780039432	0	−772.39931115285856
1	56.745779662204956	1	55.035945629838865
2	−1.4151912824514722	2	$-6.6200435063658172 \times 10^{-1}$
3	$-6.4450542898862474 \times 10^{-3}$	3	$1.1521769679666886 \times 10^{-2}$
4	$2.9070789286951036 \times 10^{-2}$	4	$6.2133289262853687 \times 10^{-3}$
5	$-2.9452746011903531 \times 10^{-3}$	5	$-3.6905070990448694 \times 10^{-4}$
6	$-2.2975471087654026 \times 10^{-4}$	6	$-2.0737261724725023 \times 10^{-5}$
7	$1.0250873105834216 \times 10^{-4}$	7	$6.3489474788585453 \times 10^{-6}$
8	$-8.6186519412855560 \times 10^{-6}$	8	$-3.0358345277994089 \times 10^{-7}$
9	$-1.3521780550224043 \times 10^{-6}$	9	$-4.2843166489555694 \times 10^{-8}$

	ω_A (deg)		I (deg)
0	23.391048943288834	0	24.130690880152385
1	2.7322340017983781	1	1.6543576261802516
2	$1.8650000640892184 \times 10^{-1}$	2	$-5.9738058023769151 \times 10^{-3}$
3	$-8.3883890359749032 \times 10^{-2}$	3	$-3.0793661701731645 \times 10^{-2}$
4	$-1.8214659188295530 \times 10^{-3}$	4	$-3.3446508501358282 \times 10^{-4}$
5	$1.1285976572535966 \times 10^{-3}$	5	$2.0688798559365392 \times 10^{-4}$
6	$-8.5475390549334154 \times 10^{-5}$	6	$-6.3404774810512887 \times 10^{-6}$
7	$-6.0280208188479357 \times 10^{-6}$	7	$-7.6415684415233045 \times 10^{-7}$
8	$2.5356906364271589 \times 10^{-6}$	8	$1.3119746930006933 \times 10^{-7}$
9	$-2.2710856675088868 \times 10^{-7}$	9	$-3.2793418444015345 \times 10^{-9}$

	χ_A (deg)		L (deg)
0	6.3219010010467799	0	−3.9791715673969933
1	1.7700473342421428	1	$1.3261313288472506 \times 10^{-1}$
2	−1.2648476136824357	2	$4.5958324228112785 \times 10^{-1}$
3	$-2.8390752484396164 \times 10^{-2}$	3	$2.1365112577474752 \times 10^{-3}$
4	$2.8858443891725863 \times 10^{-2}$	4	$-4.9902552957433060 \times 10^{-3}$
5	$-2.7709758495782557 \times 10^{-3}$	5	$2.9388451289396786 \times 10^{-4}$
6	$-2.3857948923746791 \times 10^{-4}$	6	$2.2571046657716778 \times 10^{-5}$
7	$1.0178750973931084 \times 10^{-4}$	7	$-6.1597965697083012 \times 10^{-6}$
8	$-8.5253809334469028 \times 10^{-6}$	8	$2.2180906780281153 \times 10^{-7}$
9	$-1.3520356539289742 \times 10^{-6}$	9	$4.4083698532167454 \times 10^{-8}$

Table 4–2, continued.

Interval 57: Central time $T_c = -480$, covering the time span $-520 \leq T \leq -440$

	ε (deg)		p_A (deg)
0	24.334829547668370	0	-670.46352797321232
1	$-5.6383865783127021 \times 10^{-2}$	1	54.158248559655660
2	$-9.7753404196519787 \times 10^{-2}$	2	$1.4712878178280189 \times 10^{-2}$
3	$-2.7498108851475866 \times 10^{-4}$	3	$2.8202836832577981 \times 10^{-2}$
4	$7.6373065585593352 \times 10^{-4}$	4	$2.4903391254862161 \times 10^{-4}$
5	$1.1863341814698269 \times 10^{-5}$	5	$-1.0834663627034685 \times 10^{-4}$
6	$-2.8299194660246311 \times 10^{-6}$	6	$-2.1490654116030907 \times 10^{-6}$
7	$-5.2371765776553057 \times 10^{-8}$	7	$3.2546531395659094 \times 10^{-7}$
8	$2.5065062150571465 \times 10^{-8}$	8	$-1.3111730090599406 \times 10^{-9}$
9	$-8.2341847863770434 \times 10^{-12}$	9	$-5.9460660207190853 \times 10^{-9}$

	ψ_A (deg)		Δ (deg)
0	-668.92233379315063	0	-666.34924201285460
1	49.111347219282783	1	51.447201283858703
2	$-3.0877851876596262 \times 10^{-1}$	2	$-1.5189983525652938 \times 10^{-1}$
3	$1.1764279426479075 \times 10^{-1}$	3	$6.1133270330073132 \times 10^{-2}$
4	$-2.8491389820247250 \times 10^{-4}$	4	$1.4440389487996652 \times 10^{-3}$
5	$2.9935165152392913 \times 10^{-4}$	5	$2.5568272901917228 \times 10^{-6}$
6	$6.2092596775360438 \times 10^{-5}$	6	$1.4990961511126476 \times 10^{-5}$
7	$-7.7141425837762042 \times 10^{-6}$	7	$-1.4191799725938754 \times 10^{-6}$
8	$5.4838785232914021 \times 10^{-7}$	8	$-5.1577493520386814 \times 10^{-8}$
9	$-5.8831899436860253 \times 10^{-8}$	9	$-1.1133078337401267 \times 10^{-9}$

	ω_A (deg)		I (deg)
0	27.521499855119417	0	26.324432058761920
1	$7.5749589870886980 \times 10^{-1}$	1	$2.8841774315346667 \times 10^{-1}$
2	$-5.6973644392864703 \times 10^{-1}$	2	$-3.0247147852890394 \times 10^{-1}$
3	$-2.5091940747921943 \times 10^{-2}$	3	$-1.2503505934158088 \times 10^{-2}$
4	$5.8217956859003554 \times 10^{-3}$	4	$2.1778124028742151 \times 10^{-3}$
5	$6.1179323681796369 \times 10^{-5}$	5	$7.6748667166536192 \times 10^{-5}$
6	$-2.0981888056249664 \times 10^{-6}$	6	$-6.8577983959236220 \times 10^{-7}$
7	$8.4356231408737258 \times 10^{-7}$	7	$2.0134974329211368 \times 10^{-7}$
8	$-1.1500682425454475 \times 10^{-7}$	8	$-3.4897948733879030 \times 10^{-8}$
9	$1.4788586152238459 \times 10^{-8}$	9	$-2.9102684260153469 \times 10^{-10}$

	χ_A (deg)		L (deg)
0	1.7054051293216586	0	$-6.2594579259485657 \times 10^{-1}$
1	-5.5971033299995071	1	2.9868597192585820
2	$-3.7178269626441417 \times 10^{-1}$	2	$1.8612168657521097 \times 10^{-1}$
3	$1.0636770181282669 \times 10^{-1}$	3	$-3.8687699534048571 \times 10^{-2}$
4	$4.6333903260811111 \times 10^{-4}$	4	$-1.5813229797454659 \times 10^{-3}$
5	$2.3402305505803410 \times 10^{-4}$	5	$-7.5156384353284620 \times 10^{-5}$
6	$5.5578455627373264 \times 10^{-5}$	6	$-1.4413503840873531 \times 10^{-5}$
7	$-7.1543313385243818 \times 10^{-6}$	7	$1.7154477302314166 \times 10^{-6}$
8	$6.0263107671404105 \times 10^{-7}$	8	$4.0385357415061448 \times 10^{-8}$
9	$-5.5128635275963624 \times 10^{-8}$	9	$-6.2923970097529668 \times 10^{-9}$

Table 4–2, continued.

Interval 58: Central time $T_c = -400$, covering the time span $-440 \leq T \leq -360$

	ε (deg)		p_A (deg)
0	23.574191469359482	0	−561.02416113078988
1	$-6.4782759045488933 \times 10^{-1}$	1	55.523641284881468
2	$-3.0686217148761622 \times 10^{-2}$	2	$3.0113565076564857 \times 10^{-1}$
3	$1.0620513612950443 \times 10^{-2}$	3	$1.3524760641401174 \times 10^{-2}$
4	$4.3960339848863650 \times 10^{-4}$	4	$-2.1710139185470435 \times 10^{-3}$
5	$-4.0623661451170761 \times 10^{-5}$	5	$-1.3961563551206880 \times 10^{-4}$
6	$-2.7171433807954318 \times 10^{-6}$	6	$-1.5634603472208297 \times 10^{-6}$
7	$-2.0110065804206308 \times 10^{-7}$	7	$4.6852367029503444 \times 10^{-7}$
8	$-2.6649649855282858 \times 10^{-9}$	8	$1.1998299580650168 \times 10^{-7}$
9	$4.0414357660227734 \times 10^{-9}$	9	$6.2957093749796008 \times 10^{-9}$

	ψ_A (deg)		Δ (deg)
0	−568.32846440865175	0	−562.04232721097314
1	52.852135742478973	1	53.598687636722914
2	1.3443357838730611	2	$7.3341601656002549 \times 10^{-1}$
3	$1.6824819975172054 \times 10^{-1}$	3	$8.3978209026898476 \times 10^{-2}$
4	$9.3817007850730456 \times 10^{-4}$	4	$-2.8522657772278303 \times 10^{-4}$
5	$-1.9255368589278283 \times 10^{-3}$	5	$-6.0868113157361971 \times 10^{-4}$
6	$-4.5204809274121105 \times 10^{-4}$	6	$-1.0201018020386199 \times 10^{-4}$
7	$-4.9937971279918684 \times 10^{-5}$	7	$-8.5745021288593322 \times 10^{-6}$
8	$-1.1803593459583542 \times 10^{-6}$	8	$-5.5378025391536223 \times 10^{-8}$
9	$8.4205279264018807 \times 10^{-7}$	9	$1.0239033536863654 \times 10^{-7}$

	ω_A (deg)		I (deg)
0	24.705443339934489	0	24.498070488784511
1	−3.3261459766433014	1	−2.0280080578149208
2	$-2.7037993116907721 \times 10^{-1}$	2	$-1.9593119507799172 \times 10^{-1}$
3	$7.8092959513075367 \times 10^{-2}$	3	$3.3345326567036100 \times 10^{-2}$
4	$6.9069308746508562 \times 10^{-3}$	4	$3.3299616385572091 \times 10^{-3}$
5	$-6.1223236620861881 \times 10^{-5}$	5	$-9.8827808638564882 \times 10^{-6}$
6	$-4.3679936219926170 \times 10^{-5}$	6	$-1.4691362230690377 \times 10^{-5}$
7	$-9.2354874031319585 \times 10^{-6}$	7	$-1.8403921476330220 \times 10^{-6}$
8	$-1.2200218805290717 \times 10^{-6}$	8	$-1.6748399357013090 \times 10^{-7}$
9	$-7.3780357117864849 \times 10^{-8}$	9	$-7.9835458801998384 \times 10^{-9}$

	χ_A (deg)		L (deg)
0	−7.9818110561029016	0	4.9684520782295427
1	−2.8114712487400613	1	2.0475454962324707
2	1.1702693352709847	2	$-4.8692844774704878 \times 10^{-1}$
3	$1.5870325356871557 \times 10^{-1}$	3	$-7.4368205414737631 \times 10^{-2}$
4	$9.7346499909577583 \times 10^{-4}$	4	$-1.2473679262731122 \times 10^{-3}$
5	$-1.8196070728595005 \times 10^{-3}$	5	$5.0841830897478197 \times 10^{-4}$
6	$-4.3434324241272916 \times 10^{-4}$	6	$9.6742410369604366 \times 10^{-5}$
7	$-5.0352531628976773 \times 10^{-5}$	7	$8.8165322594865355 \times 10^{-6}$
8	$-1.3799221301009938 \times 10^{-6}$	8	$1.9613456558930528 \times 10^{-7}$
9	$8.3719350985046538 \times 10^{-7}$	9	$-9.4935450262122567 \times 10^{-8}$

Table 4–2, continued.

Interval 59: Central time $T_c = -320$, covering the time span $-360 \leq T \leq -280$

	ε (deg)
0	22.463147498508514
1	$-3.5290418963540869 \times 10^{-1}$
2	$9.8765309548472255 \times 10^{-2}$
3	$7.1942462746532136 \times 10^{-3}$
4	$-1.0030463335076273 \times 10^{-3}$
5	$-6.9584064103213497 \times 10^{-5}$
6	$5.6136134907975778 \times 10^{-6}$
7	$6.3343598387803981 \times 10^{-7}$
8	$-3.7461320739133972 \times 10^{-8}$
9	$-6.3737279951157934 \times 10^{-9}$

	ψ_A (deg)
0	-449.55988119309409
1	65.422761385437214
2	$6.0806384486202554 \times 10^{-1}$
3	$-4.5138370252998031 \times 10^{-1}$
4	$-2.8992406867009317 \times 10^{-2}$
5	$1.5044509700805361 \times 10^{-2}$
6	$1.5595366019086745 \times 10^{-3}$
7	$-5.4241915538895809 \times 10^{-4}$
8	$-8.3836913214917908 \times 10^{-5}$
9	$2.0544835961132659 \times 10^{-5}$

	ω_A (deg)
0	19.542939177233050
1	$-8.8705813496542645 \times 10^{-1}$
2	$8.0875207339236894 \times 10^{-1}$
3	$2.5519820939034199 \times 10^{-2}$
4	$-2.0813605477955283 \times 10^{-2}$
5	$-9.5062633390665190 \times 10^{-4}$
6	$4.9153415953003442 \times 10^{-4}$
7	$4.3778811194135337 \times 10^{-5}$
8	$-1.4518579889718029 \times 10^{-5}$
9	$-2.0677711469041782 \times 10^{-6}$

	χ_A (deg)
0	-2.1281624570717103
1	8.1628458586228472
2	$4.2972172342570941 \times 10^{-1}$
3	$-4.3501435246392860 \times 10^{-1}$
4	$-2.6357705113500049 \times 10^{-2}$
5	$1.4980242249900869 \times 10^{-2}$
6	$1.5313210692592232 \times 10^{-3}$
7	$-5.4184854567390162 \times 10^{-4}$
8	$-8.3543195662782996 \times 10^{-5}$
9	$2.0535830638245434 \times 10^{-5}$

	p_A (deg)
0	-447.58761870259203
1	57.817795178960645
2	$1.8563501467235109 \times 10^{-1}$
3	$-3.4351565908937352 \times 10^{-2}$
4	$-2.5240364621409054 \times 10^{-3}$
5	$2.4726384312956641 \times 10^{-4}$
6	$2.6479810304737464 \times 10^{-5}$
7	$-1.3848285594655007 \times 10^{-6}$
8	$-2.8681821639477914 \times 10^{-7}$
9	$9.2573862502563005 \times 10^{-9}$

	Δ (deg)
0	-447.06368040453614
1	61.532791823608563
2	$8.5455397462722174 \times 10^{-1}$
3	$-1.4075858891382246 \times 10^{-1}$
4	$-2.5527796300677951 \times 10^{-2}$
5	$1.1582125284532692 \times 10^{-3}$
6	$6.3113456661667656 \times 10^{-4}$
7	$2.3620637715044409 \times 10^{-5}$
8	$-1.3321592382665690 \times 10^{-5}$
9	$-1.6870536612641162 \times 10^{-6}$

	I (deg)
0	20.621641650672797
1	-1.3451794183657602
2	$3.9162179829662917 \times 10^{-1}$
3	$4.3242644746695211 \times 10^{-2}$
4	$-5.2542087768089580 \times 10^{-3}$
5	$-8.2214980507330157 \times 10^{-4}$
6	$2.6017779117076013 \times 10^{-5}$
7	$1.5203671457293705 \times 10^{-5}$
8	$6.9869804237203306 \times 10^{-7}$
9	$-2.5767195177114539 \times 10^{-7}$

	L (deg)
0	3.2247613952157315
1	-4.0203144084596442
2	$-7.0140274694816031 \times 10^{-1}$
3	$1.1341797764779054 \times 10^{-1}$
4	$2.3418459695737019 \times 10^{-2}$
5	$-9.6471077228283791 \times 10^{-4}$
6	$-6.0677308395712403 \times 10^{-4}$
7	$-2.4814644592863414 \times 10^{-5}$
8	$1.3037324866351453 \times 10^{-5}$
9	$1.6966082736926935 \times 10^{-6}$

Table 4-2, continued.

Interval 60: Central time $T_c = -240$, covering the time span $-280 \leq T \leq -200$

	ε (deg)		p_A (deg)
0	22.598104164804944	0	-331.93506501025057
1	$4.6690898113276798 \times 10^{-1}$	1	57.456986285829320
2	$7.4431346171800529 \times 10^{-2}$	2	$-2.5539586837992226 \times 10^{-1}$
3	$-1.0228899171043822 \times 10^{-2}$	3	$-2.3474621132353963 \times 10^{-2}$
4	$-6.3567055144871685 \times 10^{-4}$	4	$3.3066981975698494 \times 10^{-3}$
5	$8.5474967242113882 \times 10^{-5}$	5	$9.9838389963510629 \times 10^{-5}$
6	$1.6701288441491667 \times 10^{-6}$	6	$-2.6086609173075964 \times 10^{-5}$
7	$-4.7636630231916178 \times 10^{-7}$	7	$4.1920298177428293 \times 10^{-7}$
8	$8.0303787887357666 \times 10^{-9}$	8	$1.4887830308422896 \times 10^{-7}$
9	$5.0880955033667750 \times 10^{-10}$	9	$-1.0387630676221894 \times 10^{-8}$

	ψ_A (deg)		Δ (deg)
0	-325.65857610493582	0	-324.17239775242410
1	56.703475324144286	1	59.619447181894713
2	-1.4977402316023704	2	-1.1101298116805860
3	$1.4975437135902907 \times 10^{-1}$	3	$-3.6335558725635137 \times 10^{-2}$
4	$1.0967184072960645 \times 10^{-2}$	4	$2.3424158300830290 \times 10^{-2}$
5	$-4.4776437035836965 \times 10^{-3}$	5	$-1.7008559022637721 \times 10^{-3}$
6	$6.5790197174121400 \times 10^{-4}$	6	$-2.4261868168695060 \times 10^{-4}$
7	$-2.4473037641650113 \times 10^{-5}$	7	$6.4629764842133340 \times 10^{-5}$
8	$-1.1793858467666097 \times 10^{-5}$	8	$-2.7159161765901813 \times 10^{-6}$
9	$3.1140077366765821 \times 10^{-6}$	9	$-1.1050052362623751 \times 10^{-6}$

	ω_A (deg)		I (deg)
0	22.257511107936989	0	21.325577422337379
1	2.8042522336869273	1	1.9066070748154363
2	$-7.5585543994549134 \times 10^{-2}$	2	$2.3171033859304919 \times 10^{-1}$
3	$-8.3894172978007407 \times 10^{-2}$	3	$-5.6332055342495801 \times 10^{-2}$
4	$7.7632878184521002 \times 10^{-3}$	4	$-9.5773007245280124 \times 10^{-4}$
5	$2.8904563904764768 \times 10^{-4}$	5	$7.1788082020033978 \times 10^{-4}$
6	$-1.1594764696798879 \times 10^{-4}$	6	$-5.1210324388732068 \times 10^{-5}$
7	$1.7245365717698811 \times 10^{-5}$	7	$-4.5809040229656911 \times 10^{-6}$
8	$-1.2597531358655018 \times 10^{-6}$	8	$1.3956439384984607 \times 10^{-6}$
9	$-1.3875974920243878 \times 10^{-7}$	9	$-8.2456737156360610 \times 10^{-8}$

	χ_A (deg)		L (deg)
0	6.7850166249801857	0	-4.5975295091966272
1	$-7.4236540042557649 \times 10^{-1}$	1	-2.3679972036158352
2	-1.3460915501529934	2	$9.0729879644969719 \times 10^{-1}$
3	$1.7711573177807018 \times 10^{-1}$	3	$1.7192678262369842 \times 10^{-2}$
4	$9.4396918650313929 \times 10^{-3}$	4	$-2.0790451328764896 \times 10^{-2}$
5	$-4.6456368774917111 \times 10^{-3}$	5	$1.7705109817391301 \times 10^{-3}$
6	$6.7020316171431714 \times 10^{-4}$	6	$2.2040727224368029 \times 10^{-4}$
7	$-2.4490405303318885 \times 10^{-5}$	7	$-6.4101216219050969 \times 10^{-5}$
8	$-1.1867319378632323 \times 10^{-5}$	8	$2.8482836367796071 \times 10^{-6}$
9	$3.1236569815756107 \times 10^{-6}$	9	$1.0944843690937131 \times 10^{-6}$

Table 4–2, continued.

Interval 61: Central time $T_c = -160$, covering the time span $-200 \leq T \leq -120$

	ε (deg)
0	23.699391439256386
1	$5.2330816033981775 \times 10^{-1}$
2	$-5.6259493384864815 \times 10^{-2}$
3	$-8.2033318431602032 \times 10^{-3}$
4	$6.6774163554156385 \times 10^{-4}$
5	$2.4931584012812606 \times 10^{-5}$
6	$-3.1313623302407878 \times 10^{-6}$
7	$2.0343814827951515 \times 10^{-7}$
8	$2.9182026615852936 \times 10^{-9}$
9	$-4.1118760893281951 \times 10^{-9}$

	p_A (deg)
0	-219.33324374500648
1	55.162402175733518
2	$-2.3845757488118081 \times 10^{-1}$
3	$2.1088634258865188 \times 10^{-2}$
4	$1.4694510660810244 \times 10^{-3}$
5	$-1.5091212247173072 \times 10^{-4}$
6	$3.9419854674750420 \times 10^{-6}$
7	$2.7669007558290217 \times 10^{-7}$
8	$-1.0124724406180637 \times 10^{-7}$
9	$3.6181634267406468 \times 10^{-9}$

	ψ_A (deg)
0	-218.57864954903122
1	51.752257487741612
2	$1.3304715765661958 \times 10^{-1}$
3	$9.2048123521890745 \times 10^{-2}$
4	$-6.0877528127241278 \times 10^{-3}$
5	$-7.0013893644531700 \times 10^{-5}$
6	$-4.9217728385458495 \times 10^{-5}$
7	$-1.8578234189053723 \times 10^{-6}$
8	$7.4396426162029877 \times 10^{-7}$
9	$-5.9157528981843864 \times 10^{-9}$

	Δ (deg)
0	-212.53649464223264
1	52.601179136870115
2	$-4.5828988085167103 \times 10^{-1}$
3	$8.0588172629387582 \times 10^{-2}$
4	$-1.5193849105236105 \times 10^{-3}$
5	$-1.4488131973252688 \times 10^{-4}$
6	$4.5296047939943364 \times 10^{-5}$
7	$-6.1646187803246070 \times 10^{-6}$
8	$4.2022248222522481 \times 10^{-7}$
9	$-6.9101125561169980 \times 10^{-9}$

	ω_A (deg)
0	25.541291140949806
1	$2.3778895112721623 \times 10^{-1}$
2	$-3.7337334723142133 \times 10^{-1}$
3	$2.4579295485161534 \times 10^{-2}$
4	$4.3840999514263623 \times 10^{-3}$
5	$-3.1126873333599556 \times 10^{-4}$
6	$-9.8443045771748915 \times 10^{-6}$
7	$-7.9403103080496923 \times 10^{-7}$
8	$1.0840116743893556 \times 10^{-9}$
9	$9.2865105216887919 \times 10^{-9}$

	I (deg)
0	25.113293357688533
1	1.4548120719905701
2	$-2.7313930989863579 \times 10^{-1}$
3	$-1.8698492046169593 \times 10^{-2}$
4	$3.1828200121792599 \times 10^{-3}$
5	$-5.0231835441076053 \times 10^{-5}$
6	$-4.2588784801858854 \times 10^{-6}$
7	$7.0016072001988432 \times 10^{-7}$
8	$-9.8027621808098010 \times 10^{-8}$
9	$9.5163891399824242 \times 10^{-9}$

	χ_A (deg)
0	$8.2378850337329404 \times 10^{-1}$
1	-3.7443109739678667
2	$4.0143936898854026 \times 10^{-1}$
3	$8.1822830214590811 \times 10^{-2}$
4	$-8.5978790792656293 \times 10^{-3}$
5	$-2.8350488448426132 \times 10^{-5}$
6	$-4.2474671728156727 \times 10^{-5}$
7	$-1.6214840884656678 \times 10^{-6}$
8	$7.8560442001953050 \times 10^{-7}$
9	$-1.0320166416967075 \times 10^{-8}$

	L (deg)
0	-3.5954857535537836
1	2.7893472819993091
2	$2.5680743608790162 \times 10^{-1}$
3	$-6.5588744937483328 \times 10^{-2}$
4	$2.6350864315171937 \times 10^{-3}$
5	$4.6607869327948861 \times 10^{-5}$
6	$-3.9809272397874433 \times 10^{-5}$
7	$6.2024562749848758 \times 10^{-6}$
8	$-5.3075941703763470 \times 10^{-7}$
9	$1.1169782184523872 \times 10^{-8}$

Table 4–2, continued.

Interval 62: Central time $T_c = -80$, covering the time span $-120 \leq T \leq -40$

	ε (deg)		p_A (deg)
0	24.124759551704588	0	-109.96448951413929
1	$-1.2094875596566286 \times 10^{-1}$	1	54.467715869189325
2	$-8.3914869653015218 \times 10^{-2}$	2	$7.0812041661422475 \times 10^{-2}$
3	$3.5357075322387405 \times 10^{-3}$	3	$2.4751250747271759 \times 10^{-2}$
4	$6.4557467824807032 \times 10^{-4}$	4	$-8.3645216079071869 \times 10^{-4}$
5	$-2.5092064378707704 \times 10^{-5}$	5	$-9.0691561755105921 \times 10^{-5}$
6	$-1.7631607274450848 \times 10^{-6}$	6	$2.6888489324899396 \times 10^{-6}$
7	$1.3363622791424094 \times 10^{-7}$	7	$1.6441130200543915 \times 10^{-7}$
8	$1.5577817511054047 \times 10^{-8}$	8	$-2.6912609333408592 \times 10^{-8}$
9	$-2.4613907093017122 \times 10^{-9}$	9	$-5.5832760667537456 \times 10^{-9}$

	ψ_A (deg)		Δ (deg)
0	-111.94350527506128	0	-108.25884858484431
1	55.175558131675861	1	52.349027226042447
2	$4.7366115762797613 \times 10^{-1}$	2	$3.4893381308555388 \times 10^{-1}$
3	$-4.7701750975398538 \times 10^{-2}$	3	$5.3833786104036534 \times 10^{-2}$
4	$-9.2445765329325809 \times 10^{-3}$	4	$-1.9954650556701174 \times 10^{-3}$
5	$7.0962838707454917 \times 10^{-4}$	5	$-1.5268691030085015 \times 10^{-4}$
6	$1.5140455277814658 \times 10^{-4}$	6	$-2.4190557815900544 \times 10^{-5}$
7	$-7.7813159018954928 \times 10^{-7}$	7	$-7.0673251488384895 \times 10^{-7}$
8	$-2.4729402281953378 \times 10^{-6}$	8	$1.2570991731562238 \times 10^{-7}$
9	$-1.0898887008726418 \times 10^{-7}$	9	$1.4907548140220873 \times 10^{-8}$

	ω_A (deg)		I (deg)
0	24.429357654237926	0	25.675797175777276
1	$-9.5205745947740161 \times 10^{-1}$	1	$-8.5759662165341842 \times 10^{-1}$
2	$8.6738296270534816 \times 10^{-2}$	2	$-2.3425450978840602 \times 10^{-1}$
3	$3.0061543426062955 \times 10^{-2}$	3	$2.1572234877403812 \times 10^{-2}$
4	$-4.1532480523019988 \times 10^{-3}$	4	$1.7385086245295523 \times 10^{-3}$
5	$-3.7920928393860939 \times 10^{-4}$	5	$-9.4433708492985720 \times 10^{-5}$
6	$3.5117012399609737 \times 10^{-5}$	6	$-3.8153398284061494 \times 10^{-6}$
7	$4.6811877283079217 \times 10^{-6}$	7	$-4.0978220999957025 \times 10^{-7}$
8	$-8.1836046585546861 \times 10^{-8}$	8	$-1.8054745227662793 \times 10^{-8}$
9	$-6.1803706664211173 \times 10^{-8}$	9	$2.8035012061081817 \times 10^{-9}$

	χ_A (deg)		L (deg)
0	-2.1726062070318606	0	2.0020744183260302
1	$7.8470515033132925 \times 10^{-1}$	1	2.3353002424891801
2	$4.4044931004195718 \times 10^{-1}$	2	$-3.1342008401332741 \times 10^{-1}$
3	$-8.0671247169971653 \times 10^{-2}$	3	$-3.2738907275091193 \times 10^{-2}$
4	$-8.9672662444325007 \times 10^{-3}$	4	$1.6910241935722453 \times 10^{-3}$
5	$9.2248978383109719 \times 10^{-4}$	5	$7.9103449586165224 \times 10^{-5}$
6	$1.5143472266372874 \times 10^{-4}$	6	$2.3872301666332189 \times 10^{-5}$
7	$-1.6387009056475679 \times 10^{-6}$	7	$8.7654392349113181 \times 10^{-7}$
8	$-2.4405558979328144 \times 10^{-6}$	8	$-1.4963759356787623 \times 10^{-7}$
9	$-1.0148113464009015 \times 10^{-7}$	9	$-2.1528281786737335 \times 10^{-8}$

Table 4–2, continued.

Interval 63: Central time $T_c = 0$, covering the time span $-40 \leq T \leq 40$

	ε (deg)		p_A (deg)
0	23.439103144206208	0	$2.4087337790992488 \times 10^{-1}$
1	$-4.9386077073143590 \times 10^{-1}$	1	55.879597096104303
2	$-2.3965445283267805 \times 10^{-4}$	2	$2.3881374134310689 \times 10^{-1}$
3	$8.6637485629656489 \times 10^{-3}$	3	$2.0316360191747588 \times 10^{-4}$
4	$-5.2828151901367600 \times 10^{-5}$	4	$-2.0532670699176742 \times 10^{-3}$
5	$-4.3951004595359217 \times 10^{-5}$	5	$-2.6127438417453855 \times 10^{-5}$
6	$-1.1058785949914705 \times 10^{-6}$	6	$6.3912563991077935 \times 10^{-6}$
7	$6.2431490022621172 \times 10^{-8}$	7	$8.2072342047739535 \times 10^{-7}$
8	$3.4725376218710764 \times 10^{-8}$	8	$2.1759498799003301 \times 10^{-8}$
9	$1.3658853127005757 \times 10^{-9}$	9	$-1.2292742158783186 \times 10^{-8}$

	ψ_A (deg)		Δ (deg)
0	$-2.0414520115294410 \times 10^{-1}$	0	$6.2615291005289149 \times 10^{-1}$
1	55.969995858494106	1	56.771213709867143
2	$-1.9295093699770936 \times 10^{-1}$	2	$6.1835281382298593 \times 10^{-1}$
3	$-5.6819574830421158 \times 10^{-3}$	3	$-2.5128752776055107 \times 10^{-2}$
4	$1.1073687302518981 \times 10^{-2}$	4	$-7.7222489343378689 \times 10^{-3}$
5	$-9.0868489896815619 \times 10^{-5}$	5	$-3.4208049847247442 \times 10^{-6}$
6	$-1.1999773777895820 \times 10^{-4}$	6	$7.7231678081801024 \times 10^{-5}$
7	$9.9748697306154409 \times 10^{-6}$	7	$5.3289056505918707 \times 10^{-6}$
8	$5.7911493603430550 \times 10^{-7}$	8	$-6.1561748589433599 \times 10^{-7}$
9	$-2.3647526839778175 \times 10^{-7}$	9	$-1.0593850942161246 \times 10^{-7}$

	ω_A (deg)		I (deg)
0	23.450465062489337	0	23.114556988665967
1	$-9.7259278279739817 \times 10^{-2}$	1	-1.4162765443398956
2	$1.1082286925130981 \times 10^{-2}$	2	$1.0404207539493511 \times 10^{-1}$
3	$-3.1469883339372219 \times 10^{-2}$	3	$2.6315522885852885 \times 10^{-2}$
4	$-1.0041906996819648 \times 10^{-4}$	4	$-1.6218541171038529 \times 10^{-3}$
5	$5.6455168475133958 \times 10^{-4}$	5	$-2.1656331976588425 \times 10^{-4}$
6	$-8.4403910211030209 \times 10^{-6}$	6	$5.0785607890280076 \times 10^{-6}$
7	$-3.8269157371098435 \times 10^{-6}$	7	$1.8483549770338827 \times 10^{-6}$
8	$3.1422585261198437 \times 10^{-7}$	8	$7.4962416484795782 \times 10^{-8}$
9	$9.3481729116773404 \times 10^{-9}$	9	$-1.3869774297054672 \times 10^{-8}$

	χ_A (deg)		L (deg)
0	$-4.8518673570735556 \times 10^{-1}$	0	3.4380222430370099
1	$1.0016737299946743 \times 10^{-1}$	1	$-9.8690360675407852 \times 10^{-1}$
2	$-4.7074888613099918 \times 10^{-1}$	2	$-4.0859346678513061 \times 10^{-1}$
3	$-5.8604054305076092 \times 10^{-3}$	3	$2.9203376233848717 \times 10^{-2}$
4	$1.4300208240553435 \times 10^{-2}$	4	$5.8849879702565663 \times 10^{-3}$
5	$-6.7127991650300028 \times 10^{-5}$	5	$-6.2994538219749156 \times 10^{-5}$
6	$-1.3703764889645475 \times 10^{-4}$	6	$-7.1439474118375438 \times 10^{-5}$
7	$9.0505213684444634 \times 10^{-6}$	7	$-4.2764500331174845 \times 10^{-6}$
8	$6.0368690647808607 \times 10^{-7}$	8	$6.4051126256827420 \times 10^{-7}$
9	$-2.2135404747652171 \times 10^{-7}$	9	$9.1918098138014373 \times 10^{-8}$

Table 4–2, continued.

Interval 64: Central time $T_c = 80$, covering the time span $40 \leq T \leq 120$

	ε (deg)		p_A (deg)
0	22.724671295125046	0	113.54876640151571
1	$-1.6041813558650337 \times 10^{-1}$	1	57.282789498972980
2	$7.0646783888132504 \times 10^{-2}$	2	$6.7803317258278676 \times 10^{-2}$
3	$1.4967806745062837 \times 10^{-3}$	3	$-2.4077742741672229 \times 10^{-2}$
4	$-6.6857270989190734 \times 10^{-4}$	4	$-2.7789234921969609 \times 10^{-4}$
5	$5.7578378071604775 \times 10^{-6}$	5	$1.6342157486452080 \times 10^{-4}$
6	$3.3738508454638728 \times 10^{-6}$	6	$-2.5014526964897600 \times 10^{-6}$
7	$-2.2917813537654764 \times 10^{-7}$	7	$-9.8604231978433443 \times 10^{-7}$
8	$-2.1019907929218137 \times 10^{-8}$	8	$8.0899174511758399 \times 10^{-8}$
9	$4.3139832091694682 \times 10^{-9}$	9	$8.2099058553618403 \times 10^{-9}$

	ψ_A (deg)		Δ (deg)
0	111.61366860604471	0	117.09044608667767
1	56.404525305162447	1	59.060661124248742
2	$4.4403302410703782 \times 10^{-1}$	2	$-1.1131569628378585 \times 10^{-1}$
3	$7.1490030578883907 \times 10^{-2}$	3	$-5.9650051230704656 \times 10^{-2}$
4	$-4.9184559079790816 \times 10^{-3}$	4	$5.3930420949652662 \times 10^{-3}$
5	$-1.3912698949042046 \times 10^{-3}$	5	$4.7847834528190912 \times 10^{-4}$
6	$-6.8490613661884005 \times 10^{-5}$	6	$-9.0393920612077870 \times 10^{-5}$
7	$1.2394328562905297 \times 10^{-6}$	7	$-2.1155140324960377 \times 10^{-6}$
8	$1.7719847841480384 \times 10^{-6}$	8	$1.2181773104988007 \times 10^{-6}$
9	$2.4889095220628068 \times 10^{-7}$	9	$-2.9359115676942150 \times 10^{-8}$

	ω_A (deg)		I (deg)
0	22.581778052947806	0	21.662229672877546
1	$-8.7069701538602037 \times 10^{-1}$	1	$3.1936441409272980 \times 10^{-2}$
2	$-9.8140710050197307 \times 10^{-2}$	2	$1.8546649576387279 \times 10^{-1}$
3	$2.6025931340678079 \times 10^{-2}$	3	$-1.2926246425312430 \times 10^{-2}$
4	$4.8165322168786755 \times 10^{-3}$	4	$-1.6598870615338844 \times 10^{-3}$
5	$-1.9065587721933634 \times 10^{-4}$	5	$2.2905728674660311 \times 10^{-4}$
6	$-4.6838759635421777 \times 10^{-5}$	6	$7.2898885304771316 \times 10^{-6}$
7	$-1.6608525315998471 \times 10^{-6}$	7	$-2.3050414336141165 \times 10^{-6}$
8	$-3.2347811293516124 \times 10^{-8}$	8	$1.0207190221594289 \times 10^{-8}$
9	$2.8104728109642000 \times 10^{-8}$	9	$2.1267933912813754 \times 10^{-8}$

	χ_A (deg)		L (deg)
0	−2.0950740076326087	0	$9.5437065030987305 \times 10^{-3}$
1	$-9.4447359463206877 \times 10^{-1}$	1	−1.9176927322805243
2	$4.0940512860493755 \times 10^{-1}$	2	$1.9333186102851091 \times 10^{-1}$
3	$1.0261699700263508 \times 10^{-1}$	3	$3.7496899517722484 \times 10^{-2}$
4	$-5.3133241571955160 \times 10^{-3}$	4	$-6.0027489222746884 \times 10^{-3}$
5	$-1.6634631550720911 \times 10^{-3}$	5	$-3.1915287891149424 \times 10^{-4}$
6	$-5.9477519536647907 \times 10^{-5}$	6	$9.0034532891154015 \times 10^{-5}$
7	$2.9651387319208926 \times 10^{-6}$	7	$1.1080885332971737 \times 10^{-6}$
8	$1.6434499452070584 \times 10^{-6}$	8	$-1.1372233207379882 \times 10^{-6}$
9	$2.3720647656961084 \times 10^{-7}$	9	$3.7803189897816002 \times 10^{-8}$

Table 4–2, continued.

Interval 65: Central time $T_c = 160$, covering the time span $120 \leq T \leq 200$

	ε (deg)
0	22.914636050333696
1	$3.2123508304962416 \times 10^{-1}$
2	$3.6633220173792710 \times 10^{-2}$
3	$-5.9228324767696043 \times 10^{-3}$
4	$-1.8823791073793285 \times 10^{-4}$
5	$3.2274552870236244 \times 10^{-5}$
6	$4.9052463646336507 \times 10^{-7}$
7	$-5.9064298731578425 \times 10^{-8}$
8	$-2.0485712675098837 \times 10^{-8}$
9	$-6.2163304813908160 \times 10^{-10}$

	p_A (deg)
0	227.82379125957118
1	56.796821557877339
2	$-1.6313931531416873 \times 10^{-1}$
3	$-9.7843402892868513 \times 10^{-3}$
4	$1.5713899130418396 \times 10^{-3}$
5	$1.6064725289735939 \times 10^{-5}$
6	$-6.5702311622027896 \times 10^{-6}$
7	$-1.2438648974944445 \times 10^{-7}$
8	$6.9998450072857530 \times 10^{-9}$
9	$7.1668796858429270 \times 10^{-9}$

	ψ_A (deg)
0	228.40683531269390
1	60.056143904919826
2	$2.9583200718478960 \times 10^{-2}$
3	$-1.5710838319490748 \times 10^{-1}$
4	$-7.0017356811600801 \times 10^{-3}$
5	$3.3009615142224537 \times 10^{-3}$
6	$2.0318123852537664 \times 10^{-4}$
7	$-6.5840216067828310 \times 10^{-5}$
8	$-5.9077673352976155 \times 10^{-6}$
9	$1.3983942185303064 \times 10^{-6}$

	Δ (deg)
0	233.15704592339831
1	56.853168369398135
2	$-3.0539872356159629 \times 10^{-1}$
3	$1.4060654910629863 \times 10^{-2}$
4	$1.1109823645741077 \times 10^{-3}$
5	$-3.7130414834222326 \times 10^{-4}$
6	$2.2973779820402573 \times 10^{-5}$
7	$1.4971679533428247 \times 10^{-6}$
8	$-2.6022651367242367 \times 10^{-7}$
9	$1.5190509398213489 \times 10^{-8}$

	ω_A (deg)
0	21.518861835737142
1	$2.0494789509441385 \times 10^{-1}$
2	$3.5193604846503161 \times 10^{-1}$
3	$1.5305977982348925 \times 10^{-2}$
4	$-7.5015367726336455 \times 10^{-3}$
5	$-4.0322553186065610 \times 10^{-4}$
6	$1.0655320434844041 \times 10^{-4}$
7	$7.1792339586935752 \times 10^{-6}$
8	$-1.6038746975430208 \times 10^{-6}$
9	$-1.6135634628135124 \times 10^{-7}$

	I (deg)
0	22.614813166560928
1	$7.6653571384938099 \times 10^{-1}$
2	$6.2866045410161332 \times 10^{-3}$
3	$-9.5364769546644450 \times 10^{-3}$
4	$1.1751452872783874 \times 10^{-3}$
5	$-1.7726246438498393 \times 10^{-5}$
6	$-1.2197719270429943 \times 10^{-5}$
7	$6.9895194269472982 \times 10^{-7}$
8	$3.7435936837838101 \times 10^{-8}$
9	$-3.6671444523199732 \times 10^{-9}$

	χ_A (deg)
0	$6.3315163285678715 \times 10^{-1}$
1	3.5241082918420464
2	$2.1223076605364606 \times 10^{-1}$
3	$-1.5648122502767368 \times 10^{-1}$
4	$-9.1964075390801980 \times 10^{-3}$
5	$3.3896161239812411 \times 10^{-3}$
6	$2.1485178626085787 \times 10^{-4}$
7	$-6.6261759864793735 \times 10^{-5}$
8	$-5.9257969712852667 \times 10^{-6}$
9	$1.3918759086160525 \times 10^{-6}$

	L (deg)
0	-1.9320943594035367
1	$-6.9879024986415078 \times 10^{-2}$
2	$1.5320475639045154 \times 10^{-1}$
3	$-2.5440846435685551 \times 10^{-2}$
4	$4.5995057921348084 \times 10^{-4}$
5	$4.1426663618896325 \times 10^{-4}$
6	$-3.0181311030904185 \times 10^{-5}$
7	$-1.8296328903418022 \times 10^{-6}$
8	$2.6920761034203793 \times 10^{-7}$
9	$-6.2874055435225260 \times 10^{-9}$

Table 4–2, continued.

Interval 66: Central time $T_c = 240$, covering the time span $200 \leq T \leq 280$

	ε (deg)		p_A (deg)
0	23.620048819541617	0	340.02507479226217
1	$3.2564049837437033 \times 10^{-1}$	1	55.421486119152083
2	$-3.2686490632424118 \times 10^{-2}$	2	$-1.4457515055703654 \times 10^{-1}$
3	$-4.4410216723393906 \times 10^{-3}$	3	$1.0518613011892852 \times 10^{-2}$
4	$2.6834134321874344 \times 10^{-4}$	4	$7.1788951829175359 \times 10^{-4}$
5	$3.9470526453015540 \times 10^{-6}$	5	$-5.3844470721134160 \times 10^{-5}$
6	$-1.0731142764254488 \times 10^{-6}$	6	$3.5314316476125000 \times 10^{-6}$
7	$2.1553035606317601 \times 10^{-7}$	7	$1.6399544950047553 \times 10^{-7}$
8	$6.6115990122320467 \times 10^{-9}$	8	$-7.5577119610702621 \times 10^{-8}$
9	$-3.6107650968390020 \times 10^{-9}$	9	$-6.4322908460251293 \times 10^{-10}$

	ψ_A (deg)		Δ (deg)
0	344.16918848460536	0	344.93159314252976
1	54.889896552644593	1	55.038136954558513
2	$-9.1524281984994033 \times 10^{-1}$	2	$-1.6815268323026177 \times 10^{-1}$
3	$4.5344728705302101 \times 10^{-2}$	3	$4.9740519719885990 \times 10^{-3}$
4	$1.1688295626550651 \times 10^{-2}$	4	$-5.7151142721135480 \times 10^{-4}$
5	$-1.4802462079668693 \times 10^{-3}$	5	$1.2601387427791232 \times 10^{-4}$
6	$3.6868826817591717 \times 10^{-5}$	6	$6.9958494903652407 \times 10^{-6}$
7	$2.0314566097090983 \times 10^{-5}$	7	$-1.1758076198722669 \times 10^{-6}$
8	$-3.2883667669827023 \times 10^{-6}$	8	$3.2903521315458975 \times 10^{-8}$
9	$9.8514296357951999 \times 10^{-8}$	9	$3.7546515257179439 \times 10^{-9}$

	ω_A (deg)		I (deg)
0	24.010934194609892	0	24.001533096328788
1	1.9331531229381335	1	$5.8611875168997349 \times 10^{-1}$
2	$-5.1677906702220989 \times 10^{-2}$	2	$-3.7801123221603511 \times 10^{-2}$
3	$-5.6248945537501128 \times 10^{-2}$	3	$-1.8860805396657119 \times 10^{-3}$
4	$1.4460822025717588 \times 10^{-3}$	4	$-3.1485401991065393 \times 10^{-4}$
5	$4.9772960743843795 \times 10^{-4}$	5	$-5.4470026825394331 \times 10^{-5}$
6	$-4.1477005015630228 \times 10^{-5}$	6	$6.2608422188121149 \times 10^{-6}$
7	$5.2268414589958931 \times 10^{-7}$	7	$1.7926628190325202 \times 10^{-7}$
8	$4.2946315781388555 \times 10^{-7}$	8	$-3.5687903992898311 \times 10^{-8}$
9	$-6.9305537041157127 \times 10^{-8}$	9	$1.8128594652594354 \times 10^{-9}$

	χ_A (deg)		L (deg)
0	4.5273669877727498	0	-1.4865054783732750
1	$-5.4533649002493734 \times 10^{-1}$	1	$4.0890102614996216 \times 10^{-1}$
2	$-8.4562860130179433 \times 10^{-1}$	2	$2.6560686648193652 \times 10^{-2}$
3	$3.3437107893214359 \times 10^{-2}$	3	$6.1221515556220007 \times 10^{-3}$
4	$1.2310363102966642 \times 10^{-2}$	4	$1.4119136993602258 \times 10^{-3}$
5	$-1.3989093387407467 \times 10^{-3}$	5	$-1.9507633777099785 \times 10^{-4}$
6	$2.2859874442206551 \times 10^{-5}$	6	$-4.4578780125956498 \times 10^{-6}$
7	$1.9904379723040017 \times 10^{-5}$	7	$1.4602971636465574 \times 10^{-6}$
8	$-3.1649442549018760 \times 10^{-6}$	8	$-1.1281995213706160 \times 10^{-7}$
9	$1.0117980952289590 \times 10^{-7}$	9	$-5.1451628073297890 \times 10^{-9}$

Table 4–2, continued.

Interval 67: Central time $T_c = 320$, covering the time span $280 \leq T \leq 360$

	ε (deg)		p_A (deg)
0	23.895624902078270	0	450.20980239440306
1	$-7.1361666411040255 \times 10^{-2}$	1	54.906227820549062
2	$-5.8058696285206076 \times 10^{-2}$	2	$2.5821999431832539 \times 10^{-2}$
3	$4.5066525163128019 \times 10^{-4}$	3	$1.6294317184718176 \times 10^{-2}$
4	$3.6176319784492551 \times 10^{-4}$	4	$2.8908903047466991 \times 10^{-5}$
5	$5.1649074710649745 \times 10^{-6}$	5	$-4.2512266823816218 \times 10^{-5}$
6	$-6.1566304015779076 \times 10^{-7}$	6	$-1.5349640355203935 \times 10^{-6}$
7	$-3.4409643127282012 \times 10^{-8}$	7	$1.0777423587693449 \times 10^{-7}$
8	$1.4867030334665535 \times 10^{-8}$	8	$-2.5900736824819579 \times 10^{-9}$
9	$-5.6752167015377751 \times 10^{-10}$	9	$-6.1807700314064093 \times 10^{-9}$

	ψ_A (deg)		Δ (deg)
0	449.52863759074464	0	453.87832067621742
1	51.338364568092531	1	53.982687625219434
2	$1.0684320894858805 \times 10^{-1}$	2	$-7.4519994481748487 \times 10^{-2}$
3	$9.7710001116765678 \times 10^{-2}$	3	$1.6701430118380110 \times 10^{-2}$
4	$-1.7363144558755095 \times 10^{-5}$	4	$1.7596460305939294 \times 10^{-3}$
5	$-6.8051939768839820 \times 10^{-5}$	5	$5.1799395157131089 \times 10^{-5}$
6	$-1.4559878986009605 \times 10^{-5}$	6	$-7.7047811671123266 \times 10^{-6}$
7	$-6.6647303227457230 \times 10^{-6}$	7	$-4.1020365121778701 \times 10^{-8}$
8	$-4.6297787927054273 \times 10^{-8}$	8	$1.3540358224410398 \times 10^{-8}$
9	$-1.1231663743497003 \times 10^{-8}$	9	$-2.1642620641522189 \times 10^{-9}$

	ω_A (deg)		I (deg)
0	25.923376886578422	0	24.719231699517619
1	$-3.1264377909615403 \times 10^{-1}$	1	$7.9575161585556005 \times 10^{-2}$
2	$-3.9617076913184114 \times 10^{-1}$	2	$-9.9383689143307087 \times 10^{-2}$
3	$6.6102467721712945 \times 10^{-3}$	3	$-7.3237560877084378 \times 10^{-3}$
4	$4.8104133827305182 \times 10^{-3}$	4	$8.2818077840574234 \times 10^{-5}$
5	$-2.3742604533864581 \times 10^{-5}$	5	$8.1127742782625422 \times 10^{-5}$
6	$-1.0198819955698486 \times 10^{-5}$	6	$3.1486706018973325 \times 10^{-6}$
7	$-1.7075886580325112 \times 10^{-7}$	7	$-2.9213656151589990 \times 10^{-7}$
8	$-9.6404961677771162 \times 10^{-8}$	8	$-8.6998496962272925 \times 10^{-9}$
9	$-1.6632882148376425 \times 10^{-9}$	9	$8.7462261500207324 \times 10^{-10}$

	χ_A (deg)		L (deg)
0	$-7.4794733047388196 \times 10^{-1}$	0	$-1.4585325258750244 \times 10^{-1}$
1	-3.9284926443228909	1	1.0069686418641683
2	$9.3561240041583510 \times 10^{-2}$	2	$1.1022940152394585 \times 10^{-1}$
3	$9.3177533439876288 \times 10^{-2}$	3	$-7.2023784532498247 \times 10^{-4}$
4	$-2.9160877507837897 \times 10^{-4}$	4	$-1.9468009935934732 \times 10^{-3}$
5	$-1.5753494890940225 \times 10^{-4}$	5	$-1.0373411586309489 \times 10^{-4}$
6	$-1.0736289398082974 \times 10^{-5}$	6	$7.6919922189465732 \times 10^{-6}$
7	$-5.9976154229936056 \times 10^{-6}$	7	$2.4588171833203968 \times 10^{-7}$
8	$-5.7034632588795900 \times 10^{-8}$	8	$-2.1534976019289917 \times 10^{-8}$
9	$-7.6772387904275242 \times 10^{-9}$	9	$-5.1807338118556910 \times 10^{-9}$

Table 4-2, continued.

Interval 68: Central time $T_c = 400$, covering the time span $360 \leq T \leq 440$

	ε (deg)		p_A (deg)
0	23.376773227788863	0	560.80316102619723
1	$-4.1339704722505826 \times 10^{-1}$	1	55.825698802744117
2	$-1.7271772615314392 \times 10^{-2}$	2	$1.8972079561847892 \times 10^{-1}$
3	$6.1759924674256505 \times 10^{-3}$	3	$7.9130182016474489 \times 10^{-3}$
4	$2.8137340864662625 \times 10^{-4}$	4	$-1.2212325166786959 \times 10^{-3}$
5	$-1.9718644793018601 \times 10^{-5}$	5	$-8.3009752321859833 \times 10^{-5}$
6	$-2.1539226193262955 \times 10^{-6}$	6	$2.4205098033788420 \times 10^{-7}$
7	$-2.5870795525285031 \times 10^{-8}$	7	$5.6368904179297484 \times 10^{-7}$
8	$2.3749348225124654 \times 10^{-8}$	8	$2.7992706516293689 \times 10^{-8}$
9	$7.1634878506531948 \times 10^{-10}$	9	$-7.7840209139749202 \times 10^{-9}$

	ψ_A (deg)		Δ (deg)
0	556.43894051227333	0	562.22860597346599
1	56.350075534306952	1	54.676456368662051
2	1.0087515213873974	2	$2.9403985513992185 \times 10^{-1}$
3	$1.1073620247129134 \times 10^{-2}$	3	$4.1955325232622616 \times 10^{-2}$
4	$-1.6626871878017507 \times 10^{-2}$	4	$5.4928286476152095 \times 10^{-4}$
5	$-1.4827240226571201 \times 10^{-3}$	5	$-2.3339826137039369 \times 10^{-4}$
6	$8.4204931737918165 \times 10^{-5}$	6	$-2.5088014258290304 \times 10^{-5}$
7	$3.7630960207197108 \times 10^{-5}$	7	$-1.7713245187774290 \times 10^{-6}$
8	$3.0313360047494498 \times 10^{-6}$	8	$-3.4698306989288828 \times 10^{-8}$
9	$-3.3512060408387977 \times 10^{-7}$	9	$1.6375523393190503 \times 10^{-8}$

	ω_A (deg)		I (deg)
0	23.203925352672997	0	23.910075854524963
1	-2.0144800215636101	1	$-9.0916199881696073 \times 10^{-1}$
2	$6.5460289379550779 \times 10^{-2}$	2	$-1.2053522272328164 \times 10^{-1}$
3	$5.8175277561209974 \times 10^{-2}$	3	$8.1362692836592530 \times 10^{-3}$
4	$-3.4341425747889358 \times 10^{-4}$	4	$1.7568969661235896 \times 10^{-3}$
5	$-6.4177721624846611 \times 10^{-4}$	5	$5.1281523411523941 \times 10^{-5}$
6	$-3.8303735390950682 \times 10^{-5}$	6	$-6.2761306408386292 \times 10^{-6}$
7	$2.4297594532538843 \times 10^{-6}$	7	$-5.4515259066307685 \times 10^{-7}$
8	$8.4307008133533232 \times 10^{-7}$	8	$-3.6791451120842055 \times 10^{-8}$
9	$7.0335231999509562 \times 10^{-8}$	9	$-1.3934012329748364 \times 10^{-9}$

	χ_A (deg)		L (deg)
0	-4.7539110863729893	0	2.2898750792591437
1	$6.1034609987354616 \times 10^{-1}$	1	1.2387813656301225
2	$8.8765405583247846 \times 10^{-1}$	2	$-1.1759518379762810 \times 10^{-1}$
3	$-1.5845809403593206 \times 10^{-3}$	3	$-3.6994966354079956 \times 10^{-2}$
4	$-1.6540268321548870 \times 10^{-2}$	4	$-1.7568856056724305 \times 10^{-3}$
5	$-1.3329303226486928 \times 10^{-3}$	5	$1.7848997976561848 \times 10^{-4}$
6	$9.2057079085925329 \times 10^{-5}$	6	$2.5952016468244699 \times 10^{-5}$
7	$3.6615103042490263 \times 10^{-5}$	7	$2.2266478059869645 \times 10^{-6}$
8	$2.9771784507957539 \times 10^{-6}$	8	$5.8270024992533997 \times 10^{-8}$
9	$-3.2466211058395564 \times 10^{-7}$	9	$-2.4765335302510538 \times 10^{-8}$

Table 4–2, continued.

Interval 69: Central time $T_c = 480$, covering the time span $440 \le T \le 520$

	ε (deg)		p_A (deg)
0	22.671118644355308	0	673.97212540138386
1	$-2.2372059121316524 \times 10^{-1}$	1	57.292247155926687
2	$6.3517955344959348 \times 10^{-2}$	2	$1.2737546002628038 \times 10^{-1}$
3	$5.4080024069619579 \times 10^{-3}$	3	$-1.9714661675699461 \times 10^{-2}$
4	$-4.5349991104885460 \times 10^{-4}$	4	$-1.7137507558157918 \times 10^{-3}$
5	$-4.4189156856757343 \times 10^{-5}$	5	$7.5967454336389540 \times 10^{-5}$
6	$1.6084135660276729 \times 10^{-7}$	6	$1.2019034115168478 \times 10^{-5}$
7	$1.6814723182869252 \times 10^{-7}$	7	$4.2276830151945201 \times 10^{-7}$
8	$3.1121360997410952 \times 10^{-8}$	8	$-3.2530199218308771 \times 10^{-8}$
9	$1.7157126161411189 \times 10^{-9}$	9	$-1.3756567361780936 \times 10^{-8}$

	ψ_A (deg)		Δ (deg)
0	674.33203618014561	0	675.23483758044152
1	60.388426832047038	1	58.575713538925255
2	$-3.0201970734705563 \times 10^{-1}$	2	$5.6673370369267938 \times 10^{-1}$
3	$-1.3589045233845425 \times 10^{-1}$	3	$-2.5232557417217039 \times 10^{-2}$
4	$1.5982856894894217 \times 10^{-2}$	4	$-1.0267993807687029 \times 10^{-2}$
5	$2.2111453572452575 \times 10^{-3}$	5	$-4.4531143967925103 \times 10^{-4}$
6	$-4.3205046512254267 \times 10^{-4}$	6	$9.2847222677895754 \times 10^{-5}$
7	$-2.5500755526482525 \times 10^{-5}$	7	$1.4141417189072720 \times 10^{-5}$
8	$1.0874660258829151 \times 10^{-5}$	8	$-2.0889007729185422 \times 10^{-9}$
9	$5.1567805372763652 \times 10^{-9}$	9	$-2.0546768284921790 \times 10^{-7}$

	ω_A (deg)		I (deg)
0	21.191243166565812	0	21.771518786789374
1	$2.3139532975125961 \times 10^{-1}$	1	-1.0137047000828946
2	$3.2746430905395608 \times 10^{-1}$	2	$1.3457968858326593 \times 10^{-1}$
3	$-3.1088173618360900 \times 10^{-2}$	3	$2.9460904854476933 \times 10^{-2}$
4	$-5.6744568395778564 \times 10^{-3}$	4	$-3.2389033635718518 \times 10^{-4}$
5	$7.3952817765694899 \times 10^{-4}$	5	$-3.1455162836611437 \times 10^{-4}$
6	$6.2024182644520008 \times 10^{-5}$	6	$-1.5168551390541896 \times 10^{-5}$
7	$-1.3070686879740596 \times 10^{-5}$	7	$1.8255963748213325 \times 10^{-6}$
8	$-4.2848827653021681 \times 10^{-7}$	8	$2.8833026832500698 \times 10^{-7}$
9	$2.5484508648193498 \times 10^{-7}$	9	$4.7857376317772618 \times 10^{-9}$

	χ_A (deg)		L (deg)
0	$3.8774946977594797 \times 10^{-1}$	0	2.4434648617321869
1	3.3396211032963283	1	-1.3965972345470491
2	$-4.6233397439111536 \times 10^{-1}$	2	$-4.6915034036953127 \times 10^{-1}$
3	$-1.2299896348908912 \times 10^{-1}$	3	$6.7691695584123648 \times 10^{-3}$
4	$1.8591668050003141 \times 10^{-2}$	4	$8.9618248570045920 \times 10^{-3}$
5	$2.2078965345678627 \times 10^{-3}$	5	$5.1844780184142893 \times 10^{-4}$
6	$-4.5183595456533001 \times 10^{-4}$	6	$-8.3314770089756392 \times 10^{-5}$
7	$-2.6413743133175517 \times 10^{-5}$	7	$-1.3670372739011818 \times 10^{-5}$
8	$1.0931738828703675 \times 10^{-5}$	8	$-1.7316057692863692 \times 10^{-8}$
9	$2.2173478334773482 \times 10^{-8}$	9	$1.9041243187975801 \times 10^{-7}$

Table 4–2, continued.

Interval 70: Central time $T_c = 560$, covering the time span $520 \le T \le 600$

	ε (deg)		p_A (deg)
0	22.815121316379889	0	788.63499478607532
1	$3.6848213118368214 \times 10^{-1}$	1	57.110942479027214
2	$6.3942077464749522 \times 10^{-2}$	2	$-1.7697490973679922 \times 10^{-1}$
3	$-5.9408216587302148 \times 10^{-3}$	3	$-2.1730654304254416 \times 10^{-2}$
4	$-6.5617166369698892 \times 10^{-4}$	4	$1.7592893221801482 \times 10^{-3}$
5	$4.3070626917397725 \times 10^{-5}$	5	$1.4500771033706425 \times 10^{-4}$
6	$3.4201826873226995 \times 10^{-6}$	6	$-1.5217898244371374 \times 10^{-5}$
7	$-4.4617819137770438 \times 10^{-7}$	7	$-6.6816262055607445 \times 10^{-7}$
8	$-1.8399779194707716 \times 10^{-8}$	8	$1.8675932141903464 \times 10^{-7}$
9	$6.3383511236566433 \times 10^{-9}$	9	$2.3758092105490385 \times 10^{-9}$

	ψ_A (deg)		Δ (deg)
0	790.87896447181127	0	794.25869205335927
1	56.128261258881956	1	59.563611661198012
2	$-3.8161285958220295 \times 10^{-1}$	2	$-4.5282222223231346 \times 10^{-1}$
3	$7.0218326794273613 \times 10^{-2}$	3	$-9.1986657952354955 \times 10^{-2}$
4	$2.4274850558539932 \times 10^{-4}$	4	$8.4160966104199078 \times 10^{-3}$
5	$-1.1013418864920394 \times 10^{-3}$	5	$1.0687614168560109 \times 10^{-3}$
6	$8.1031722446774163 \times 10^{-5}$	6	$-1.5898982401257751 \times 10^{-4}$
7	$-3.7308242913686745 \times 10^{-6}$	7	$-9.6553461340774577 \times 10^{-6}$
8	$-5.0840724846651039 \times 10^{-7}$	8	$2.9478133138230043 \times 10^{-6}$
9	$1.8772876176206150 \times 10^{-7}$	9	$3.4932160335267587 \times 10^{-8}$

	ω_A (deg)		I (deg)
0	22.767321532972282	0	21.566218905693554
1	$9.6534967042703716 \times 10^{-1}$	1	$9.2099688479925922 \times 10^{-1}$
2	$-1.0668805565841280 \times 10^{-1}$	2	$2.6227373703648223 \times 10^{-1}$
3	$-1.4233223890907194 \times 10^{-2}$	3	$-1.7483187381056377 \times 10^{-2}$
4	$4.6168033636766294 \times 10^{-3}$	4	$-3.5858662008283616 \times 10^{-3}$
5	$-3.5068181826782068 \times 10^{-5}$	5	$2.2263490822425859 \times 10^{-4}$
6	$-3.8339743759942371 \times 10^{-5}$	6	$2.9602830327783818 \times 10^{-5}$
7	$1.9806018270286221 \times 10^{-6}$	7	$-3.4059719641617461 \times 10^{-6}$
8	$-1.0318230795365238 \times 10^{-7}$	8	$-2.1122336331718307 \times 10^{-7}$
9	$-4.1479730687646518 \times 10^{-9}$	9	$5.3441220177304704 \times 10^{-8}$

	χ_A (deg)		L (deg)
0	2.4314373608283076	0	-2.2701416181977390
1	-1.0544442163173245	1	-2.6548393425839613
2	$-2.2458936552840283 \times 10^{-1}$	2	$2.8989609572494310 \times 10^{-1}$
3	$9.9059978818807569 \times 10^{-2}$	3	$7.5093570700771764 \times 10^{-2}$
4	$-1.3185969084526021 \times 10^{-3}$	4	$-6.7218767982007351 \times 10^{-3}$
5	$-1.3593677242027033 \times 10^{-3}$	5	$-9.5989624461442550 \times 10^{-4}$
6	$9.5536586411624056 \times 10^{-5}$	6	$1.4347275639609421 \times 10^{-4}$
7	$-2.2465367435408145 \times 10^{-6}$	7	$9.1063388667996780 \times 10^{-6}$
8	$-7.0176575552058380 \times 10^{-7}$	8	$-2.7507363765118312 \times 10^{-6}$
9	$1.8098032009858198 \times 10^{-7}$	9	$-3.2448450175711897 \times 10^{-8}$

Table 4–2, continued.

Interval 71: Central time $T_c = 640$, covering the time span $600 \leq T \leq 680$

	ε (deg)		p_A (deg)
0	23.761872916983302	0	901.02106747480556
1	$4.9223945213206809 \times 10^{-1}$	1	55.229056190130252
2	$-3.7429223610641067 \times 10^{-2}$	2	$-2.3256097798264586 \times 10^{-1}$
3	$-8.5432013277068777 \times 10^{-3}$	3	$1.2187700819201916 \times 10^{-2}$
4	$2.8488942494134193 \times 10^{-4}$	4	$1.7895707238813616 \times 10^{-3}$
5	$3.7116864017506752 \times 10^{-5}$	5	$-6.9716787287198876 \times 10^{-5}$
6	$-5.8809869323176251 \times 10^{-7}$	6	$-1.5466157289866043 \times 10^{-6}$
7	$4.6177843890963244 \times 10^{-8}$	7	$-4.3131857843914258 \times 10^{-9}$
8	$-1.7967255203000779 \times 10^{-8}$	8	$-4.2710278021097196 \times 10^{-8}$
9	$-1.3904471685778561 \times 10^{-9}$	9	$7.2035521282146387 \times 10^{-9}$

	ψ_A (deg)		Δ (deg)
0	902.03632094166538	0	908.17377576247473
1	55.379067719587217	1	54.211797567149243
2	$2.4016797365477720 \times 10^{-2}$	2	$-6.3489615894762208 \times 10^{-1}$
3	$-2.1114693205156539 \times 10^{-2}$	3	$4.4332976379023105 \times 10^{-2}$
4	$-6.9497556534809252 \times 10^{-3}$	4	$3.4940373076757548 \times 10^{-3}$
5	$4.7351992554599188 \times 10^{-4}$	5	$-5.3243659019934645 \times 10^{-4}$
6	$8.8079155940684988 \times 10^{-5}$	6	$4.0196060888477568 \times 10^{-5}$
7	$-1.0489558311735345 \times 10^{-6}$	7	$1.1032662350521673 \times 10^{-6}$
8	$-1.0410500587196762 \times 10^{-6}$	8	$-5.4223060846473768 \times 10^{-7}$
9	$-6.0612601755458037 \times 10^{-8}$	9	$5.4485517377238315 \times 10^{-8}$

	ω_A (deg)		I (deg)
0	23.996311727851434	0	24.439083011971384
1	$3.8151549674961002 \times 10^{-1}$	1	1.6380309213141972
2	$2.1796934258458666 \times 10^{-2}$	2	$-1.0396022308803627 \times 10^{-1}$
3	$1.9009441526019979 \times 10^{-2}$	3	$-3.0269433918659028 \times 10^{-2}$
4	$-1.7025622237744326 \times 10^{-3}$	4	$1.4349168158416658 \times 10^{-3}$
5	$-4.0100796102590638 \times 10^{-4}$	5	$1.1332810492203655 \times 10^{-4}$
6	$1.6242674062464866 \times 10^{-5}$	6	$-1.3026676621643429 \times 10^{-5}$
7	$3.2582888060190577 \times 10^{-6}$	7	$7.4937146993281252 \times 10^{-7}$
8	$1.0966203117812581 \times 10^{-8}$	8	$2.5763892764112597 \times 10^{-8}$
9	$-2.7636660916777695 \times 10^{-8}$	9	$-8.0173252443632977 \times 10^{-9}$

	χ_A (deg)		L (deg)
0	1.1105764663463185	0	-3.9531058118089825
1	$1.6828000388088919 \times 10^{-1}$	1	1.0918621946467574
2	$2.8076972853541474 \times 10^{-1}$	2	$4.4713095347673522 \times 10^{-1}$
3	$-3.5923220168451761 \times 10^{-2}$	3	$-3.3032914060375739 \times 10^{-2}$
4	$-9.6264995418894868 \times 10^{-3}$	4	$-2.2137141214345614 \times 10^{-3}$
5	$5.8240670586809205 \times 10^{-4}$	5	$4.6807712251651502 \times 10^{-4}$
6	$9.7734659526438877 \times 10^{-5}$	6	$-3.9058915796778692 \times 10^{-5}$
7	$-1.2813538046125679 \times 10^{-6}$	7	$-1.1836934618185457 \times 10^{-6}$
8	$-1.0257487843373270 \times 10^{-6}$	8	$4.8784532477498117 \times 10^{-7}$
9	$-6.7246880235868101 \times 10^{-8}$	9	$-4.6051225829226831 \times 10^{-8}$

Table 4–2, continued.

Interval 72: Central time $T_c = 720$, covering the time span $680 \leq T \leq 760$

	ε (deg)
0	24.209806942830020
1	$-9.0413951493907485 \times 10^{-2}$
2	$-9.1836645795972867 \times 10^{-2}$
3	$6.6274967868054222 \times 10^{-4}$
4	$7.4159508112329300 \times 10^{-4}$
5	$-5.2132672375332189 \times 10^{-7}$
6	$-2.1507485077594801 \times 10^{-6}$
7	$8.6003081076015190 \times 10^{-8}$
8	$8.5696754159619576 \times 10^{-9}$
9	$-2.1122219545351456 \times 10^{-9}$

	p_A (deg)
0	1010.3456856657479
1	54.318653219196444
2	$3.1609190587155056 \times 10^{-2}$
3	$2.7244800452306539 \times 10^{-2}$
4	$-3.2127112566095777 \times 10^{-5}$
5	$-1.0366994441909822 \times 10^{-4}$
6	$3.7763980606618761 \times 10^{-7}$
7	$6.8775297094485034 \times 10^{-8}$
8	$-3.4941490875638429 \times 10^{-8}$
9	$-1.8322279233344606 \times 10^{-9}$

	ψ_A (deg)
0	1011.6209311154096
1	53.845273972781579
2	$-3.7634402951562033 \times 10^{-1}$
3	$-5.2088346995823424 \times 10^{-3}$
4	$7.6214402196379646 \times 10^{-3}$
5	$1.6535134927371269 \times 10^{-4}$
6	$-7.3481823076425963 \times 10^{-5}$
7	$4.9085217822113428 \times 10^{-6}$
8	$7.0685940179783108 \times 10^{-7}$
9	$-1.0502556222580746 \times 10^{-7}$

	Δ (deg)
0	1013.5376359472292
1	51.713131873885032
2	$3.3043513735354886 \times 10^{-2}$
3	$5.9779741857081645 \times 10^{-2}$
4	$2.7210766435550411 \times 10^{-4}$
5	$2.8411555044774549 \times 10^{-5}$
6	$-6.2172339703012529 \times 10^{-7}$
7	$-1.6442609271152915 \times 10^{-6}$
8	$-1.1009938938248401 \times 10^{-8}$
9	$-6.4955452908707276 \times 10^{-9}$

	ω_A (deg)
0	25.085742126549202
1	$6.4943523801868842 \times 10^{-1}$
2	$-4.8353049203303958 \times 10^{-2}$
3	$-3.0707035882671156 \times 10^{-2}$
4	$-1.7553301429669398 \times 10^{-3}$
5	$3.7131148881096352 \times 10^{-4}$
6	$1.2457190151416070 \times 10^{-5}$
7	$-2.7451060893651322 \times 10^{-6}$
8	$9.8379160480279455 \times 10^{-8}$
9	$1.9760370318158243 \times 10^{-8}$

	I (deg)
0	26.074013660446027
1	$-1.5965660032173261 \times 10^{-1}$
2	$-2.9075340251939498 \times 10^{-1}$
3	$6.0677831111024395 \times 10^{-4}$
4	$2.1180231198249174 \times 10^{-3}$
5	$7.8574148904195214 \times 10^{-6}$
6	$4.0292212575385156 \times 10^{-9}$
7	$-4.2206011690071864 \times 10^{-8}$
8	$-2.9362975787119798 \times 10^{-8}$
9	$8.7338097564129681 \times 10^{-10}$

	χ_A (deg)
0	1.4025835010723402
1	$-5.1804833684314325 \times 10^{-1}$
2	$-4.5019601279969591 \times 10^{-1}$
3	$-3.6214954692369316 \times 10^{-2}$
4	$8.5338154185934370 \times 10^{-3}$
5	$3.4643607946080342 \times 10^{-4}$
6	$-8.0497153437472463 \times 10^{-5}$
7	$4.2809478255107637 \times 10^{-6}$
8	$7.6422658482044414 \times 10^{-7}$
9	$-1.0079559520485457 \times 10^{-7}$

	L (deg)
0	$4.0043515082777909 \times 10^{-1}$
1	2.8833677784297053
2	$-3.9971919288117285 \times 10^{-3}$
3	$-3.8198268677926032 \times 10^{-2}$
4	$-2.9451792989148763 \times 10^{-4}$
5	$-9.5565832898311388 \times 10^{-5}$
6	$1.1479785633865374 \times 10^{-6}$
7	$1.6264171177238367 \times 10^{-6}$
8	$-2.9931214585215664 \times 10^{-8}$
9	$4.2403692552935693 \times 10^{-9}$

Table 4–2, continued.

Interval 73: Central time $T_c = 800$, covering the time span $760 \leq T \leq 840$

	ε (deg)
0	23.451907449363895
1	$-6.0729269201336286 \times 10^{-1}$
2	$-2.0410201960209887 \times 10^{-2}$
3	$1.0186919687566865 \times 10^{-2}$
4	$3.1110723918679772 \times 10^{-4}$
5	$-4.6869824952310152 \times 10^{-5}$
6	$-2.9366724203985261 \times 10^{-6}$
7	$-2.5241502404568929 \times 10^{-8}$
8	$2.7415427783042717 \times 10^{-8}$
9	$2.0928839392159553 \times 10^{-9}$

	p_A (deg)
0	1120.1606928536748
1	55.712065788517090
2	$2.8625628484846150 \times 10^{-1}$
3	$9.7092124616510738 \times 10^{-3}$
4	$-2.2364967476308788 \times 10^{-3}$
5	$-1.1808945323020388 \times 10^{-4}$
6	$2.6228865720716652 \times 10^{-6}$
7	$9.4207332989943313 \times 10^{-7}$
8	$7.1183309108109482 \times 10^{-8}$
9	$-6.3215092342812115 \times 10^{-9}$

	ψ_A (deg)
0	1117.5122330931925
1	52.568558385470047
2	$2.4525360397455221 \times 10^{-1}$
3	$1.0215680322767455 \times 10^{-1}$
4	$5.5264771377954098 \times 10^{-3}$
5	$-8.2893018360111678 \times 10^{-5}$
6	$-1.7651102870776380 \times 10^{-5}$
7	$-8.8840059503792558 \times 10^{-6}$
8	$-1.1316944048079728 \times 10^{-6}$
9	$-6.4243186642193987 \times 10^{-8}$

	Δ (deg)
0	1119.5231028654360
1	54.869607858275465
2	$7.4288498176153218 \times 10^{-1}$
3	$4.9211738696697652 \times 10^{-2}$
4	$-3.9222744525249114 \times 10^{-3}$
5	$-7.0227018638652364 \times 10^{-4}$
6	$-5.9146463064557238 \times 10^{-5}$
7	$4.4368556040577933 \times 10^{-7}$
8	$7.5716457023915182 \times 10^{-7}$
9	$1.0053894150605814 \times 10^{-7}$

	ω_A (deg)
0	24.875847587233515
1	-1.1107092348879002
2	$-3.3659562777956484 \times 10^{-1}$
3	$5.0899541168752269 \times 10^{-5}$
4	$5.0849896779704927 \times 10^{-3}$
5	$2.4285379875965675 \times 10^{-4}$
6	$-9.4513847233631103 \times 10^{-6}$
7	$-2.0430402135434488 \times 10^{-7}$
8	$-1.1093814457143086 \times 10^{-7}$
9	$-1.8390306082939151 \times 10^{-8}$

	I (deg)
0	23.860761482025632
1	-1.8777223024094897
2	$-8.0347554816013579 \times 10^{-2}$
3	$3.3550727306269487 \times 10^{-2}$
4	$1.6527436397911245 \times 10^{-3}$
5	$-1.1216445057450217 \times 10^{-4}$
6	$-1.5300625667969596 \times 10^{-5}$
7	$-1.1685021138409232 \times 10^{-6}$
8	$-1.8528392250761167 \times 10^{-8}$
9	$9.9669178872659268 \times 10^{-9}$

	χ_A (deg)
0	-2.8910178863659428
1	-3.4233289360175269
2	$-3.0013724510658668 \times 10^{-2}$
3	$1.0345829980644760 \times 10^{-1}$
4	$8.0098744413576244 \times 10^{-3}$
5	$-9.6521081780355187 \times 10^{-5}$
6	$-2.4456020500275464 \times 10^{-5}$
7	$-9.0434030677540434 \times 10^{-6}$
8	$-1.1734169946490482 \times 10^{-6}$
9	$-6.0637375041409946 \times 10^{-8}$

	L (deg)
0	4.5859205703497508
1	$8.8196186160874798 \times 10^{-1}$
2	$-5.0449057320594526 \times 10^{-1}$
3	$-4.0084224844712153 \times 10^{-2}$
4	$2.1955139128335687 \times 10^{-3}$
5	$5.8594283905941445 \times 10^{-4}$
6	$5.9292931979122458 \times 10^{-5}$
7	$5.2138719355606957 \times 10^{-7}$
8	$-6.7704472333641863 \times 10^{-7}$
9	$-1.0763918671644098 \times 10^{-7}$

Table 4–2, continued.

Interval 74: Central time $T_c = 880$, covering the time span $840 \leq T \leq 920$

	ε (deg)		p_A (deg)
0	22.461973166319833	0	1233.7406632621140
1	$-2.8683264654076722 \times 10^{-1}$	1	57.752589935180919
2	$9.2066437418014227 \times 10^{-2}$	2	$1.4849282639008472 \times 10^{-1}$
3	$5.3570467227275578 \times 10^{-3}$	3	$-3.1783115639827575 \times 10^{-2}$
4	$-9.0921946977057167 \times 10^{-4}$	4	$-1.7666480155781431 \times 10^{-3}$
5	$-4.2558720481407962 \times 10^{-5}$	5	$2.2582943256696355 \times 10^{-4}$
6	$4.5083184763644644 \times 10^{-6}$	6	$1.5462704397941810 \times 10^{-5}$
7	$2.6338741312598424 \times 10^{-7}$	7	$-1.0410504923135518 \times 10^{-6}$
8	$-7.4774956181525465 \times 10^{-9}$	8	$-1.1211542383785890 \times 10^{-7}$
9	$1.7365261709186460 \times 10^{-10}$	9	$3.1533050623360769 \times 10^{-11}$

	ψ_A (deg)		Δ (deg)
0	1228.9306062533576	0	1235.4998056874912
1	59.924541026247941	1	60.869897238448587
2	1.4596313559412356	2	$4.3869241586374054 \times 10^{-1}$
3	$4.3912292219142443 \times 10^{-4}$	3	$-1.2300982934418648 \times 10^{-1}$
4	$-3.6407524125098237 \times 10^{-2}$	4	$-1.0079752044673833 \times 10^{-2}$
5	$-4.7152064279523499 \times 10^{-3}$	5	$1.6470384867518487 \times 10^{-3}$
6	$1.8682186653816323 \times 10^{-4}$	6	$2.2640288269174835 \times 10^{-4}$
7	$1.5513373238866828 \times 10^{-4}$	7	$-1.9976031971041075 \times 10^{-5}$
8	$1.9571665482399405 \times 10^{-5}$	8	$-5.0479345685103366 \times 10^{-6}$
9	$-1.3367066843039255 \times 10^{-6}$	9	$1.8024242971948718 \times 10^{-7}$

	ω_A (deg)		I (deg)
0	21.086547251514504	0	20.840734135139837
1	-2.2071857912016290	1	$-7.9228044596130068 \times 10^{-1}$
2	$2.2507765528334261 \times 10^{-1}$	2	$3.2705306428353759 \times 10^{-1}$
3	$8.7461350289426957 \times 10^{-2}$	3	$1.8182615091647922 \times 10^{-2}$
4	$1.4708524181151628 \times 10^{-3}$	4	$-4.4340071388126821 \times 10^{-3}$
5	$-1.2430928484489023 \times 10^{-3}$	5	$-2.7848097318295696 \times 10^{-4}$
6	$-1.5386266941095108 \times 10^{-4}$	6	$4.2320182068613566 \times 10^{-5}$
7	$6.0252270772087234 \times 10^{-7}$	7	$5.0573444691565105 \times 10^{-6}$
8	$3.3733315639159049 \times 10^{-6}$	8	$-4.1061805592549864 \times 10^{-7}$
9	$5.1854930494962029 \times 10^{-7}$	9	$-9.7259417154390394 \times 10^{-8}$

	χ_A (deg)		L (deg)
0	-5.1885815407482939	0	1.9510857328889596
1	2.3798134910549359	1	-3.3658445218726956
2	1.3951556395227807	2	$-3.0403319549058177 \times 10^{-1}$
3	$2.8555748045176866 \times 10^{-2}$	3	$9.6378370303719473 \times 10^{-2}$
4	$-3.6196899204804762 \times 10^{-2}$	4	$8.3979193135410826 \times 10^{-3}$
5	$-4.9199669132633853 \times 10^{-3}$	5	$-1.4565563973528670 \times 10^{-3}$
6	$1.8350227654549670 \times 10^{-4}$	6	$-2.1108225487812761 \times 10^{-4}$
7	$1.5601954661152246 \times 10^{-4}$	7	$1.9082185440324451 \times 10^{-5}$
8	$1.9628216289208209 \times 10^{-5}$	8	$4.9388838437212675 \times 10^{-6}$
9	$-1.3344649030286443 \times 10^{-6}$	9	$-1.8117322362766383 \times 10^{-7}$

Table 4-2, continued.

Interval 75: Central time $T_c = 960$, covering the time span $920 \leq T \leq 1000$

	ε (deg)		p_A (deg)
0	22.640454329655351	0	1349.1511910094691
1	$4.4141396428561371 \times 10^{-1}$	1	57.335435871657839
2	$6.4345675045374057 \times 10^{-2}$	2	$-2.2902872609527333 \times 10^{-1}$
3	$-8.7491328828957354 \times 10^{-3}$	3	$-1.9040938832909505 \times 10^{-2}$
4	$-4.4650001267309400 \times 10^{-4}$	4	$2.7425835671249168 \times 10^{-3}$
5	$7.2376639245314669 \times 10^{-5}$	5	$5.0966835467588091 \times 10^{-5}$
6	$1.0400551868994941 \times 10^{-7}$	6	$-2.2056662469599691 \times 10^{-5}$
7	$-5.7383640474313051 \times 10^{-7}$	7	$7.2439266992835961 \times 10^{-7}$
8	$2.2445357432976020 \times 10^{-8}$	8	$1.9802387016365279 \times 10^{-7}$
9	$6.8017667965943929 \times 10^{-9}$	9	$-1.4782935797825861 \times 10^{-8}$

	ψ_A (deg)		Δ (deg)
0	1353.4666908699757	0	1356.0851386579452
1	62.075382009758140	1	58.660121459600750
2	-1.3507519008326613	2	$-7.7805442593929684 \times 10^{-1}$
3	$-1.8661712737107539 \times 10^{-1}$	3	$-1.7193539123918718 \times 10^{-2}$
4	$4.9687934909164842 \times 10^{-2}$	4	$1.2255954306802174 \times 10^{-2}$
5	$8.0925846165744686 \times 10^{-4}$	5	$-8.4309739128142681 \times 10^{-4}$
6	$-1.5086852955517712 \times 10^{-3}$	6	$-7.9051184117797063 \times 10^{-5}$
7	$1.3215099679201100 \times 10^{-4}$	7	$2.1075425916097697 \times 10^{-5}$
8	$3.5042803002960380 \times 10^{-5}$	8	$-1.0993329475714138 \times 10^{-6}$
9	$-8.2487595288611645 \times 10^{-6}$	9	$-2.2077583374326507 \times 10^{-7}$

	ω_A (deg)		I (deg)
0	20.607198741648159	0	21.701279042298752
1	1.9869105611450860	1	1.4913499637852710
2	$5.0784611085384829 \times 10^{-1}$	2	$1.4339446757560358 \times 10^{-1}$
3	$-7.4208056522525843 \times 10^{-2}$	3	$-3.6194780389054620 \times 10^{-2}$
4	$-8.6628367611849826 \times 10^{-3}$	4	$-1.3292680471144578 \times 10^{-4}$
5	$1.7657821740357268 \times 10^{-3}$	5	$3.3923639014489738 \times 10^{-4}$
6	$2.1714497415205522 \times 10^{-5}$	6	$-2.3964380079958858 \times 10^{-5}$
7	$-4.0464110005675249 \times 10^{-5}$	7	$-1.1203801403984482 \times 10^{-6}$
8	$3.6926442140186856 \times 10^{-6}$	8	$4.0099431728249288 \times 10^{-7}$
9	$7.4300638439786001 \times 10^{-7}$	9	$-2.9813131031381623 \times 10^{-8}$

	χ_A (deg)		L (deg)
0	4.6618247106268814	0	-3.6517821268519992
1	5.1376559098292615	1	-1.4622654748722360
2	-1.1760862474504434	2	$5.8457004884808660 \times 10^{-1}$
3	$-1.8087345010108745 \times 10^{-1}$	3	$-6.9837726267249917 \times 10^{-6}$
4	$4.7709703071569554 \times 10^{-2}$	4	$-9.9124353387234844 \times 10^{-3}$
5	$9.0301050433313950 \times 10^{-4}$	5	$8.8751846671545123 \times 10^{-4}$
6	$-1.4908057855145497 \times 10^{-3}$	6	$5.8650804031553267 \times 10^{-5}$
7	$1.3067897008869359 \times 10^{-4}$	7	$-2.0332549925982509 \times 10^{-5}$
8	$3.4843763741198042 \times 10^{-5}$	8	$1.3019694243833951 \times 10^{-6}$
9	$-8.2323074360780660 \times 10^{-6}$	9	$2.0577212099380216 \times 10^{-7}$

Table 4–2, continued.

Interval 76: Central time $T_c = 1040$, covering the time span $1000 \leq T \leq 1080$

	ε (deg)		p_A (deg)
0	23.682785466419668	0	1461.7616144447012
1	$5.0929864586609079 \times 10^{-1}$	1	55.283965720866310
2	$-4.3841824907256810 \times 10^{-2}$	2	$-2.2166656212667452 \times 10^{-1}$
3	$-6.9930929364417431 \times 10^{-3}$	3	$1.5964851776694859 \times 10^{-2}$
4	$4.7295052952355662 \times 10^{-4}$	4	$1.2075173572152054 \times 10^{-3}$
5	$1.9221708619938181 \times 10^{-5}$	5	$-1.0348827442331366 \times 10^{-4}$
6	$-1.5378819961943825 \times 10^{-6}$	6	$1.2490407346557093 \times 10^{-6}$
7	$-1.1465902165023935 \times 10^{-8}$	7	$-5.1689958445668701 \times 10^{-8}$
8	$-1.6608086729718534 \times 10^{-8}$	8	$8.2893483611977309 \times 10^{-9}$
9	$2.2332323855607302 \times 10^{-9}$	9	$7.0106955990621377 \times 10^{-9}$

	ψ_A (deg)		Δ (deg)
0	1466.5041008538046	0	1468.0007845246153
1	51.576868174825975	1	53.618753972091284
2	$-7.3705433993996927 \times 10^{-1}$	2	$-3.6785159559647088 \times 10^{-1}$
3	$1.3397089078472118 \times 10^{-1}$	3	$5.2398556982702267 \times 10^{-2}$
4	$-2.8632539320924097 \times 10^{-3}$	4	$-6.0870687949473519 \times 10^{-4}$
5	$-3.3900141967017654 \times 10^{-4}$	5	$-1.6496855024259635 \times 10^{-4}$
6	$1.4586093024277532 \times 10^{-4}$	6	$2.6485017146412255 \times 10^{-5}$
7	$-2.0748070905277434 \times 10^{-5}$	7	$-2.4818129803749814 \times 10^{-6}$
8	$1.4941135940512505 \times 10^{-6}$	8	$9.0980649076971606 \times 10^{-8}$
9	$-5.7893716380863343 \times 10^{-9}$	9	$5.8951079135560017 \times 10^{-9}$

	ω_A (deg)		I (deg)
0	25.510879251611448	0	24.647160599473855
1	2.1415303405956635	1	1.1864866259368193
2	$-3.9616877704161209 \times 10^{-1}$	2	$-1.7479242147447500 \times 10^{-1}$
3	$-4.1578821771661610 \times 10^{-2}$	3	$-1.2182472875156282 \times 10^{-2}$
4	$6.0843815066316671 \times 10^{-3}$	4	$1.9130622247586372 \times 10^{-3}$
5	$-7.9009936723254571 \times 10^{-6}$	5	$-3.7904163479081121 \times 10^{-5}$
6	$-1.2510789989190204 \times 10^{-5}$	6	$-4.1556731914702609 \times 10^{-6}$
7	$2.3253156566040820 \times 10^{-6}$	7	$5.1345593911229941 \times 10^{-7}$
8	$-3.7675831135551795 \times 10^{-7}$	8	$-3.1912512022077756 \times 10^{-8}$
9	$3.6806754891489845 \times 10^{-8}$	9	$1.0073405646031801 \times 10^{-9}$

	χ_A (deg)		L (deg)
0	5.1941463627979215	0	−2.9517164566709200
1	−4.0216514320323178	1	1.7966517787012157
2	$-5.9512793010199888 \times 10^{-1}$	2	$1.6855845224242792 \times 10^{-1}$
3	$1.2864479535881207 \times 10^{-1}$	3	$-3.9770681819166561 \times 10^{-2}$
4	$-2.7323845980335201 \times 10^{-3}$	4	$1.6857319513329324 \times 10^{-3}$
5	$-3.5174613747729481 \times 10^{-4}$	5	$8.8664682954733111 \times 10^{-5}$
6	$1.3410723424550624 \times 10^{-4}$	6	$-2.5234358385106540 \times 10^{-5}$
7	$-2.0028663809570309 \times 10^{-5}$	7	$2.2802587135939607 \times 10^{-6}$
8	$1.5385291985071092 \times 10^{-6}$	8	$-7.6467695738275428 \times 10^{-8}$
9	$-1.5785268232649657 \times 10^{-8}$	9	$2.3981749732876377 \times 10^{-9}$

Table 4–2, continued.

Interval 77: Central time $T_c = 1120$, covering the time span $1080 \leq T \leq 1160$

	ε (deg)		p_A (deg)
0	24.185990385957100	0	1571.2991676415517
1	$-3.3518143392947276 \times 10^{-2}$	1	54.461181020095863
2	$-7.6781570045570584 \times 10^{-2}$	2	$2.4330529031518686 \times 10^{-2}$
3	$1.4724185200603163 \times 10^{-3}$	3	$2.0993118053174376 \times 10^{-2}$
4	$4.7258389740847817 \times 10^{-4}$	4	$-3.9095175098997661 \times 10^{-4}$
5	$-1.2807512253273602 \times 10^{-5}$	5	$-4.8466885445426111 \times 10^{-5}$
6	$-3.3175205427667213 \times 10^{-8}$	6	$1.8510236064368119 \times 10^{-6}$
7	$7.9856486912680365 \times 10^{-8}$	7	$-3.2005169370412149 \times 10^{-7}$
8	$-1.7276455500913164 \times 10^{-8}$	8	$-1.6812046041461972 \times 10^{-8}$
9	$-1.0715217744948835 \times 10^{-9}$	9	$4.8808257556731374 \times 10^{-9}$

	ψ_A (deg)		Δ (deg)
0	1568.3233873573694	0	1574.0896042001125
1	51.372571071217823	1	52.907591800057612
2	$6.2260722290476118 \times 10^{-1}$	2	$1.5615782438272639 \times 10^{-1}$
3	$9.6012227231377343 \times 10^{-2}$	3	$3.3461805218131924 \times 10^{-2}$
4	$-3.1089118136863789 \times 10^{-3}$	4	$-1.3949847927601416 \times 10^{-3}$
5	$-6.1713804218552954 \times 10^{-4}$	5	$-2.5604441945917037 \times 10^{-5}$
6	$-1.1545393103769441 \times 10^{-4}$	6	$-3.1715847418599835 \times 10^{-6}$
7	$-5.4274806037339537 \times 10^{-6}$	7	$2.9710014825628239 \times 10^{-8}$
8	$5.9352449743504739 \times 10^{-7}$	8	$4.2670250929771641 \times 10^{-8}$
9	$1.4982046466201184 \times 10^{-7}$	9	$-2.4988912746747248 \times 10^{-9}$

	ω_A (deg)		I (deg)
0	26.167698885396835	0	25.476319867583512
1	-1.4391957540522824	1	$-3.5234065525738848 \times 10^{-1}$
2	$-3.4476698337315194 \times 10^{-1}$	2	$-1.7097358928839565 \times 10^{-1}$
3	$4.5895550407819842 \times 10^{-2}$	3	$1.0015252704970152 \times 10^{-2}$
4	$4.3006196529881823 \times 10^{-3}$	4	$8.1337025645063736 \times 10^{-4}$
5	$-2.1564435280722205 \times 10^{-4}$	5	$-5.8954773157039310 \times 10^{-5}$
6	$-2.2539370092602438 \times 10^{-5}$	6	$-2.1122314900205294 \times 10^{-7}$
7	$-2.2528675982190303 \times 10^{-6}$	7	$-1.3607328638043526 \times 10^{-8}$
8	$-1.2843384438402285 \times 10^{-7}$	8	$8.9006423654831873 \times 10^{-9}$
9	$6.2566934857249077 \times 10^{-9}$	9	$1.1620662211277490 \times 10^{-9}$

	χ_A (deg)		L (deg)
0	-3.2779858284502235	0	$8.1133915804449726 \times 10^{-1}$
1	-3.3918599222679854	1	1.7049242926525069
2	$6.7589101441590055 \times 10^{-1}$	2	$-1.4660258607517117 \times 10^{-1}$
3	$8.3027701024353352 \times 10^{-2}$	3	$-1.4405796632087732 \times 10^{-2}$
4	$-4.2372644287442031 \times 10^{-3}$	4	$1.2345911446501025 \times 10^{-3}$
5	$-6.4305777690155433 \times 10^{-4}$	5	$-1.7710167110394160 \times 10^{-5}$
6	$-1.0512432214050085 \times 10^{-4}$	6	$3.7798884857711813 \times 10^{-6}$
7	$-4.7820460894155656 \times 10^{-6}$	7	$-3.7086836596644148 \times 10^{-7}$
8	$5.5272962885938804 \times 10^{-7}$	8	$-5.9008791328947195 \times 10^{-8}$
9	$1.4440977182234195 \times 10^{-7}$	9	$8.0900753238310705 \times 10^{-9}$

Table 4–2, continued.

Interval 78: Central time $T_c = 1200$, covering the time span $1160 \le T \le 1240$

	ε (deg)
0	23.638194684122333
1	$-4.6867385218458513 \times 10^{-1}$
2	$-2.2660176208053532 \times 10^{-2}$
3	$6.7521433864543346 \times 10^{-3}$
4	$1.4100263152323125 \times 10^{-4}$
5	$-2.6669625152897276 \times 10^{-5}$
6	$-9.0492562603234110 \times 10^{-7}$
7	$1.0195108899025493 \times 10^{-7}$
8	$1.2126796991238476 \times 10^{-8}$
9	$-1.5379331186385628 \times 10^{-9}$

	p_A (deg)
0	1681.0923208541829
1	55.486461428287795
2	$2.0783781275967746 \times 10^{-1}$
3	$7.1503779328115643 \times 10^{-3}$
4	$-1.3585731026727770 \times 10^{-3}$
5	$-4.3565970420016577 \times 10^{-5}$
6	$2.9284694616139559 \times 10^{-6}$
7	$3.5732267256767184 \times 10^{-7}$
8	$-1.9517167108954008 \times 10^{-8}$
9	$-3.9172078114273636 \times 10^{-9}$

	ψ_A (deg)
0	1677.9807342198580
1	58.403346454828354
2	$7.2095039916314727 \times 10^{-1}$
3	$-1.3249123570492278 \times 10^{-1}$
4	$-1.8512928460998493 \times 10^{-2}$
5	$2.0831167771970261 \times 10^{-3}$
6	$4.9445039431132137 \times 10^{-4}$
7	$-1.5828594410431202 \times 10^{-5}$
8	$-1.2297060134667052 \times 10^{-5}$
9	$-3.5421265082276256 \times 10^{-7}$

	Δ (deg)
0	1682.1204428803614
1	55.326531766653015
2	$3.9846906408027988 \times 10^{-1}$
3	$4.8411246917401518 \times 10^{-3}$
4	$-2.1732092822738858 \times 10^{-3}$
5	$-2.2670259105090237 \times 10^{-5}$
6	$3.5948719316804906 \times 10^{-6}$
7	$1.6233739862960717 \times 10^{-7}$
8	$-6.7683161711008563 \times 10^{-9}$
9	$1.0070898973373088 \times 10^{-9}$

	ω_A (deg)
0	22.705463824455221
1	-1.4386461633913943
2	$3.3194498466114480 \times 10^{-1}$
3	$3.5276662661126493 \times 10^{-2}$
4	$-7.8210804538235510 \times 10^{-3}$
5	$-6.6735147226085203 \times 10^{-4}$
6	$8.1611177089806552 \times 10^{-5}$
7	$1.4227321919187256 \times 10^{-5}$
8	$-5.0639378699329981 \times 10^{-7}$
9	$-3.0093810445306819 \times 10^{-7}$

	I (deg)
0	23.881655681398943
1	-1.1074384058323594
2	$-1.1242976632554486 \times 10^{-2}$
3	$1.3847293696977758 \times 10^{-2}$
4	$-2.6888032502246227 \times 10^{-4}$
5	$-3.7992445927593354 \times 10^{-5}$
6	$2.1789314905030619 \times 10^{-6}$
7	$1.9470452476815764 \times 10^{-8}$
8	$-1.1620751102943077 \times 10^{-8}$
9	$4.0473951619254621 \times 10^{-10}$

	χ_A (deg)
0	-3.3955099355677799
1	3.1968145308604539
2	$5.4332690083437794 \times 10^{-1}$
3	$-1.5232699110452528 \times 10^{-1}$
4	$-1.7467803312751131 \times 10^{-2}$
5	$2.2762347004595947 \times 10^{-3}$
6	$4.9305803976253687 \times 10^{-4}$
7	$-1.6916335463204888 \times 10^{-5}$
8	$-1.2277792955365016 \times 10^{-5}$
9	$-3.4920784089353703 \times 10^{-7}$

	L (deg)
0	2.7273283146541951
1	$1.6016332098218148 \times 10^{-1}$
2	$-2.0826446117036230 \times 10^{-1}$
3	$3.3484685201455387 \times 10^{-3}$
4	$8.8891452841844147 \times 10^{-4}$
5	$-3.4951898655294054 \times 10^{-5}$
6	$-3.2968172151471136 \times 10^{-7}$
7	$2.7993930686268161 \times 10^{-7}$
8	$-1.9196764425819302 \times 10^{-8}$
9	$-5.4779019978192859 \times 10^{-9}$

Table 4–2, continued.

Interval 79: Central time $T_c = 1280$, covering the time span $1240 \leq T \leq 1320$

	ε (deg)
0	22.775015524072690
1	$-3.3023702650632199 \times 10^{-1}$
2	$5.3439598975306248 \times 10^{-2}$
3	$4.6072969991626233 \times 10^{-3}$
4	$-3.7888762274083078 \times 10^{-4}$
5	$-1.9845658824226854 \times 10^{-5}$
6	$5.2612827622779591 \times 10^{-7}$
7	$8.1990214318308678 \times 10^{-9}$
8	$1.2315124877279107 \times 10^{-8}$
9	$1.0987727226294108 \times 10^{-9}$

	ψ_A (deg)
0	1795.0868234088571
1	57.489988963182456
2	$-6.4655056410956084 \times 10^{-1}$
3	$1.4102984997637766 \times 10^{-2}$
4	$1.9019318123400783 \times 10^{-2}$
5	$-1.5403472701135745 \times 10^{-3}$
6	$-1.1943217305873281 \times 10^{-4}$
7	$4.2208907292401716 \times 10^{-5}$
8	$-3.3970575826382640 \times 10^{-6}$
9	$-4.7316496387821197 \times 10^{-7}$

	ω_A (deg)
0	22.258735791086699
1	$7.2666845296830905 \times 10^{-1}$
2	$3.6566606850465666 \times 10^{-2}$
3	$-5.6001243970526689 \times 10^{-2}$
4	$1.8569911628027052 \times 10^{-3}$
5	$7.9110500252999261 \times 10^{-4}$
6	$-5.8539747028703093 \times 10^{-5}$
7	$-2.2495566610719037 \times 10^{-6}$
8	$1.0203230633361786 \times 10^{-6}$
9	$-1.0239478005334193 \times 10^{-7}$

	χ_A (deg)
0	1.4832591880332621
1	$4.3353700756876937 \times 10^{-1}$
2	$-8.6348165297636188 \times 10^{-1}$
3	$3.1849359098421666 \times 10^{-2}$
4	$2.1738606251207469 \times 10^{-2}$
5	$-1.6289019760853530 \times 10^{-3}$
6	$-1.3724270914279351 \times 10^{-4}$
7	$4.2112449295838405 \times 10^{-5}$
8	$-3.3362027755851289 \times 10^{-6}$
9	$-4.6585795040485745 \times 10^{-7}$

	p_A (deg)
0	1793.7167513269025
1	57.091058921132168
2	$1.5185155485387401 \times 10^{-1}$
3	$-1.6083097785191000 \times 10^{-2}$
4	$-1.1977773301936101 \times 10^{-3}$
5	$6.2361166003773576 \times 10^{-5}$
6	$4.7813569478679069 \times 10^{-6}$
7	$1.1091528261683066 \times 10^{-7}$
8	$9.9775412133302130 \times 10^{-10}$
9	$-5.0952559132069847 \times 10^{-9}$

	Δ (deg)
0	1795.7283047568761
1	58.157850682914809
2	$2.5154786896081070 \times 10^{-1}$
3	$-2.7690573767634966 \times 10^{-2}$
4	$-1.5128100091815261 \times 10^{-3}$
5	$9.0970151152301860 \times 10^{-5}$
6	$3.7062388497894411 \times 10^{-6}$
7	$-2.7549819367255229 \times 10^{-7}$
8	$-6.3952618546707840 \times 10^{-9}$
9	$4.1780039641229809 \times 10^{-9}$

	I (deg)
0	22.024314028629934
1	$-6.4623459122188649 \times 10^{-1}$
2	$1.1258659103770341 \times 10^{-1}$
3	$6.0370085671629255 \times 10^{-3}$
4	$-5.8142279416657394 \times 10^{-4}$
5	$6.2763479054505468 \times 10^{-7}$
6	$6.4969806343712260 \times 10^{-7}$
7	$-1.1367125847171968 \times 10^{-7}$
8	$-8.1657782647022565 \times 10^{-9}$
9	$5.0959791311115093 \times 10^{-11}$

	L (deg)
0	1.6461147319071856
1	−1.1538711732905298
2	$-1.0444954275372453 \times 10^{-1}$
3	$1.2397051609740128 \times 10^{-2}$
4	$2.6171898650708690 \times 10^{-4}$
5	$-2.7218502667856129 \times 10^{-5}$
6	$1.6513483038427714 \times 10^{-6}$
7	$3.8596889545242535 \times 10^{-7}$
8	$7.6965685255846256 \times 10^{-9}$
9	$-9.7994393698233158 \times 10^{-9}$

Table 4–2, continued.

Interval 80: Central time $T_c = 1360$, covering the time span $1320 \leq T \leq 1400$

	ε (deg)		p_A (deg)
0	22.629230090967074	0	1908.3597600322008
1	$1.9284793600078548 \times 10^{-1}$	1	57.341842623607089
2	$6.3286048209443302 \times 10^{-2}$	2	$-9.6058928949151965 \times 10^{-2}$
3	$-3.2538816436266385 \times 10^{-3}$	3	$-1.9805453626904665 \times 10^{-2}$
4	$-4.8014715984561283 \times 10^{-4}$	4	$8.9109086991383114 \times 10^{-4}$
5	$1.6475239669412727 \times 10^{-5}$	5	$1.0212546595633555 \times 10^{-4}$
6	$1.6362404474023142 \times 10^{-6}$	6	$-5.1137121196139461 \times 10^{-6}$
7	$-1.1413861020213016 \times 10^{-7}$	7	$-3.3301718178315238 \times 10^{-7}$
8	$-2.2113323838555775 \times 10^{-9}$	8	$5.9274516454533232 \times 10^{-8}$
9	$2.5871022696901139 \times 10^{-9}$	9	$1.4108388262285837 \times 10^{-10}$

	ψ_A (deg)		Δ (deg)
0	1907.5394030895625	0	1912.8186091842078
1	55.841470925424445	1	58.589546929695843
2	$3.5869538860358333 \times 10^{-1}$	2	$-1.5540997083257204 \times 10^{-1}$
3	$8.8530449979619298 \times 10^{-2}$	3	$-3.4240400156611984 \times 10^{-2}$
4	$-7.6022846227671295 \times 10^{-3}$	4	$8.3882603982366026 \times 10^{-4}$
5	$-1.3309421603674369 \times 10^{-3}$	5	$1.3827750889457353 \times 10^{-4}$
6	$-6.5477996966475595 \times 10^{-5}$	6	$7.6218992066298051 \times 10^{-7}$
7	$-2.1169328587272098 \times 10^{-7}$	7	$-3.8518282098399795 \times 10^{-7}$
8	$2.4192519991931616 \times 10^{-6}$	8	$-5.3866039955896789 \times 10^{-8}$
9	$2.4295535148104914 \times 10^{-7}$	9	$2.2242463994017219 \times 10^{-9}$

	ω_A (deg)		I (deg)
0	22.729218961806438	0	21.750697888994891
1	$-4.3003848424211314 \times 10^{-1}$	1	$3.8648600587901239 \times 10^{-1}$
2	$-1.6535679807661908 \times 10^{-1}$	2	$1.2908528869019511 \times 10^{-1}$
3	$3.1232751628569886 \times 10^{-2}$	3	$-3.4785167947003156 \times 10^{-3}$
4	$5.6170010478490607 \times 10^{-3}$	4	$-6.6538576223908824 \times 10^{-4}$
5	$-3.1720931736454744 \times 10^{-4}$	5	$-1.2432476587675938 \times 10^{-5}$
6	$-4.6173310985826405 \times 10^{-5}$	6	$2.5945390272458834 \times 10^{-7}$
7	$-1.9994115022721064 \times 10^{-6}$	7	$2.3218647338917292 \times 10^{-7}$
8	$-5.7889798261732696 \times 10^{-8}$	8	$4.5220991796896971 \times 10^{-9}$
9	$4.3194419653364264 \times 10^{-8}$	9	$-2.6608733560258816 \times 10^{-9}$

	χ_A (deg)		L (deg)
0	$-8.8846838528029926 \times 10^{-1}$	0	$-9.8951476197706224 \times 10^{-1}$
1	-1.6252099670398771	1	-1.3430706587492787
2	$4.9377257281219908 \times 10^{-1}$	2	$6.1904309872157373 \times 10^{-2}$
3	$1.1740293523860353 \times 10^{-1}$	3	$1.5169200187403546 \times 10^{-2}$
4	$-9.4464748330609512 \times 10^{-3}$	4	$1.1126122494598453 \times 10^{-4}$
5	$-1.5601795197671586 \times 10^{-3}$	5	$-3.1785457413074175 \times 10^{-5}$
6	$-5.1389460847865492 \times 10^{-5}$	6	$-6.5628420177757664 \times 10^{-6}$
7	$9.2156612568171562 \times 10^{-7}$	7	$8.1034587577543929 \times 10^{-9}$
8	$2.3003197012020577 \times 10^{-6}$	8	$1.1896021929252403 \times 10^{-7}$
9	$2.3963396680477640 \times 10^{-7}$	9	$-2.0167670074801018 \times 10^{-9}$

Table 4–2, continued.

Interval 81: Central time $T_c = 1440$, covering the time span $1400 \leq T \leq 1480$

	ε (deg)		p_A (deg)
0	23.328184654021058	0	2021.7651211736869
1	$4.4730292113019871 \times 10^{-1}$	1	55.971582341813862
2	$-6.1785900998806250 \times 10^{-3}$	2	$-2.0514402062589446 \times 10^{-1}$
3	$-6.8984168430689367 \times 10^{-3}$	3	$3.2362994610734995 \times 10^{-3}$
4	$7.9752305384561819 \times 10^{-5}$	4	$1.5346520956424062 \times 10^{-3}$
5	$3.2777974821342220 \times 10^{-5}$	5	$-3.8608119077975153 \times 10^{-5}$
6	$-6.8972983336279318 \times 10^{-7}$	6	$-5.4477051102498630 \times 10^{-6}$
7	$-2.0468607940609945 \times 10^{-7}$	7	$2.8171791488276118 \times 10^{-7}$
8	$-9.2189723263956346 \times 10^{-10}$	8	$6.0589504600180135 \times 10^{-8}$
9	$3.9078519354386005 \times 10^{-9}$	9	$9.1333552610581686 \times 10^{-10}$

	ψ_A (deg)		Δ (deg)
0	2022.7598923636808	0	2027.7418458928160
1	59.061361233734915	1	56.122725891063640
2	$-2.9915505918231862 \times 10^{-2}$	2	$-4.0491941207835538 \times 10^{-1}$
3	$-1.5941880244890800 \times 10^{-1}$	3	$-2.6499789199466777 \times 10^{-3}$
4	$-5.5968153193026192 \times 10^{-3}$	4	$2.6440285690865640 \times 10^{-3}$
5	$3.5582811032340700 \times 10^{-3}$	5	$3.2335758149863712 \times 10^{-6}$
6	$1.5723673731707683 \times 10^{-4}$	6	$-8.0113550252519659 \times 10^{-6}$
7	$-7.3508333171807532 \times 10^{-5}$	7	$2.0190246541076564 \times 10^{-7}$
8	$-4.8106499033282226 \times 10^{-6}$	8	$1.9992534759742873 \times 10^{-8}$
9	$1.6651705905528763 \times 10^{-6}$	9	$-1.5062766889568093 \times 10^{-9}$

	ω_A (deg)		I (deg)
0	22.236658311409688	0	23.291152496716040
1	$3.8690930112274288 \times 10^{-1}$	1	1.0639837900243624
2	$3.4335510634246895 \times 10^{-1}$	2	$2.1455618020583269 \times 10^{-2}$
3	$1.4692818958496422 \times 10^{-2}$	3	$-1.3988191406889911 \times 10^{-2}$
4	$-8.5945876981765505 \times 10^{-3}$	4	$-4.5877047920423816 \times 10^{-4}$
5	$-3.7568865441211045 \times 10^{-4}$	5	$4.3451201528526073 \times 10^{-5}$
6	$1.2386778686162352 \times 10^{-4}$	6	$3.0254376999367281 \times 10^{-6}$
7	$6.4356907253877900 \times 10^{-6}$	7	$-3.1244942879378548 \times 10^{-8}$
8	$-1.9441723165303878 \times 10^{-6}$	8	$-1.0353813272014230 \times 10^{-8}$
9	$-1.4887791978747450 \times 10^{-7}$	9	$-1.2649257181752413 \times 10^{-9}$

	χ_A (deg)		L (deg)
0	1.0840940896810125	0	−2.6383677643291304
1	3.3566288439994835	1	$-1.7495857989739211 \times 10^{-1}$
2	$1.9629244763772083 \times 10^{-1}$	2	$2.1627011830787042 \times 10^{-1}$
3	$-1.7401812338886941 \times 10^{-1}$	3	$7.1970512497595875 \times 10^{-3}$
4	$-7.8368365343602601 \times 10^{-3}$	4	$-1.1619604441927762 \times 10^{-3}$
5	$3.7328612340578252 \times 10^{-3}$	5	$-5.7373933484227123 \times 10^{-5}$
6	$1.6850292740711863 \times 10^{-4}$	6	$1.9353481819125793 \times 10^{-6}$
7	$-7.4506438143650538 \times 10^{-5}$	7	$1.5353468674351386 \times 10^{-7}$
8	$-4.8893958517718727 \times 10^{-6}$	8	$5.2019800244808679 \times 10^{-8}$
9	$1.6654726562052264 \times 10^{-6}$	9	$2.6433132561302072 \times 10^{-9}$

Table 4-2, continued.

Interval 82: Central time $T_c = 1520$, covering the time span $1480 \leq T \leq 1560$

	ε (deg)		p_A (deg)
0	23.952350871193955	0	2132.4304972546433
1	$1.2707905954643375 \times 10^{-1}$	1	54.820491884801017
2	$-6.5267604060452435 \times 10^{-2}$	2	$-5.5308067893164312 \times 10^{-2}$
3	$-2.0850668000270020 \times 10^{-3}$	3	$1.8905164814156980 \times 10^{-2}$
4	$4.4547753520243682 \times 10^{-4}$	4	$4.4825112311012146 \times 10^{-4}$
5	$1.3117581932024381 \times 10^{-5}$	5	$-4.7393603669162226 \times 10^{-5}$
6	$5.4188471986221990 \times 10^{-7}$	6	$-1.6608440696780022 \times 10^{-6}$
7	$-7.0446058891880248 \times 10^{-8}$	7	$-6.0865970112194029 \times 10^{-7}$
8	$-3.8523075303398242 \times 10^{-8}$	8	$1.6067691632236440 \times 10^{-8}$
9	$1.0895153974392191 \times 10^{-9}$	9	$1.1916848328128935 \times 10^{-8}$

	ψ_A (deg)		Δ (deg)
0	2136.2434902185775	0	2137.1240691707438
1	53.685639558048456	1	53.426742268372060
2	$-8.8518011454922270 \times 10^{-1}$	2	$-2.0650501812713332 \times 10^{-1}$
3	$5.5427249254154402 \times 10^{-2}$	3	$3.2817335485681951 \times 10^{-2}$
4	$1.1234922778621747 \times 10^{-2}$	4	$1.5723885174072173 \times 10^{-3}$
5	$-1.4405475955988470 \times 10^{-3}$	5	$-6.1745414889913424 \times 10^{-5}$
6	$6.4169279642971738 \times 10^{-5}$	6	$3.4649339542374810 \times 10^{-6}$
7	$1.8091549481227456 \times 10^{-5}$	7	$2.2800831687979876 \times 10^{-7}$
8	$-3.3443031957444956 \times 10^{-6}$	8	$-5.9863034488693549 \times 10^{-8}$
9	$1.6276941009200069 \times 10^{-7}$	9	$-4.3061487448239157 \times 10^{-9}$

	ω_A (deg)		I (deg)
0	24.865742324764669	0	25.027774251055615
1	1.8343325340476655	1	$5.2602545232782053 \times 10^{-1}$
2	$-1.2180539229569407 \times 10^{-1}$	2	$-1.5190815209237611 \times 10^{-1}$
3	$-6.1977233686714163 \times 10^{-2}$	3	$-1.1586350008881338 \times 10^{-2}$
4	$1.7617369625122137 \times 10^{-3}$	4	$8.3123444408215129 \times 10^{-4}$
5	$5.2620340185864056 \times 10^{-4}$	5	$6.5656532170193971 \times 10^{-5}$
6	$-4.2860023259877726 \times 10^{-5}$	6	$-1.2156120327305572 \times 10^{-6}$
7	$1.3102786646053105 \times 10^{-6}$	7	$6.5578373530396841 \times 10^{-9}$
8	$4.0074990720092000 \times 10^{-7}$	8	$2.2822029511855388 \times 10^{-8}$
9	$-7.5791498627803341 \times 10^{-8}$	9	$-1.4420665051942012 \times 10^{-9}$

	χ_A (deg)		L (deg)
0	4.1825106296854972	0	-1.2467799787335687
1	-1.2170310521281812	1	1.5270185758832922
2	$-9.1849723153950047 \times 10^{-1}$	2	$1.6848349709186038 \times 10^{-1}$
3	$3.5933551810183851 \times 10^{-2}$	3	$-1.5638204181213900 \times 10^{-2}$
4	$1.2476973085826995 \times 10^{-2}$	4	$-1.3773315621248915 \times 10^{-3}$
5	$-1.3647773697205110 \times 10^{-3}$	5	$1.9398994471551171 \times 10^{-5}$
6	$5.3335161236507402 \times 10^{-5}$	6	$-3.7404213570493140 \times 10^{-6}$
7	$1.8458245215035855 \times 10^{-5}$	7	$-9.0258301191432397 \times 10^{-7}$
8	$-3.3191082098518909 \times 10^{-6}$	8	$7.4292203965736894 \times 10^{-8}$
9	$1.5153224025859364 \times 10^{-7}$	9	$1.8054063150970926 \times 10^{-8}$

Table 4–2, continued.

Interval 83: Central time $T_c = 1600$, covering the time span $1560 \le T \le 1640$

	ε (deg)
0	23.701386203580928
1	$-3.5819557390551729 \times 10^{-1}$
2	$-4.1428668539255292 \times 10^{-2}$
3	$6.0668074112627432 \times 10^{-3}$
4	$4.2405177652731719 \times 10^{-4}$
5	$-2.8053961759453255 \times 10^{-5}$
6	$-1.9668213600074581 \times 10^{-6}$
7	$1.5084518454480717 \times 10^{-7}$
8	$-3.4671826614196698 \times 10^{-9}$
9	$-2.8616578085789912 \times 10^{-9}$

	p_A (deg)
0	2242.3680281976381
1	55.306770774291436
2	$1.6900080220700603 \times 10^{-1}$
3	$1.4133318168727990 \times 10^{-2}$
4	$-1.2379065311655249 \times 10^{-3}$
5	$-9.5472083244518172 \times 10^{-5}$
6	$2.6553457281523655 \times 10^{-6}$
7	$2.3829847980059349 \times 10^{-7}$
8	$-3.4297849560746287 \times 10^{-8}$
9	$3.3209629819665580 \times 10^{-9}$

	ψ_A (deg)
0	2239.8664534734011
1	50.945532459433858
2	$3.1228210778148090 \times 10^{-1}$
3	$1.2726201319677426 \times 10^{-1}$
4	$3.4845770570108328 \times 10^{-3}$
5	$1.5171396573684819 \times 10^{-4}$
6	$-3.5626102662499776 \times 10^{-5}$
7	$-1.2935710049814231 \times 10^{-5}$
8	$-1.0257549012040344 \times 10^{-6}$
9	$-1.1031354141839243 \times 10^{-7}$

	Δ (deg)
0	2243.8259306410137
1	53.704141944047531
2	$3.0627967136758908 \times 10^{-1}$
3	$4.9836677083862026 \times 10^{-2}$
4	$2.4431697208665552 \times 10^{-4}$
5	$-1.8549118377308989 \times 10^{-4}$
6	$-2.5627557713041808 \times 10^{-5}$
7	$-2.1950159057549837 \times 10^{-6}$
8	$-2.7119238123792989 \times 10^{-8}$
9	$1.1394702235910894 \times 10^{-8}$

	ω_A (deg)
0	25.900033055359438
1	-1.1033251043254720
2	$-4.7752502507340466 \times 10^{-1}$
3	$1.3612043978730625 \times 10^{-2}$
4	$6.4234750269783858 \times 10^{-3}$
5	$1.0082557795038153 \times 10^{-4}$
6	$-5.0373421507952070 \times 10^{-6}$
7	$-3.9674958362181518 \times 10^{-7}$
8	$-1.6847251121976795 \times 10^{-7}$
9	$-1.7391386426484625 \times 10^{-8}$

	I (deg)
0	24.644719751858905
1	$-9.2728535588399461 \times 10^{-1}$
2	$-1.7162746797735417 \times 10^{-1}$
3	$1.1058643382405837 \times 10^{-2}$
4	$1.9188822289160592 \times 10^{-3}$
5	$3.2674309797480344 \times 10^{-5}$
6	$-4.8903551550036037 \times 10^{-6}$
7	$-6.2815508985897522 \times 10^{-7}$
8	$-3.6968932805856393 \times 10^{-8}$
9	$5.4906143219443121 \times 10^{-10}$

	χ_A (deg)
0	-2.7412470837336648
1	-4.7829304882296827
2	$1.7675158179889236 \times 10^{-1}$
3	$1.2856101854534652 \times 10^{-1}$
4	$4.3151690338738465 \times 10^{-3}$
5	$6.7464481692128804 \times 10^{-5}$
6	$-3.6119348904929573 \times 10^{-5}$
7	$-1.2040587934979937 \times 10^{-5}$
8	$-9.9391276792153676 \times 10^{-7}$
9	$-1.1974658016030303 \times 10^{-7}$

	L (deg)
0	2.2906946749491682
1	1.7398256265364454
2	$-1.5690471879006869 \times 10^{-1}$
3	$-3.9225556326704289 \times 10^{-2}$
4	$-1.3757864707198011 \times 10^{-3}$
5	$1.1914015795667677 \times 10^{-4}$
6	$2.8759819819705368 \times 10^{-5}$
7	$2.2997688824883843 \times 10^{-6}$
8	$-1.9731718204389414 \times 10^{-8}$
9	$-7.1947138894557216 \times 10^{-9}$

Table 4–2, continued.

Interval 84: Central time $T_c = 1680$, covering the time span $1640 \leq T \leq 1720$

	ε (deg)		p_A (deg)
0	22.929793369920922	0	2354.5572401511866
1	$-3.3771533039194556 \times 10^{-1}$	1	56.887388555359269
2	$4.7740283939653945 \times 10^{-2}$	2	$1.7337034148754926 \times 10^{-1}$
3	$6.6549089362618382 \times 10^{-3}$	3	$-1.5387920278171675 \times 10^{-2}$
4	$-4.2029074864507069 \times 10^{-4}$	4	$-1.9530299965987609 \times 10^{-3}$
5	$-4.8628845206933392 \times 10^{-5}$	5	$6.6994824856647674 \times 10^{-5}$
6	$7.9057954177411167 \times 10^{-7}$	6	$1.3086350452424255 \times 10^{-5}$
7	$2.7661241331285474 \times 10^{-7}$	7	$2.6323083173003019 \times 10^{-7}$
8	$1.4402407168217715 \times 10^{-8}$	8	$-8.7292808731727081 \times 10^{-8}$
9	$-2.2748140153064273 \times 10^{-9}$	9	$-7.7597351998846838 \times 10^{-9}$

	ψ_A (deg)		Δ (deg)
0	2349.1678020924500	0	2355.2571735879241
1	59.515773125511402	1	58.041299929994168
2	1.6175184329316144	2	$6.6046077578752072 \times 10^{-1}$
3	$-2.3904330390745688 \times 10^{-2}$	3	$-2.0156338772397479 \times 10^{-2}$
4	$-4.3422994620522840 \times 10^{-2}$	4	$-1.0910059928958229 \times 10^{-2}$
5	$-4.8502074942345383 \times 10^{-3}$	5	$-5.6678520513314078 \times 10^{-4}$
6	$4.1886779989870036 \times 10^{-4}$	6	$8.8364813498365032 \times 10^{-5}$
7	$2.0356793322270160 \times 10^{-4}$	7	$1.6491265394481210 \times 10^{-5}$
8	$1.9140868441345233 \times 10^{-5}$	8	$3.3212167120344436 \times 10^{-7}$
9	$-3.2630850579159903 \times 10^{-6}$	9	$-2.1756699389077037 \times 10^{-7}$

	ω_A (deg)		I (deg)
0	21.634103897168638	0	22.189803418135090
1	-2.5199933961705184	1	-1.2765255682590145
2	$2.8324626073190489 \times 10^{-1}$	2	$1.2558005711186428 \times 10^{-1}$
3	$9.8639274692249363 \times 10^{-2}$	3	$3.3006283338949242 \times 10^{-2}$
4	$-7.9310697445186006 \times 10^{-4}$	4	$-4.0788354839955658 \times 10^{-4}$
5	$-1.5638863601482233 \times 10^{-3}$	5	$-3.3568417824106307 \times 10^{-4}$
6	$-1.5824681929122650 \times 10^{-4}$	6	$-1.7562454057134580 \times 10^{-5}$
7	$7.4822029532670732 \times 10^{-6}$	7	$1.7553527916050325 \times 10^{-6}$
8	$4.8786931589381109 \times 10^{-6}$	8	$3.4224180157469077 \times 10^{-7}$
9	$5.5784683392107908 \times 10^{-7}$	9	$1.2243254148530071 \times 10^{-8}$

	χ_A (deg)		L (deg)
0	-5.8374233026052146	0	3.0796591880772111
1	2.8945573909515762	1	-1.2732522932874726
2	1.5384132129335062	2	$-5.2177183063100428 \times 10^{-1}$
3	$-1.7096022758057724 \times 10^{-2}$	3	$6.6903346023124110 \times 10^{-3}$
4	$-4.3121335589626611 \times 10^{-2}$	4	$9.4028039298559515 \times 10^{-3}$
5	$-4.8241102925784802 \times 10^{-3}$	5	$6.2542297675369418 \times 10^{-4}$
6	$4.1734189961803974 \times 10^{-4}$	6	$-7.7859731309665949 \times 10^{-5}$
7	$2.0271828005816212 \times 10^{-4}$	7	$-1.6196645695936493 \times 10^{-5}$
8	$1.9195647736999223 \times 10^{-5}$	8	$-4.1194002372749776 \times 10^{-7}$
9	$-3.2521952473120054 \times 10^{-6}$	9	$2.0934048429006552 \times 10^{-7}$

Table 4–2, continued.

Interval 85: Central time $T_c = 1760$, covering the time span $1720 \leq T \leq 1800$

	ε (deg)
0	22.771903064956467
1	$1.9505210152466768 \times 10^{-1}$
2	$6.5491683141501732 \times 10^{-2}$
3	$-4.3596813484935252 \times 10^{-3}$
4	$-6.6540894806633232 \times 10^{-4}$
5	$3.4973644940701056 \times 10^{-5}$
6	$3.2260957483180273 \times 10^{-6}$
7	$-2.3027317075343068 \times 10^{-7}$
8	$-7.7718710471248617 \times 10^{-9}$
9	$1.7952474321786903 \times 10^{-9}$

	p_A (deg)
0	2468.8880260750176
1	57.203188161426134
2	$-1.0800182665215813 \times 10^{-1}$
3	$-2.2703719480956722 \times 10^{-2}$
4	$1.3649434424987874 \times 10^{-3}$
5	$1.5630262143075669 \times 10^{-4}$
6	$-1.0863074953651352 \times 10^{-5}$
7	$-6.7377262844442325 \times 10^{-7}$
8	$8.9834471407130588 \times 10^{-8}$
9	$6.6119622321734112 \times 10^{-10}$

	ψ_A (deg)
0	2472.9646147509961
1	61.529687703634698
2	-1.4096828297770300
3	$-1.5122320869745422 \times 10^{-1}$
4	$5.3465550696002422 \times 10^{-2}$
5	$-1.2110461179027417 \times 10^{-3}$
6	$-1.4118927050061731 \times 10^{-3}$
7	$2.0834496225497037 \times 10^{-4}$
8	$2.0926465292307443 \times 10^{-5}$
9	$-9.7200102817139952 \times 10^{-6}$

	Δ (deg)
0	2473.9647911735675
1	59.753486941955786
2	$-3.9069434086785365 \times 10^{-1}$
3	$-9.9331725884924896 \times 10^{-2}$
4	$8.8653275470087784 \times 10^{-3}$
5	$1.2378138983629945 \times 10^{-3}$
6	$-1.7904596643377625 \times 10^{-4}$
7	$-1.2501680303769610 \times 10^{-5}$
8	$3.5649503894924746 \times 10^{-6}$
9	$7.0508759817593140 \times 10^{-8}$

	ω_A (deg)
0	20.886209558751014
1	1.9337847364571539
2	$4.6054133128644589 \times 10^{-1}$
3	$-8.8991946060384585 \times 10^{-2}$
4	$-6.4332847841176535 \times 10^{-3}$
5	$2.0139700904424297 \times 10^{-3}$
6	$-5.9841805289908853 \times 10^{-5}$
7	$-3.7094898071468565 \times 10^{-5}$
8	$5.9493931298392547 \times 10^{-6}$
9	$3.3224071397806952 \times 10^{-7}$

	I (deg)
0	21.479444535904168
1	$6.9441226668212773 \times 10^{-1}$
2	$2.7149962746686396 \times 10^{-1}$
3	$-1.8648688547567148 \times 10^{-2}$
4	$-3.8438348081095269 \times 10^{-3}$
5	$2.6908864917927271 \times 10^{-4}$
6	$3.3623078919441768 \times 10^{-5}$
7	$-4.2164552584492404 \times 10^{-6}$
8	$-2.6086130462059353 \times 10^{-7}$
9	$7.0468018242334040 \times 10^{-8}$

	χ_A (deg)
0	4.4069649142180625
1	4.6923870332751784
2	-1.3768385961480029
3	$-1.4029512269901103 \times 10^{-1}$
4	$5.3451555484832307 \times 10^{-2}$
5	$-1.2440383363326986 \times 10^{-3}$
6	$-1.4103939687926921 \times 10^{-3}$
7	$2.0834903890019691 \times 10^{-4}$
8	$2.0859780291459742 \times 10^{-5}$
9	$-9.7181841150752613 \times 10^{-6}$

	L (deg)
0	-1.6749848439691737
1	-2.7650962421919520
2	$2.9919508792900146 \times 10^{-1}$
3	$8.1658825631861172 \times 10^{-2}$
4	$-7.6629026369452297 \times 10^{-3}$
5	$-1.1211897965854017 \times 10^{-3}$
6	$1.6882184526791583 \times 10^{-4}$
7	$1.1997351407307768 \times 10^{-5}$
8	$-3.4759666355974256 \times 10^{-6}$
9	$-7.0484327485603986 \times 10^{-8}$

Table 4–2, continued.

Interval 86: Central time $T_c = 1840$, covering the time span $1800 \le T \le 1880$

	ε (deg)		p_A (deg)
0	23.437939228452918	0	2581.9212963441344
1	$3.9802453471070651 \times 10^{-1}$	1	55.755025566579499
2	$-2.0133442588961546 \times 10^{-2}$	2	$-1.9758296906242274 \times 10^{-1}$
3	$-7.4323714640569560 \times 10^{-3}$	3	$8.4542500310803874 \times 10^{-3}$
4	$2.8629381673525791 \times 10^{-4}$	4	$1.6994461910762573 \times 10^{-3}$
5	$3.7971893118672552 \times 10^{-5}$	5	$-9.0032784870321674 \times 10^{-5}$
6	$-2.3263802923696404 \times 10^{-6}$	6	$-3.9094220342745576 \times 10^{-6}$
7	$-9.0949267143795235 \times 10^{-8}$	7	$6.3961010008611317 \times 10^{-7}$
8	$1.4906631721284312 \times 10^{-8}$	8	$-1.1273762783474734 \times 10^{-9}$
9	$1.1995587624575275 \times 10^{-9}$	9	$-2.7447923125733329 \times 10^{-9}$

	ψ_A (deg)		Δ (deg)
0	2585.5200075408769	0	2588.6220294907426
1	51.847724329939233	1	54.750540477170015
2	$-5.4833331665364898 \times 10^{-1}$	2	$-5.9037311212735023 \times 10^{-1}$
3	$1.3620901189675695 \times 10^{-1}$	3	$4.7208175289258419 \times 10^{-2}$
4	$-3.5711586735067933 \times 10^{-3}$	4	$3.6340547253821398 \times 10^{-3}$
5	$-1.5972867973070271 \times 10^{-4}$	5	$-5.8287964295054871 \times 10^{-4}$
6	$1.0713098132585635 \times 10^{-4}$	6	$4.3951424224938669 \times 10^{-5}$
7	$-2.0281584663785434 \times 10^{-5}$	7	$6.5133388735736752 \times 10^{-7}$
8	$1.6361506580319022 \times 10^{-6}$	8	$-6.1326006590441240 \times 10^{-7}$
9	$-7.8336462715235928 \times 10^{-8}$	9	$6.9326352926987118 \times 10^{-8}$

	ω_A (deg)		I (deg)
0	25.172793647130977	0	23.926876044087207
1	1.6000130935442425	1	1.4309213712240237
2	$-4.2090849921337391 \times 10^{-1}$	2	$-1.0336210444698886 \times 10^{-1}$
3	$-2.8332927615868008 \times 10^{-2}$	3	$-2.8883331340818315 \times 10^{-2}$
4	$6.7512842449382390 \times 10^{-3}$	4	$1.8169009332081732 \times 10^{-3}$
5	$-8.9286207360152868 \times 10^{-5}$	5	$1.1552461617134612 \times 10^{-4}$
6	$-9.8495580448879310 \times 10^{-6}$	6	$-1.4160218213702370 \times 10^{-5}$
7	$1.6101467729448750 \times 10^{-6}$	7	$9.0792892202002102 \times 10^{-7}$
8	$-3.4459569801001571 \times 10^{-7}$	8	$-6.0519481309241469 \times 10^{-9}$
9	$3.7469974575864175 \times 10^{-8}$	9	$-9.7020707796521835 \times 10^{-9}$

	χ_A (deg)		L (deg)
0	3.9317270056335215	0	-3.4686500766638522
1	-4.2539749406874149	1	1.0710311117755559
2	$-4.0704770495756702 \times 10^{-1}$	2	$4.3432776947019500 \times 10^{-1}$
3	$1.4134866354880619 \times 10^{-1}$	3	$-4.0512445267930227 \times 10^{-2}$
4	$-4.4296281476444404 \times 10^{-3}$	4	$-2.4422496397691930 \times 10^{-3}$
5	$-2.3221852413231469 \times 10^{-4}$	5	$5.0638356252266649 \times 10^{-4}$
6	$1.0500528523430528 \times 10^{-4}$	6	$-4.5111649480824360 \times 10^{-5}$
7	$-1.9920973138171085 \times 10^{-5}$	7	$-4.7877503072120982 \times 10^{-8}$
8	$1.6678407717617414 \times 10^{-6}$	8	$6.0621369698426866 \times 10^{-7}$
9	$-8.0170815043595923 \times 10^{-8}$	9	$-7.2173773842898052 \times 10^{-8}$

Table 4–2, continued.

Interval 87: Central time $T_c = 1920$, covering the time span $1880 \leq T \leq 1960$

	ε (deg)		p_A (deg)
0	23.871131686306798	0	2692.4022963821271
1	$-3.7608544356473053 \times 10^{-3}$	1	54.897034115467915
2	$-6.6360000707387302 \times 10^{-2}$	2	$4.1987009038533817 \times 10^{-3}$
3	$3.7879617484375340 \times 10^{-4}$	3	$2.0642706620514211 \times 10^{-2}$
4	$5.6647617719666601 \times 10^{-4}$	4	$-9.9062920626764738 \times 10^{-5}$
5	$-3.5319029297113361 \times 10^{-7}$	5	$-8.1252568595512613 \times 10^{-5}$
6	$-8.6288980257334214 \times 10^{-7}$	6	$-1.4849624353576059 \times 10^{-6}$
7	$-1.2975854743180847 \times 10^{-7}$	7	$-3.2996444765083419 \times 10^{-7}$
8	$-2.4329901659449575 \times 10^{-8}$	8	$4.4686077235682056 \times 10^{-8}$
9	$2.1694566912771303 \times 10^{-9}$	9	$8.8739367606827012 \times 10^{-9}$

	ψ_A (deg)		Δ (deg)
0	2689.3255484938360	0	2695.5092927105701
1	53.033231063719374	1	52.713259181513849
2	$7.3646015541448705 \times 10^{-1}$	2	$9.3853139115743868 \times 10^{-2}$
3	$6.9513226826883821 \times 10^{-2}$	3	$5.6604778278250653 \times 10^{-2}$
4	$-8.2690717459462972 \times 10^{-3}$	4	$-8.6522929978261566 \times 10^{-4}$
5	$-1.0588461291036089 \times 10^{-3}$	5	$-8.8341319004000712 \times 10^{-5}$
6	$-8.3061748322284904 \times 10^{-5}$	6	$-6.2660051104723292 \times 10^{-6}$
7	$7.9397493333548055 \times 10^{-6}$	7	$-1.4912846972031238 \times 10^{-6}$
8	$2.1256232624187166 \times 10^{-6}$	8	$3.6074203924800038 \times 10^{-8}$
9	$1.7129615945481927 \times 10^{-7}$	9	$-1.7146683757664814 \times 10^{-9}$

	ω_A (deg)		I (deg)
0	25.095218906779932	0	25.274481733905228
1	-1.4912099460814999	1	$-1.9804679578608201 \times 10^{-1}$
2	$-2.0466057465832265 \times 10^{-1}$	2	$-2.4055662122861362 \times 10^{-1}$
3	$5.4592514810821943 \times 10^{-2}$	3	$6.6938910841968937 \times 10^{-3}$
4	$2.6525773555183063 \times 10^{-3}$	4	$2.1529376476188981 \times 10^{-3}$
5	$-4.2094284552767464 \times 10^{-4}$	5	$-4.3973037087908658 \times 10^{-5}$
6	$-3.2156135823131490 \times 10^{-5}$	6	$-3.7799645245705351 \times 10^{-6}$
7	$-1.5210507416152220 \times 10^{-6}$	7	$1.3505815796475073 \times 10^{-9}$
8	$1.5951539320604215 \times 10^{-7}$	8	$-1.0533907393112270 \times 10^{-8}$
9	$4.2677085049401082 \times 10^{-8}$	9	$-8.1529984273552187 \times 10^{-10}$

	χ_A (deg)		L (deg)
0	-3.3748378754575485	0	$4.5220310834301546 \times 10^{-1}$
1	-2.0292414641444069	1	2.4003059289148463
2	$8.1360504923091764 \times 10^{-1}$	2	$-9.9536962839440603 \times 10^{-2}$
3	$5.1619043414693633 \times 10^{-2}$	3	$-4.0939674905931693 \times 10^{-2}$
4	$-9.8204015594104909 \times 10^{-3}$	4	$9.5935561449319047 \times 10^{-4}$
5	$-9.8315837272234716 \times 10^{-4}$	5	$4.5517009102925578 \times 10^{-5}$
6	$-6.8215723063807811 \times 10^{-5}$	6	$3.0955664787347973 \times 10^{-6}$
7	$8.1425947125339305 \times 10^{-6}$	7	$9.6776941187333787 \times 10^{-7}$
8	$2.0195330747152641 \times 10^{-6}$	8	$2.2033092962643216 \times 10^{-8}$
9	$1.6368407088423961 \times 10^{-7}$	9	$1.2180549159425571 \times 10^{-8}$

Table 4-2, continued.

Interval 88: Central time $T_c = 2000$, covering the time span $1960 \leq T \leq 2040$

	ε (deg)		p_A (deg)
0	23.446424034371124	0	2802.9069269775124
1	$-3.7754587771641539 \times 10^{-1}$	1	55.760338254187094
2	$-1.5207616135435268 \times 10^{-2}$	2	$1.8423209532086273 \times 10^{-1}$
3	$6.9427076199496388 \times 10^{-3}$	3	$5.1991590453226544 \times 10^{-3}$
4	$9.2485705141588464 \times 10^{-5}$	4	$-1.6645435605283704 \times 10^{-3}$
5	$-4.2442955682726430 \times 10^{-5}$	5	$-2.4838911493060373 \times 10^{-5}$
6	$5.1738198245693501 \times 10^{-7}$	6	$7.7975112462920259 \times 10^{-6}$
7	$2.7774082815512775 \times 10^{-7}$	7	$-1.5994953660284866 \times 10^{-7}$
8	$-2.5250537723390925 \times 10^{-8}$	8	$-7.6663418872191525 \times 10^{-8}$
9	$-4.1331207171414641 \times 10^{-9}$	9	$7.4857893474639785 \times 10^{-9}$

	ψ_A (deg)		Δ (deg)
0	2801.3915720032312	0	2803.5154005680672
1	58.622320495989987	1	55.703016678778684
2	$2.4651005813139153 \times 10^{-1}$	2	$5.7591660061494304 \times 10^{-1}$
3	$-1.4512871073451875 \times 10^{-1}$	3	$1.0578227934722249 \times 10^{-2}$
4	$-2.5510483951693135 \times 10^{-3}$	4	$-6.0655917798932264 \times 10^{-3}$
5	$3.1195667261531927 \times 10^{-3}$	5	$-3.8212798452074337 \times 10^{-4}$
6	$7.4047851713330530 \times 10^{-5}$	6	$1.5248893147111616 \times 10^{-5}$
7	$-6.1916131810190029 \times 10^{-5}$	7	$5.9782548421157190 \times 10^{-6}$
8	$-2.0669628958855307 \times 10^{-6}$	8	$4.1512618641008523 \times 10^{-7}$
9	$1.3611642745778002 \times 10^{-6}$	9	$-3.1991025876417783 \times 10^{-8}$

	ω_A (deg)		I (deg)
0	22.488737175607248	0	23.554846351706499
1	$-6.5778576885864366 \times 10^{-1}$	1	-1.3188275962238345
2	$3.2045997267058818 \times 10^{-1}$	2	$-1.1546737228764702 \times 10^{-3}$
3	$2.8237345021168294 \times 10^{-3}$	3	$2.8075839216965425 \times 10^{-2}$
4	$-8.1067980363083562 \times 10^{-3}$	4	$5.1050346009282206 \times 10^{-5}$
5	$2.2858032678462550 \times 10^{-5}$	5	$-1.9172667284307731 \times 10^{-4}$
6	$1.1209538583655578 \times 10^{-4}$	6	$-8.6243772124393739 \times 10^{-6}$
7	$8.9509992213814255 \times 10^{-7}$	7	$3.2306762613905083 \times 10^{-7}$
8	$-1.6984225024509288 \times 10^{-6}$	8	$1.2783549360109210 \times 10^{-7}$
9	$-3.1938047525617486 \times 10^{-8}$	9	$9.7778879862134697 \times 10^{-9}$

	χ_A (deg)		L (deg)
0	-1.6516641957593365	0	3.1792144238634466
1	3.1161515555057135	1	$4.7776786249377269 \times 10^{-2}$
2	$6.0129387276411571 \times 10^{-2}$	2	$-4.2696459761165256 \times 10^{-1}$
3	$-1.6166340470382285 \times 10^{-1}$	3	$-4.1023843947358285 \times 10^{-3}$
4	$-5.3748349037951307 \times 10^{-4}$	4	$4.8121808371943682 \times 10^{-3}$
5	$3.2777923908258796 \times 10^{-3}$	5	$3.4260862907197840 \times 10^{-4}$
6	$6.2502102287221593 \times 10^{-5}$	6	$-9.2219563555511522 \times 10^{-6}$
7	$-6.2363056195782457 \times 10^{-5}$	7	$-6.1313634346997189 \times 10^{-6}$
8	$-1.9735196659321007 \times 10^{-6}$	8	$-4.9541944205847570 \times 10^{-7}$
9	$1.3532724788226876 \times 10^{-6}$	9	$4.0907293770160728 \times 10^{-8}$

Table 4–2, continued.

Interval 89: Central time $T_c = 2080$, covering the time span $2040 \leq T \leq 2120$

	ε (deg)		p_A (deg)
0	22.814845788602114	0	2915.7878812071720
1	$-1.9556365232003553 \times 10^{-1}$	1	57.045808998829566
2	$5.3865587191250838 \times 10^{-2}$	2	$9.6095963809860430 \times 10^{-2}$
3	$3.1463307889335326 \times 10^{-3}$	3	$-1.7768888880785557 \times 10^{-2}$
4	$-4.8178463769962543 \times 10^{-4}$	4	$-8.2549334098542992 \times 10^{-4}$
5	$-1.5690149713603139 \times 10^{-5}$	5	$1.0237257494471546 \times 10^{-4}$
6	$1.6280474535843951 \times 10^{-6}$	6	$5.3035953357455140 \times 10^{-6}$
7	$1.7939984943759180 \times 10^{-7}$	7	$-2.6116486424345690 \times 10^{-7}$
8	$6.8372030949472159 \times 10^{-10}$	8	$-8.8371349013264061 \times 10^{-8}$
9	$-4.4600286774643476 \times 10^{-9}$	9	$-4.7165400296360697 \times 10^{-10}$

	ψ_A (deg)		Δ (deg)
0	2916.8563156826677	0	2918.6406478816185
1	56.256592178383049	1	59.003061556565896
2	$-4.3613144240388325 \times 10^{-1}$	2	$8.6029063127101846 \times 10^{-2}$
3	$5.9123956635973109 \times 10^{-2}$	3	$-7.5181083204609147 \times 10^{-2}$
4	$1.0032826674870446 \times 10^{-2}$	4	$6.8616180470966409 \times 10^{-4}$
5	$-1.3417931138749875 \times 10^{-3}$	5	$9.9070431917814059 \times 10^{-4}$
6	$2.4876521890515271 \times 10^{-5}$	6	$-1.9090576256606716 \times 10^{-5}$
7	$1.0985241343358655 \times 10^{-5}$	7	$-1.2962979711654595 \times 10^{-5}$
8	$-2.7020894411219213 \times 10^{-6}$	8	$3.4737409938613373 \times 10^{-7}$
9	$1.2335429019881855 \times 10^{-7}$	9	$1.8273310848423021 \times 10^{-7}$

	ω_A (deg)		I (deg)
0	22.746522962312973	0	21.777507395040837
1	$5.5096094151880376 \times 10^{-1}$	1	$-2.7946785065739432 \times 10^{-1}$
2	$-8.9975912551976626 \times 10^{-2}$	2	$2.0725607595762147 \times 10^{-1}$
3	$-3.7071337151879893 \times 10^{-2}$	3	$-1.4697789399579975 \times 10^{-3}$
4	$4.0736178495510911 \times 10^{-3}$	4	$-2.8606844546300119 \times 10^{-3}$
5	$4.1987103964182921 \times 10^{-4}$	5	$7.7222275146971115 \times 10^{-5}$
6	$-4.7994093916352877 \times 10^{-5}$	6	$2.7305599363464944 \times 10^{-5}$
7	$1.0283491975594769 \times 10^{-6}$	7	$-8.7622590693729755 \times 10^{-7}$
8	$2.0727069605479103 \times 10^{-7}$	8	$-2.8285245629586326 \times 10^{-7}$
9	$-5.7011324893322901 \times 10^{-8}$	9	$1.1885133546135916 \times 10^{-8}$

	χ_A (deg)		L (deg)
0	1.1582842938440390	0	$7.3777769711122364 \times 10^{-1}$
1	$-8.5474387490430910 \times 10^{-1}$	1	-2.1117218443963987
2	$-5.7786538363246691 \times 10^{-1}$	2	$1.3347958338353444 \times 10^{-2}$
3	$8.2934237246142274 \times 10^{-2}$	3	$6.1026704385757108 \times 10^{-2}$
4	$1.1937089825689666 \times 10^{-2}$	4	$-1.6886707983603212 \times 10^{-3}$
5	$-1.5322276393776323 \times 10^{-3}$	5	$-9.1816812695582286 \times 10^{-4}$
6	$9.7321875134701123 \times 10^{-6}$	6	$2.6041755103757337 \times 10^{-5}$
7	$1.1754472000882647 \times 10^{-5}$	7	$1.2860436979113820 \times 10^{-5}$
8	$-2.5505211144849741 \times 10^{-6}$	8	$-4.4825283664587955 \times 10^{-7}$
9	$1.2251088789373622 \times 10^{-7}$	9	$-1.8388666111257817 \times 10^{-7}$

Table 4–2, continued.

Interval 90: Central time $T_c = 2160$, covering the time span $2120 \leq T \leq 2200$

	ε (deg)		p_A (deg)
0	22.877491525305014	0	3029.9292526063128
1	$2.4979114081113937 \times 10^{-1}$	1	56.911015270606287
2	$4.4154165225977929 \times 10^{-2}$	2	$-1.2125736878100975 \times 10^{-1}$
3	$-4.2693601557119264 \times 10^{-3}$	3	$-1.3188089515533971 \times 10^{-2}$
4	$-3.0010001874260339 \times 10^{-4}$	4	$1.1835579812464353 \times 10^{-3}$
5	$2.5831998908349010 \times 10^{-5}$	5	$4.6319582914321053 \times 10^{-5}$
6	$3.3481551374843522 \times 10^{-7}$	6	$-6.0587165188874843 \times 10^{-6}$
7	$-5.9031831783847059 \times 10^{-8}$	7	$1.2942069108235024 \times 10^{-7}$
8	$9.9356528626375688 \times 10^{-9}$	8	$2.6558607393825531 \times 10^{-9}$
9	$-1.5049329396648195 \times 10^{-9}$	9	$-4.3940177140869310 \times 10^{-9}$

	ψ_A (deg)		Δ (deg)
0	3028.8785777294415	0	3035.2703889874485
1	56.559562571840776	1	57.242352141366155
2	$4.7496670054376847 \times 10^{-1}$	2	$-3.6952908869789444 \times 10^{-1}$
3	$3.7997851228395905 \times 10^{-2}$	3	$6.9118293953477213 \times 10^{-3}$
4	$-1.2583157099253607 \times 10^{-2}$	4	$4.1465262162397299 \times 10^{-3}$
5	$-1.2097003243395150 \times 10^{-3}$	5	$-4.7050753972269136 \times 10^{-4}$
6	$3.0734886683018316 \times 10^{-5}$	6	$-5.0135482990979142 \times 10^{-6}$
7	$1.9172241326049467 \times 10^{-5}$	7	$4.7868184409484562 \times 10^{-6}$
8	$2.5088221384357712 \times 10^{-6}$	8	$-4.0534773372589308 \times 10^{-7}$
9	$-7.0263332165482964 \times 10^{-8}$	9	$-4.7513526861945402 \times 10^{-9}$

	ω_A (deg)		I (deg)
0	22.697897294227336	0	22.443237032403296
1	$-5.5796041608336282 \times 10^{-1}$	1	$7.9396391137605010 \times 10^{-1}$
2	$-4.2444025467393790 \times 10^{-2}$	2	$3.4040986619497975 \times 10^{-2}$
3	$4.1838585885951270 \times 10^{-2}$	3	$-1.6904688515509532 \times 10^{-2}$
4	$2.8091378861663991 \times 10^{-3}$	4	$9.8090404114282761 \times 10^{-4}$
5	$-5.4323188948019781 \times 10^{-4}$	5	$1.1556605321424469 \times 10^{-4}$
6	$-4.4049111905770612 \times 10^{-5}$	6	$-1.5678351980805001 \times 10^{-5}$
7	$4.6579050128421708 \times 10^{-7}$	7	$1.5987439097128503 \times 10^{-8}$
8	$4.3603992966993063 \times 10^{-7}$	8	$9.0270262956621426 \times 10^{-8}$
9	$6.2338758200929462 \times 10^{-8}$	9	$-7.5467320429621129 \times 10^{-9}$

	χ_A (deg)		L (deg)
0	-1.1393845499761955	0	-1.9483958236809943
1	$-3.8022208201058182 \times 10^{-1}$	1	$-3.6136412065016561 \times 10^{-1}$
2	$6.4685188318652853 \times 10^{-1}$	2	$2.6741268748906756 \times 10^{-1}$
3	$5.4997711205359880 \times 10^{-2}$	3	$-2.1214528397206982 \times 10^{-2}$
4	$-1.4998670524099979 \times 10^{-2}$	4	$-3.1995612049437897 \times 10^{-3}$
5	$-1.3090222226173688 \times 10^{-3}$	5	$5.3963237987863280 \times 10^{-4}$
6	$4.7901353690215897 \times 10^{-5}$	6	$4.3507413110398560 \times 10^{-7}$
7	$1.9278128247316370 \times 10^{-5}$	7	$-4.7935165085073153 \times 10^{-6}$
8	$2.4491657007860737 \times 10^{-6}$	8	$3.9921592750122270 \times 10^{-7}$
9	$-6.5283789034648774 \times 10^{-8}$	9	$5.1098601456761042 \times 10^{-10}$

Table 4–2, continued.

Interval 91: Central time $T_c = 2240$, covering the time span $2200 \leq T \leq 2280$

	ε (deg)		p_A (deg)
0	23.536724701569014	0	3142.5247775194792
1	$3.5626327106307407 \times 10^{-1}$	1	55.654480298452341
2	$-1.8717584785991717 \times 10^{-2}$	2	$-1.5783129679009321 \times 10^{-1}$
3	$-4.9922931543566974 \times 10^{-3}$	3	$6.4180540464163482 \times 10^{-3}$
4	$1.7267962957139266 \times 10^{-4}$	4	$9.5110174319178718 \times 10^{-4}$
5	$1.4049670693014776 \times 10^{-5}$	5	$-4.7898145410051200 \times 10^{-5}$
6	$-1.2687415847699188 \times 10^{-6}$	6	$-1.3243792159906692 \times 10^{-7}$
7	$2.7987739636749678 \times 10^{-8}$	7	$3.5995613876546857 \times 10^{-7}$
8	$1.4924124377011862 \times 10^{-8}$	8	$-1.6680017367064019 \times 10^{-8}$
9	$-3.5300302303018605 \times 10^{-10}$	9	$-4.1478884982998751 \times 10^{-9}$

	ψ_A (deg)		Δ (deg)
0	3144.4253162196825	0	3147.4425486932561
1	58.235932767372914	1	55.122626660294241
2	$-3.8390392304783010 \times 10^{-1}$	2	$-1.4925419250852206 \times 10^{-1}$
3	$-1.2651973110628777 \times 10^{-1}$	3	$1.5402770959993864 \times 10^{-2}$
4	$8.0088040776607277 \times 10^{-3}$	4	$-1.5158518808283263 \times 10^{-3}$
5	$2.4745547536662448 \times 10^{-3}$	5	$-3.0813051832541472 \times 10^{-5}$
6	$-2.1545032461375735 \times 10^{-4}$	6	$1.8623904886852665 \times 10^{-5}$
7	$-4.1387765968100618 \times 10^{-5}$	7	$-6.0544574319915346 \times 10^{-7}$
8	$5.6177895544745799 \times 10^{-6}$	8	$-8.4026027931115262 \times 10^{-9}$
9	$7.2360544119769495 \times 10^{-7}$	9	$2.5271430979508796 \times 10^{-10}$

	ω_A (deg)		I (deg)
0	22.701124756258217	0	23.891131338681751
1	$8.5799206506211959 \times 10^{-1}$	1	$5.8040453275553892 \times 10^{-1}$
2	$2.9083509523263801 \times 10^{-1}$	2	$-5.5445114497596654 \times 10^{-2}$
3	$-1.5009174870863495 \times 10^{-2}$	3	$4.4452507641037347 \times 10^{-4}$
4	$-7.2653099591134235 \times 10^{-3}$	4	$4.8127962023065386 \times 10^{-4}$
5	$2.5405140383019048 \times 10^{-4}$	5	$-1.0057853694544122 \times 10^{-4}$
6	$9.2087309124271903 \times 10^{-5}$	6	$-2.9682902607855657 \times 10^{-7}$
7	$-5.8807573748646157 \times 10^{-6}$	7	$5.8151892620948094 \times 10^{-7}$
8	$-1.1754957001696667 \times 10^{-6}$	8	$-1.4329511667976730 \times 10^{-8}$
9	$1.3289095450309527 \times 10^{-7}$	9	$-5.0476374335655198 \times 10^{-10}$

	χ_A (deg)		L (deg)
0	2.0726607450988107	0	-1.4937356833014631
1	2.8169933983384615	1	$5.7650952846729374 \times 10^{-1}$
2	$-2.3768493363779347 \times 10^{-1}$	2	$-8.5209389684715686 \times 10^{-3}$
3	$-1.4375932655532540 \times 10^{-1}$	3	$-9.8810528451473562 \times 10^{-3}$
4	$7.0857731268574222 \times 10^{-3}$	4	$2.6699159819196769 \times 10^{-3}$
5	$2.6547347063350508 \times 10^{-3}$	5	$-1.1321764292415642 \times 10^{-5}$
6	$-2.1529713013289071 \times 10^{-4}$	6	$-2.0844102678860360 \times 10^{-5}$
7	$-4.2444507873100290 \times 10^{-5}$	7	$9.7452215236126301 \times 10^{-7}$
8	$5.6368988241018043 \times 10^{-6}$	8	$9.2633735993228734 \times 10^{-10}$
9	$7.2935272476588542 \times 10^{-7}$	9	$-5.0645344610019834 \times 10^{-9}$

Table 4–2, continued.

Interval 92: Central time $T_c = 2320$, covering the time span $2280 \le T \le 2360$

	ε (deg)		p_A (deg)
0	23.952461279870223	0	3252.9545930739895
1	$2.8087101740642869 \times 10^{-2}$	1	54.893823603267818
2	$-5.6117851324809283 \times 10^{-2}$	2	$-1.7005570755472190 \times 10^{-2}$
3	$-8.8951327257232884 \times 10^{-4}$	3	$1.5265182681202443 \times 10^{-2}$
4	$3.2486309949898077 \times 10^{-4}$	4	$2.1492599736320592 \times 10^{-4}$
5	$6.9754101326069774 \times 10^{-6}$	5	$-3.5793153505790406 \times 10^{-5}$
6	$-2.2393882786489302 \times 10^{-7}$	6	$-1.5619711036571523 \times 10^{-6}$
7	$-9.8586738954383261 \times 10^{-8}$	7	$-7.9369550737897188 \times 10^{-8}$
8	$-2.6905666080726564 \times 10^{-9}$	8	$3.2415218795401816 \times 10^{-8}$
9	$1.9147730283559489 \times 10^{-9}$	9	$7.7357157252182710 \times 10^{-10}$

	ψ_A (deg)		Δ (deg)
0	3255.6003439603749	0	3256.8373068827669
1	52.765727727438445	1	54.334935272050770
2	$-6.0603946955058764 \times 10^{-1}$	2	$-6.9173458353685558 \times 10^{-2}$
3	$7.4158115997606187 \times 10^{-2}$	3	$3.9678519802016471 \times 10^{-3}$
4	$5.7483609361798183 \times 10^{-3}$	4	$7.3503661700203589 \times 10^{-4}$
5	$-8.1320017360463500 \times 10^{-4}$	5	$1.6714502264974126 \times 10^{-4}$
6	$8.1166370212152729 \times 10^{-5}$	6	$-3.7875103631522937 \times 10^{-6}$
7	$1.6834622346133728 \times 10^{-6}$	7	$-8.5091187769056047 \times 10^{-7}$
8	$-1.3490166839591229 \times 10^{-6}$	8	$2.5373849170630577 \times 10^{-8}$
9	$1.4735227264305616 \times 10^{-7}$	9	$3.3735639263183626 \times 10^{-9}$

	ω_A (deg)		I (deg)
0	25.294577386808407	0	24.629537238694198
1	1.2780793959494234	1	$1.5691531662186771 \times 10^{-1}$
2	$-2.3558666882989457 \times 10^{-1}$	2	$-5.8959220823634222 \times 10^{-2}$
3	$-4.4183961643548432 \times 10^{-2}$	3	$-3.8658761271257132 \times 10^{-3}$
4	$3.2497883135238472 \times 10^{-3}$	4	$-5.9477807631288422 \times 10^{-4}$
5	$2.9965245456947036 \times 10^{-4}$	5	$2.5409707909704997 \times 10^{-5}$
6	$-2.7195380757001084 \times 10^{-5}$	6	$7.8107138106372847 \times 10^{-6}$
7	$1.6243693888702937 \times 10^{-6}$	7	$-8.1828905067455562 \times 10^{-8}$
8	$2.1988262905129514 \times 10^{-8}$	8	$-2.9440583400535844 \times 10^{-8}$
9	$-2.5304124273528995 \times 10^{-8}$	9	$-2.6128211353101730 \times 10^{-10}$

	χ_A (deg)		L (deg)
0	2.9044667364724621	0	$-3.6816289660073775 \times 10^{-1}$
1	-2.3261120844110538	1	$6.0926263981910252 \times 10^{-1}$
2	$-6.5679156064489876 \times 10^{-1}$	2	$5.7092625722828728 \times 10^{-2}$
3	$6.3861277396398033 \times 10^{-2}$	3	$1.2301513112629196 \times 10^{-2}$
4	$6.8822576216625461 \times 10^{-3}$	4	$-5.8300354521679119 \times 10^{-4}$
5	$-8.2224320425790551 \times 10^{-4}$	5	$-2.2652125730491031 \times 10^{-4}$
6	$7.1938048582887528 \times 10^{-5}$	6	$2.3788504844172594 \times 10^{-6}$
7	$1.9237861007315752 \times 10^{-6}$	7	$9.1341749102553507 \times 10^{-7}$
8	$-1.3351995684221119 \times 10^{-6}$	8	$1.1604718096005930 \times 10^{-8}$
9	$1.4704016491473898 \times 10^{-7}$	9	$-2.9109564611705702 \times 10^{-9}$

Table 4–2, continued.

Interval 93: Central time $T_c = 2400$, covering the time span $2360 \leq T \leq 2440$

	ε (deg)		p_A (deg)
0	23.592107623588696	0	3363.1846869898482
1	$-3.6962647306523314 \times 10^{-1}$	1	55.483602508916690
2	$-3.3572980832826215 \times 10^{-2}$	2	$1.5793340437908075 \times 10^{-1}$
3	$4.6086526813165922 \times 10^{-3}$	3	$1.1475452128653282 \times 10^{-2}$
4	$3.1389572729890081 \times 10^{-4}$	4	$-7.4527340613841766 \times 10^{-4}$
5	$-6.4682647765447592 \times 10^{-6}$	5	$-5.8641900351250019 \times 10^{-5}$
6	$-3.6274206449477536 \times 10^{-7}$	6	$-2.2884312347807472 \times 10^{-6}$
7	$-9.3382052763484159 \times 10^{-8}$	7	$-2.9145651301064957 \times 10^{-7}$
8	$-2.5557041154930987 \times 10^{-8}$	8	$1.8641250632679087 \times 10^{-8}$
9	$-6.9460226164260199 \times 10^{-12}$	9	$9.4676481513078268 \times 10^{-9}$

	ψ_A (deg)		Δ (deg)
0	3359.7488101188940	0	3365.3742126581335
1	52.366523960250336	1	54.363398151582135
2	$5.6732243797175484 \times 10^{-1}$	2	$1.2696375206851786 \times 10^{-1}$
3	$1.1108060541286692 \times 10^{-1}$	3	$3.1837769865882384 \times 10^{-2}$
4	$3.0244165082021611 \times 10^{-4}$	4	$1.9250267373659294 \times 10^{-3}$
5	$-4.6882617713229969 \times 10^{-4}$	5	$-8.2475355890815957 \times 10^{-5}$
6	$-1.0407125479270150 \times 10^{-4}$	6	$-1.4331825493867709 \times 10^{-5}$
7	$-1.2347028978159574 \times 10^{-5}$	7	$-5.4209522603248139 \times 10^{-7}$
8	$-3.2415130132040841 \times 10^{-7}$	8	$-8.7549460269686161 \times 10^{-8}$
9	$6.7462365968720115 \times 10^{-8}$	9	$-6.8415301984441006 \times 10^{-9}$

	ω_A (deg)		I (deg)
0	25.105203085540685	0	24.271121527059174
1	-1.5548689368984898	1	$-5.6829301757047343 \times 10^{-1}$
2	$-3.3682897073348384 \times 10^{-1}$	2	$-1.1898041330266758 \times 10^{-1}$
3	$3.2131300060770599 \times 10^{-2}$	3	$-1.5973934506566976 \times 10^{-3}$
4	$5.2592381698132705 \times 10^{-3}$	4	$1.1404663218902328 \times 10^{-3}$
5	$-5.5139018949806541 \times 10^{-5}$	5	$1.0390330842686113 \times 10^{-4}$
6	$-1.7653669912603169 \times 10^{-5}$	6	$-2.6981266815390457 \times 10^{-6}$
7	$-1.7535897874688085 \times 10^{-6}$	7	$-3.8898190801599826 \times 10^{-7}$
8	$-2.2314810087368101 \times 10^{-7}$	8	$9.1003121293324298 \times 10^{-9}$
9	$-1.3119793915872296 \times 10^{-8}$	9	$-1.4333037083138080 \times 10^{-9}$

	χ_A (deg)		L (deg)
0	-3.7583681336525629	0	1.4726306651445379
1	-3.3920510502264165	1	1.2133706395274329
2	$4.6663189119835879 \times 10^{-1}$	2	$3.0716830710098292 \times 10^{-2}$
3	$1.0924855940708194 \times 10^{-1}$	3	$-2.2608157909670076 \times 10^{-2}$
4	$5.3622944364960244 \times 10^{-5}$	4	$-2.8693259169775726 \times 10^{-3}$
5	$-5.1977476442578809 \times 10^{-4}$	5	$4.1946687674808876 \times 10^{-5}$
6	$-9.3527317624879010 \times 10^{-5}$	6	$1.3968984200998039 \times 10^{-5}$
7	$-1.1382207119890183 \times 10^{-5}$	7	$1.3357055821082713 \times 10^{-7}$
8	$-3.8959759153747760 \times 10^{-7}$	8	$9.8843667552089115 \times 10^{-8}$
9	$5.4460232362871161 \times 10^{-8}$	9	$1.7676493262010762 \times 10^{-8}$

Table 4–2, continued.

Interval 94: Central time $T_c = 2480$, covering the time span $2440 \leq T \leq 2520$

	ε (deg)
0	22.806438733543670
1	$-3.5240700191777798 \times 10^{-1}$
2	$4.1750147362170103 \times 10^{-2}$
3	$6.5855055291071130 \times 10^{-3}$
4	$-2.3724934382759129 \times 10^{-4}$
5	$-5.0932996753415409 \times 10^{-5}$
6	$1.6335894954740082 \times 10^{-7}$
7	$4.6393432264970487 \times 10^{-7}$
8	$1.0656214814715420 \times 10^{-9}$
9	$-6.7086857290237563 \times 10^{-9}$

	p_A (deg)
0	3475.6384127256992
1	56.988865171085998
2	$1.7779692275134528 \times 10^{-1}$
3	$-1.1470893169864503 \times 10^{-2}$
4	$-1.9165320831036319 \times 10^{-3}$
5	$2.1047270545781164 \times 10^{-5}$
6	$1.4511613388461452 \times 10^{-5}$
7	$2.6037782965680136 \times 10^{-7}$
8	$-1.5801555684349093 \times 10^{-7}$
9	$-3.7865794899714036 \times 10^{-9}$

	ψ_A (deg)
0	3472.0647641164171
1	60.396450224934124
2	1.0283103116939941
3	$-1.2823372504571778 \times 10^{-1}$
4	$-3.1457067658387810 \times 10^{-2}$
5	$6.5577366984202211 \times 10^{-4}$
6	$7.7609132346493682 \times 10^{-4}$
7	$5.4906721529462918 \times 10^{-5}$
8	$-1.5113736367872103 \times 10^{-5}$
9	$-2.9052699160057144 \times 10^{-6}$

	Δ (deg)
0	3476.5071967443669
1	57.134816552086096
2	$5.5112006738211820 \times 10^{-1}$
3	$2.0317963582750065 \times 10^{-2}$
4	$-5.6771252618233086 \times 10^{-3}$
5	$-7.5850666308130477 \times 10^{-4}$
6	$-1.8205950254066807 \times 10^{-5}$
7	$6.5834279800672522 \times 10^{-6}$
8	$1.0684306563165745 \times 10^{-6}$
9	$4.1814082312377724 \times 10^{-8}$

	ω_A (deg)
0	21.300309714232060
1	-1.6446978389699155
2	$3.6813480084586318 \times 10^{-1}$
3	$5.7270798260460666 \times 10^{-2}$
4	$-6.1795407003483578 \times 10^{-3}$
5	$-1.1253514068972718 \times 10^{-3}$
6	$2.2426632339066409 \times 10^{-5}$
7	$2.0653065936990002 \times 10^{-5}$
8	$1.4750989310642587 \times 10^{-6}$
9	$-3.1843348968495258 \times 10^{-7}$

	I (deg)
0	22.418500438305989
1	-1.1716258483094576
2	$1.7222160898242445 \times 10^{-2}$
3	$2.5446590910519022 \times 10^{-2}$
4	$1.4626214088664747 \times 10^{-3}$
5	$-1.4894342096189291 \times 10^{-4}$
6	$-2.2320807931612753 \times 10^{-5}$
7	$-8.0476160347714215 \times 10^{-7}$
8	$1.1015537861620719 \times 10^{-7}$
9	$2.2617936749779754 \times 10^{-8}$

	χ_A (deg)
0	-3.8677958284294961
1	3.7032977187373944
2	$8.9923941612217144 \times 10^{-1}$
3	$-1.2706376592088645 \times 10^{-1}$
4	$-3.0268647212108826 \times 10^{-2}$
5	$7.5078217100729740 \times 10^{-4}$
6	$7.6576554987481908 \times 10^{-4}$
7	$5.4047165463686251 \times 10^{-5}$
8	$-1.4954093745323702 \times 10^{-5}$
9	$-2.9004605795702377 \times 10^{-6}$

	L (deg)
0	2.8789451317839060
1	$-1.7880600849164149 \times 10^{-1}$
2	$-4.0278231626609666 \times 10^{-1}$
3	$-3.2985179865646819 \times 10^{-2}$
4	$4.1099538695562104 \times 10^{-3}$
5	$8.0367077187796285 \times 10^{-4}$
6	$3.1219445246594135 \times 10^{-5}$
7	$-6.4994307037983925 \times 10^{-6}$
8	$-1.2330847541933814 \times 10^{-6}$
9	$-4.4554044141837006 \times 10^{-8}$

Table 4–2, continued.

Interval 95: Central time $T_c = 2560$, covering the time span $2520 \leq T \leq 2600$

	ε (deg)		p_A (deg)
0	22.599750971729553	0	3590.3211816033566
1	$1.7265426963849661 \times 10^{-1}$	1	57.474114507639743
2	$7.3249633119199407 \times 10^{-2}$	2	$-8.0554694004899209 \times 10^{-2}$
3	$-2.4058423367259044 \times 10^{-3}$	3	$-2.4685229214718228 \times 10^{-2}$
4	$-6.6281572798018045 \times 10^{-4}$	4	$6.8207197898972106 \times 10^{-4}$
5	$1.1745996693037980 \times 10^{-5}$	5	$1.5330827351771954 \times 10^{-4}$
6	$2.1980702760276766 \times 10^{-6}$	6	$-3.6917767492918553 \times 10^{-6}$
7	$8.2704842149056113 \times 10^{-9}$	7	$-3.5379836980915354 \times 10^{-7}$
8	$1.5557612752708081 \times 10^{-8}$	8	$-1.1179071406690571 \times 10^{-8}$
9	$-2.6470828771066795 \times 10^{-9}$	9	$-6.2684335784483427 \times 10^{-9}$

	ψ_A (deg)		Δ (deg)
0	3593.7785705295051	0	3594.3236224088953
1	59.456671549761237	1	60.182277611184407
2	−1.0514784829667098	2	$-4.2930771562362753 \times 10^{-2}$
3	$-3.1362913177451439 \times 10^{-2}$	3	$-1.1164442340641468 \times 10^{-1}$
4	$2.7789402921088915 \times 10^{-2}$	4	$-1.3796013093337378 \times 10^{-3}$
5	$-2.1411454796828028 \times 10^{-3}$	5	$1.7366350414424161 \times 10^{-3}$
6	$-3.2091306593926130 \times 10^{-4}$	6	$3.6132069094449912 \times 10^{-5}$
7	$8.8221510503129525 \times 10^{-5}$	7	$-2.8122238679601085 \times 10^{-5}$
8	$-3.8725051958024265 \times 10^{-6}$	8	$-7.8845846622965361 \times 10^{-7}$
9	$-1.6617758840447950 \times 10^{-6}$	9	$4.9583799513106582 \times 10^{-7}$

	ω_A (deg)		I (deg)
0	21.407761048838914	0	21.204339912616621
1	1.6052668256354770	1	$1.9114329737998072 \times 10^{-1}$
2	$2.0677202954807108 \times 10^{-1}$	2	$2.8515744157615368 \times 10^{-1}$
3	$-6.5927205302945303 \times 10^{-2}$	3	$3.4173799046365609 \times 10^{-3}$
4	$-3.5218806082046238 \times 10^{-4}$	4	$-4.2486417423780482 \times 10^{-3}$
5	$1.0185902783096111 \times 10^{-3}$	5	$-1.0007364045655969 \times 10^{-4}$
6	$-7.4895644036200079 \times 10^{-5}$	6	$4.7861639964432087 \times 10^{-5}$
7	$-6.7902331933433461 \times 10^{-6}$	7	$1.3408497352998133 \times 10^{-6}$
8	$2.1068453060388222 \times 10^{-6}$	8	$-6.2307987514175095 \times 10^{-7}$
9	$-1.3747515317983398 \times 10^{-7}$	9	$-2.0480388372162372 \times 10^{-8}$

	χ_A (deg)		L (deg)
0	3.7349923920699333	0	$-5.2296709681470777 \times 10^{-1}$
1	2.1597912775133754	1	−2.9227024849169463
2	−1.0366270082305283	2	$-4.2361221486192918 \times 10^{-2}$
3	$-1.0142707313085205 \times 10^{-2}$	3	$9.1868189011011297 \times 10^{-2}$
4	$2.8394864167233226 \times 10^{-2}$	4	$2.2460726732343408 \times 10^{-3}$
5	$-2.2681289528452583 \times 10^{-3}$	5	$-1.6246708927277364 \times 10^{-3}$
6	$-3.2766436850920739 \times 10^{-4}$	6	$-4.1293429192947068 \times 10^{-5}$
7	$8.8349027872102218 \times 10^{-5}$	7	$2.8015996765511329 \times 10^{-5}$
8	$-3.8124821037449145 \times 10^{-6}$	8	$7.7847435704496674 \times 10^{-7}$
9	$-1.6527483870745032 \times 10^{-6}$	9	$-5.0395830473838421 \times 10^{-7}$

Table 4–2, continued.

Interval 96: Central time $T_c = 2640$, covering the time span $2600 \le T \le 2680$

	ε (deg)		p_A (deg)
0	23.336032844206001	0	3703.9399695778440
1	$4.9956277832320352 \times 10^{-1}$	1	56.012675818548436
2	$-2.2146507539248816 \times 10^{-3}$	2	$-2.3630047558004904 \times 10^{-1}$
3	$-8.3479721544375616 \times 10^{-3}$	3	$2.1498681276393939 \times 10^{-3}$
4	$4.4038945817446081 \times 10^{-5}$	4	$2.0118515520713158 \times 10^{-3}$
5	$4.6778629314983013 \times 10^{-5}$	5	$-4.0113969031820627 \times 10^{-5}$
6	$-6.1873976730753688 \times 10^{-7}$	6	$-7.4272700184101705 \times 10^{-6}$
7	$-1.4117481096919954 \times 10^{-7}$	7	$4.5644708415561553 \times 10^{-7}$
8	$1.1119366832957683 \times 10^{-8}$	8	$1.9548244626078392 \times 10^{-9}$
9	$-5.1630375281342244 \times 10^{-10}$	9	$-5.6601338630003767 \times 10^{-9}$

	ψ_A (deg)		Δ (deg)
0	3706.1068851106843	0	3711.1604267813533
1	53.614642633565162	1	56.063902804588795
2	$-2.2290721680140835 \times 10^{-1}$	2	$-7.3490594531015595 \times 10^{-1}$
3	$8.7780938388205668 \times 10^{-2}$	3	$1.8448619262849705 \times 10^{-2}$
4	$-4.1444886306660867 \times 10^{-3}$	4	$7.7942941994226960 \times 10^{-3}$
5	$-3.5879522527629850 \times 10^{-4}$	5	$-7.3907458585317871 \times 10^{-4}$
6	$3.3932951791741831 \times 10^{-5}$	6	$-2.2988516709629150 \times 10^{-6}$
7	$-7.3602778451105908 \times 10^{-6}$	7	$9.8362350665948620 \times 10^{-6}$
8	$7.9485165549370952 \times 10^{-7}$	8	$-1.0780736156177540 \times 10^{-6}$
9	$1.8783178127216988 \times 10^{-9}$	9	$-8.2365512124364366 \times 10^{-9}$

	ω_A (deg)		I (deg)
0	24.316043593636526	0	23.296347622637963
1	$9.0936230479289312 \times 10^{-1}$	1	1.6652251279615073
2	$-2.5770859964705310 \times 10^{-1}$	2	$1.3049816842501935 \times 10^{-2}$
3	$-7.0839516559183835 \times 10^{-4}$	3	$-3.5367156413909918 \times 10^{-2}$
4	$4.5958271572778099 \times 10^{-3}$	4	$4.8308989504441300 \times 10^{-4}$
5	$-2.0010138556915966 \times 10^{-4}$	5	$2.4693744407611786 \times 10^{-4}$
6	$-1.6810033656974270 \times 10^{-5}$	6	$-1.8005533409033434 \times 10^{-5}$
7	$6.9986559390801194 \times 10^{-7}$	7	$-5.3031306566492505 \times 10^{-8}$
8	$-1.0319964509929476 \times 10^{-7}$	8	$1.8671725105410651 \times 10^{-7}$
9	$1.5376192572272700 \times 10^{-8}$	9	$-2.0425009385549999 \times 10^{-8}$

	χ_A (deg)		L (deg)
0	2.3616101574181060	0	-4.0288489629794298
1	-2.6072799182232506	1	$-8.2657699758998346 \times 10^{-2}$
2	$5.6071972195251868 \times 10^{-3}$	2	$5.4269643565957995 \times 10^{-1}$
3	$9.4718690867778524 \times 10^{-2}$	3	$-1.4930895327064128 \times 10^{-2}$
4	$-6.3706717274293110 \times 10^{-3}$	4	$-6.3127981981272876 \times 10^{-3}$
5	$-4.3523710289535674 \times 10^{-4}$	5	$6.8560035789548218 \times 10^{-4}$
6	$4.4933285500149236 \times 10^{-5}$	6	$-2.2862731590414113 \times 10^{-6}$
7	$-7.0656513845487359 \times 10^{-6}$	7	$-9.3326949010528579 \times 10^{-6}$
8	$7.6682133593496628 \times 10^{-7}$	8	$1.0712838822308749 \times 10^{-6}$
9	$4.4438347646167378 \times 10^{-9}$	9	$2.2585182418200192 \times 10^{-9}$

Table 4–2, continued.

Interval 97: Central time $T_c = 2720$, covering the time span $2680 \le T \le 2760$

	ε (deg)		p_A (deg)
0	24.049140649420202	0	3814.4745471809545
1	$1.5391251391699727 \times 10^{-1}$	1	54.667560793665090
2	$-7.2658021066317429 \times 10^{-2}$	2	$-6.5003188435570554 \times 10^{-2}$
3	$-1.9509661745511778 \times 10^{-3}$	3	$2.1895917427559711 \times 10^{-2}$
4	$6.0601776231130790 \times 10^{-4}$	4	$2.7216053473097723 \times 10^{-4}$
5	$1.5870130716151376 \times 10^{-6}$	5	$-9.8348617746141678 \times 10^{-5}$
6	$-2.3684913925242566 \times 10^{-6}$	6	$2.0583744747767464 \times 10^{-6}$
7	$9.3251411619620677 \times 10^{-8}$	7	$2.7582083738667516 \times 10^{-7}$
8	$1.7471726295441665 \times 10^{-8}$	8	$-2.5391465547968001 \times 10^{-8}$
9	$-7.9485809529716705 \times 10^{-10}$	9	$-4.2536517494544281 \times 10^{-9}$

	ψ_A (deg)		Δ (deg)
0	3813.8189675054463	0	3818.9976472982676
1	54.507379936930906	1	52.279125642394535
2	$2.6260621158865914 \times 10^{-1}$	2	$-1.4259370528038791 \times 10^{-1}$
3	$-1.4822122894916541 \times 10^{-2}$	3	$6.1094083101379200 \times 10^{-2}$
4	$-6.8198709159640613 \times 10^{-3}$	4	$2.0782048136417257 \times 10^{-5}$
5	$3.4353378890261339 \times 10^{-4}$	5	$-6.8012898869836264 \times 10^{-5}$
6	$7.5317811012504005 \times 10^{-5}$	6	$9.0405492734179470 \times 10^{-6}$
7	$1.4530906424695647 \times 10^{-6}$	7	$-2.1891486910720439 \times 10^{-6}$
8	$-8.7845011423283375 \times 10^{-7}$	8	$3.8528906825870896 \times 10^{-8}$
9	$-6.6819783252481046 \times 10^{-8}$	9	$8.3628511922234450 \times 10^{-11}$

	ω_A (deg)		I (deg)
0	24.653900811326501	0	25.645126867307141
1	$-3.5721329848194382 \times 10^{-1}$	1	$4.4907700632404805 \times 10^{-1}$
2	$-1.3705463686569065 \times 10^{-2}$	2	$-2.6501894628661084 \times 10^{-1}$
3	$2.4410295437827472 \times 10^{-2}$	3	$-6.5800953732023574 \times 10^{-3}$
4	$-1.9791496134914954 \times 10^{-3}$	4	$2.3695757577731470 \times 10^{-3}$
5	$-3.3438608707346130 \times 10^{-4}$	5	$9.8182207279994256 \times 10^{-6}$
6	$1.7455431519124439 \times 10^{-5}$	6	$-3.0975832532739547 \times 10^{-6}$
7	$2.6437141042678513 \times 10^{-6}$	7	$8.2238273373290423 \times 10^{-8}$
8	$9.9232417911051967 \times 10^{-9}$	8	$-4.0367972469723076 \times 10^{-8}$
9	$-2.2598680030503717 \times 10^{-8}$	9	$1.3225223599038720 \times 10^{-9}$

	χ_A (deg)		L (deg)
0	$-7.1958133458537897 \times 10^{-1}$	0	-1.0896929597599833
1	$-1.7534653226268217 \times 10^{-1}$	1	2.6348613763429192
2	$3.5992000428644254 \times 10^{-1}$	2	$9.0262910387698122 \times 10^{-2}$
3	$-4.0473798356335675 \times 10^{-2}$	3	$-4.4841524007079850 \times 10^{-2}$
4	$-7.8349864862529248 \times 10^{-3}$	4	$9.2246849738485118 \times 10^{-5}$
5	$5.1257232447621490 \times 10^{-4}$	5	$1.2951569607218629 \times 10^{-5}$
6	$7.8883781753659362 \times 10^{-5}$	6	$-5.8646215168042138 \times 10^{-6}$
7	$6.6170363398457875 \times 10^{-7}$	7	$2.3285668190607591 \times 10^{-6}$
8	$-8.6622323084453182 \times 10^{-7}$	8	$-7.0911453023762390 \times 10^{-8}$
9	$-5.9986879723926574 \times 10^{-8}$	9	$-4.8559783822246410 \times 10^{-9}$

Table 4–2, continued.

Interval 98: Central time $T_c = 2800$, covering the time span $2760 \leq T \leq 2840$

	ε (deg)		p_A (deg)
0	23.810729124930800	0	3924.0884720160644
1	$-3.6750033669659394 \times 10^{-1}$	1	55.143710666845768
2	$-4.3403836252535344 \times 10^{-2}$	2	$1.6901645953192230 \times 10^{-1}$
3	$6.1387057977758591 \times 10^{-3}$	3	$1.3433087323590030 \times 10^{-2}$
4	$3.4073338573992497 \times 10^{-4}$	4	$-1.2401026336436397 \times 10^{-3}$
5	$-2.3266472676925594 \times 10^{-5}$	5	$-7.3296177946008027 \times 10^{-5}$
6	$-1.5696975350618558 \times 10^{-6}$	6	$-8.8781422973983063 \times 10^{-7}$
7	$-1.6906497978447026 \times 10^{-7}$	7	$2.8963293953581579 \times 10^{-7}$
8	$7.3066690497970126 \times 10^{-9}$	8	$8.2718897330209127 \times 10^{-8}$
9	$3.8010124164991838 \times 10^{-9}$	9	$1.6407549191910204 \times 10^{-10}$

	ψ_A (deg)		Δ (deg)
0	3923.7102873973050	0	3924.6631577533100
1	55.037351143158412	1	53.955422530064941
2	$-1.2296694363591340 \times 10^{-1}$	2	$5.3431749678661962 \times 10^{-1}$
3	$-1.1070936246686500 \times 10^{-2}$	3	$4.3777731219163408 \times 10^{-2}$
4	$7.0481983080093763 \times 10^{-3}$	4	$-3.4527895360040280 \times 10^{-3}$
5	$2.7968194893104073 \times 10^{-4}$	5	$-4.3966628069379151 \times 10^{-4}$
6	$-7.6935569538445739 \times 10^{-5}$	6	$-2.8084521149076797 \times 10^{-5}$
7	$1.6084920277336665 \times 10^{-6}$	7	$1.1803324155450639 \times 10^{-6}$
8	$7.2169927575187426 \times 10^{-7}$	8	$4.1255167354281075 \times 10^{-7}$
9	$-7.8821581151131170 \times 10^{-8}$	9	$3.5609633136623483 \times 10^{-8}$

	ω_A (deg)		I (deg)
0	24.193551402908243	0	24.618696575818233
1	$-9.9913027968158758 \times 10^{-2}$	1	-1.3582336741556060
2	$-8.2477096175834913 \times 10^{-3}$	2	$-1.2587031450131334 \times 10^{-1}$
3	$-2.1475693527251618 \times 10^{-2}$	3	$2.7669487315565463 \times 10^{-2}$
4	$-1.2292654318812743 \times 10^{-3}$	4	$1.4681544978005696 \times 10^{-3}$
5	$3.7762819343981846 \times 10^{-4}$	5	$-1.3312059933107593 \times 10^{-4}$
6	$1.0614971991044800 \times 10^{-5}$	6	$-1.0942413536177098 \times 10^{-5}$
7	$-2.7230564461544152 \times 10^{-6}$	7	$-3.7616904358932940 \times 10^{-7}$
8	$5.3579199587576044 \times 10^{-8}$	8	$1.9459776471536999 \times 10^{-8}$
9	$1.7402950248958178 \times 10^{-8}$	9	$4.9099275607904585 \times 10^{-9}$

	χ_A (deg)		L (deg)
0	$-4.1384211579509318 \times 10^{-1}$	0	3.2522413445222636
1	$-1.1535709293899058 \times 10^{-1}$	1	1.2888801752994970
2	$-3.1940489288004691 \times 10^{-1}$	2	$-4.0696426332264238 \times 10^{-1}$
3	$-2.6504776160986704 \times 10^{-2}$	3	$-3.1876561100211529 \times 10^{-2}$
4	$9.1330152167061747 \times 10^{-3}$	4	$2.7675441286065523 \times 10^{-3}$
5	$3.8968995703764390 \times 10^{-4}$	5	$3.7384729807722148 \times 10^{-4}$
6	$-8.3349728848545056 \times 10^{-5}$	6	$2.3606015186654083 \times 10^{-5}$
7	$1.0274586272447872 \times 10^{-6}$	7	$-8.9291945274710412 \times 10^{-7}$
8	$6.5849385369097033 \times 10^{-7}$	8	$-3.0895522868112732 \times 10^{-7}$
9	$-7.7204662452724843 \times 10^{-8}$	9	$-3.5750494813655033 \times 10^{-8}$

Table 4–2, continued.

Interval 99: Central time $T_c = 2880$, covering the time span $2840 \le T \le 2920$

	ε (deg)		p_A (deg)
0	22.994102938658660	0	4035.9394202301871
1	$-3.7817234047556243 \times 10^{-1}$	1	56.712779573400526
2	$4.0130662126731594 \times 10^{-2}$	2	$1.7487924305763356 \times 10^{-1}$
3	$5.7207801162527819 \times 10^{-3}$	3	$-1.3402438259021360 \times 10^{-2}$
4	$-4.4073867093880550 \times 10^{-4}$	4	$-1.5156892395081550 \times 10^{-3}$
5	$-3.1886912189839476 \times 10^{-5}$	5	$9.7759188784783970 \times 10^{-5}$
6	$3.2944008988721072 \times 10^{-6}$	6	$8.6593023980822616 \times 10^{-6}$
7	$1.6777026220717967 \times 10^{-7}$	7	$-8.4794056107471523 \times 10^{-7}$
8	$-4.0715967461336107 \times 10^{-8}$	8	$-6.9388931888134719 \times 10^{-8}$
9	$-1.7370557855213751 \times 10^{-9}$	9	$1.1873210879899323 \times 10^{-8}$

	ψ_A (deg)		Δ (deg)
0	4033.6916419951371	0	4037.3752716183828
1	55.356865926781562	1	58.680227247813358
2	$3.5901197976484468 \times 10^{-1}$	2	$4.3175589736618564 \times 10^{-1}$
3	$7.4010088019662468 \times 10^{-2}$	3	$-7.3810244191334714 \times 10^{-2}$
4	$2.5653150634055206 \times 10^{-4}$	4	$-7.1438784872270214 \times 10^{-3}$
5	$-8.8302885934481613 \times 10^{-4}$	5	$8.3781732098170331 \times 10^{-4}$
6	$-6.8580980417180347 \times 10^{-5}$	6	$1.2349845275008081 \times 10^{-4}$
7	$-4.8508012570485046 \times 10^{-6}$	7	$-6.3950720963316334 \times 10^{-6}$
8	$1.9746397504096189 \times 10^{-7}$	8	$-2.1060512400240053 \times 10^{-6}$
9	$1.3111338195761618 \times 10^{-7}$	9	$4.2558977249754614 \times 10^{-9}$

	ω_A (deg)		I (deg)
0	23.247067139915795	0	22.035783259033508
1	$-9.7087137595836145 \times 10^{-1}$	1	$-9.3382192417466550 \times 10^{-1}$
2	$-1.4588896809949231 \times 10^{-1}$	2	$2.1360970376048829 \times 10^{-1}$
3	$1.3346562957296933 \times 10^{-2}$	3	$1.6656629927535331 \times 10^{-2}$
4	$4.4364533076386966 \times 10^{-3}$	4	$-3.1879051326814204 \times 10^{-3}$
5	$3.7769628158786755 \times 10^{-5}$	5	$-1.9318074362396166 \times 10^{-4}$
6	$-3.1784611202856806 \times 10^{-5}$	6	$2.5175996048872870 \times 10^{-5}$
7	$-1.5032547844618262 \times 10^{-6}$	7	$2.7951985065619240 \times 10^{-6}$
8	$-9.1236774551208427 \times 10^{-8}$	8	$-1.3995517763674779 \times 10^{-7}$
9	$-1.5508481378778947 \times 10^{-9}$	9	$-4.0668038202257179 \times 10^{-8}$

	χ_A (deg)		L (deg)
0	−2.4404307201418950	0	2.2979767260749157
1	−1.4622898083828670	1	−2.1378744932218879
2	$2.0448991703855713 \times 10^{-1}$	2	$-2.7160687113287353 \times 10^{-1}$
3	$9.4598997318479653 \times 10^{-2}$	3	$6.5168469553804880 \times 10^{-2}$
4	$1.5656121127352272 \times 10^{-3}$	4	$5.7072634712093522 \times 10^{-3}$
5	$-1.0892662033262433 \times 10^{-3}$	5	$-7.7990575833426266 \times 10^{-4}$
6	$-7.6136834578593755 \times 10^{-5}$	6	$-1.1497446333131393 \times 10^{-4}$
7	$-3.1882036542610927 \times 10^{-6}$	7	$5.6463329564335440 \times 10^{-6}$
8	$2.6289868367227823 \times 10^{-7}$	8	$2.0369013841838214 \times 10^{-6}$
9	$1.1412727386971713 \times 10^{-7}$	9	$8.7291641968013748 \times 10^{-9}$

Table 4–2, continued.

Interval 100: Central time $T_c = 2960$, covering the time span $2920 \le T \le 3000$

	ε (deg)		p_A (deg)
0	22.676530555360627	0	4150.0828461050913
1	$7.5786411916055089 \times 10^{-2}$	1	57.237370150626703
2	$5.8410704564386139 \times 10^{-2}$	2	$-5.2393259612698480 \times 10^{-2}$
3	$-2.6228011041357648 \times 10^{-3}$	3	$-1.8460641043861287 \times 10^{-2}$
4	$-4.4832818689229922 \times 10^{-4}$	4	$8.2080968669416139 \times 10^{-4}$
5	$1.7661851173208655 \times 10^{-5}$	5	$9.5184508970269595 \times 10^{-5}$
6	$1.3157694789893551 \times 10^{-6}$	6	$-2.6600480730743795 \times 10^{-6}$
7	$1.5105394323155405 \times 10^{-7}$	7	$-2.0558463037745553 \times 10^{-7}$
8	$5.4147251411594481 \times 10^{-9}$	8	$-7.6903239287209578 \times 10^{-8}$
9	$-5.3558012903329249 \times 10^{-9}$	9	$-2.4747112373588489 \times 10^{-9}$

	ψ_A (deg)		Δ (deg)
0	4148.9310373516930	0	4155.1259402946849
1	60.009877300278621	1	58.332670521255079
2	$4.6551884344885380 \times 10^{-1}$	2	$-4.1801311332663005 \times 10^{-1}$
3	$-1.1327902278872326 \times 10^{-1}$	3	$-2.3070590381812053 \times 10^{-2}$
4	$-1.8182902563222595 \times 10^{-2}$	4	$8.7192784800312862 \times 10^{-3}$
5	$1.3716497900865463 \times 10^{-3}$	5	$-3.0479415173758685 \times 10^{-4}$
6	$4.3543017269181908 \times 10^{-4}$	6	$-8.7754150780682108 \times 10^{-5}$
7	$-2.4852021385977741 \times 10^{-7}$	7	$1.0568273192421205 \times 10^{-5}$
8	$-9.2791472995047276 \times 10^{-6}$	8	$2.9011310596058449 \times 10^{-7}$
9	$-6.2430166533074493 \times 10^{-7}$	9	$-1.6898412554224930 \times 10^{-7}$

	ω_A (deg)		I (deg)
0	21.320829356132495	0	21.874169114807296
1	$-5.7483235845550951 \times 10^{-1}$	1	$6.8325612719811793 \times 10^{-1}$
2	$2.9264984686228844 \times 10^{-1}$	2	$1.1352686345712177 \times 10^{-1}$
3	$3.6348107947426265 \times 10^{-2}$	3	$-2.4948691868373907 \times 10^{-2}$
4	$-4.6578918748048912 \times 10^{-3}$	4	$-2.0293218160366155 \times 10^{-4}$
5	$-7.6470754061063873 \times 10^{-4}$	5	$3.0065066629706685 \times 10^{-4}$
6	$3.5817674738729534 \times 10^{-5}$	6	$-1.2881077231485308 \times 10^{-5}$
7	$1.2256580582079557 \times 10^{-5}$	7	$-2.0536354774000186 \times 10^{-6}$
8	$1.7064830055936756 \times 10^{-7}$	8	$2.4426685436822147 \times 10^{-7}$
9	$-2.0018629666300156 \times 10^{-7}$	9	$3.6699733345741725 \times 10^{-9}$

	χ_A (deg)		L (deg)
0	−1.2444993881053927	0	−1.6089980199821819
1	2.9935191961281081	1	−1.1894483621076444
2	$5.5451938054995527 \times 10^{-1}$	2	$3.9199398550600616 \times 10^{-1}$
3	$-1.0103690738318282 \times 10^{-1}$	3	$5.3114113951192657 \times 10^{-3}$
4	$-1.9867274010351541 \times 10^{-2}$	4	$-8.3440332443742015 \times 10^{-3}$
5	$1.3414463724809596 \times 10^{-3}$	5	$4.0385816238273523 \times 10^{-4}$
6	$4.4495259646892936 \times 10^{-4}$	6	$8.8645431312411138 \times 10^{-5}$
7	$-4.4396644382704746 \times 10^{-7}$	7	$-1.0796164540387486 \times 10^{-5}$
8	$-9.2158420233198045 \times 10^{-6}$	8	$-3.9385088364137680 \times 10^{-7}$
9	$-6.1969171423807271 \times 10^{-7}$	9	$1.6608755394208670 \times 10^{-7}$

Table 4–2, continued.

Interval 101: Central time $T_c = 3040$, covering the time span $3000 \leq T \leq 3080$

	ε (deg)		p_A (deg)
0	23.135931819475922	0	4263.6626918867728
1	$3.3562591046734666 \times 10^{-1}$	1	56.256139872136376
2	$2.1258170551103260 \times 10^{-3}$	2	$-1.5529496229790951 \times 10^{-1}$
3	$-5.0951013976756592 \times 10^{-3}$	3	$2.1988809779722613 \times 10^{-3}$
4	$1.7464560670257701 \times 10^{-4}$	4	$1.1463710423270464 \times 10^{-3}$
5	$2.2934148934534638 \times 10^{-5}$	5	$-7.4869776530684827 \times 10^{-5}$
6	$-2.9727932993007181 \times 10^{-6}$	6	$-2.9919852701724086 \times 10^{-6}$
7	$-5.8882402673740915 \times 10^{-8}$	7	$1.0194494489205940 \times 10^{-6}$
8	$4.5759086216111600 \times 10^{-8}$	8	$-1.1466975322360857 \times 10^{-8}$
9	$-5.1624112654505984 \times 10^{-10}$	9	$-1.4968907715428082 \times 10^{-8}$

	ψ_A (deg)		Δ (deg)
0	4267.4628861166771	0	4268.7336506895141
1	57.278414695372160	1	55.473879042560591
2	$-9.3921097821443865 \times 10^{-1}$	2	$-1.9977355564406149 \times 10^{-1}$
3	$-1.2187291542696049 \times 10^{-2}$	3	$3.3435855887913979 \times 10^{-2}$
4	$1.8377968924896635 \times 10^{-2}$	4	$-9.7892794387904433 \times 10^{-4}$
5	$-1.1570269261355710 \times 10^{-3}$	5	$-2.5601275061439262 \times 10^{-4}$
6	$-1.7259992014077304 \times 10^{-4}$	6	$2.4023642008688704 \times 10^{-5}$
7	$4.0939083890706591 \times 10^{-5}$	7	$-1.1927235373212999 \times 10^{-6}$
8	$-1.3415243199947995 \times 10^{-6}$	8	$1.3874782188587159 \times 10^{-8}$
9	$-6.4810631200782021 \times 10^{-7}$	9	$1.8657385827199201 \times 10^{-8}$

	ω_A (deg)		I (deg)
0	22.651761597469341	0	23.368487243998250
1	1.7507120681282261	1	$6.4515960777550678 \times 10^{-1}$
2	$1.1561825543214004 \times 10^{-1}$	2	$-8.2300223149249861 \times 10^{-2}$
3	$-5.4786921549394373 \times 10^{-2}$	3	$-3.3783028645560401 \times 10^{-3}$
4	$-1.1170615500043603 \times 10^{-3}$	4	$1.6128163350506643 \times 10^{-3}$
5	$6.9320903488776929 \times 10^{-4}$	5	$-7.5522108058917474 \times 10^{-5}$
6	$-2.9567858240800889 \times 10^{-5}$	6	$-6.9244069967340392 \times 10^{-6}$
7	$-4.5612001598724950 \times 10^{-6}$	7	$6.4475816498386468 \times 10^{-7}$
8	$9.3126957633521481 \times 10^{-7}$	8	$-3.9203406774096160 \times 10^{-8}$
9	$-3.7439356465096239 \times 10^{-8}$	9	$7.0961580001951016 \times 10^{-10}$

	χ_A (deg)		L (deg)
0	4.1291760656280809	0	-1.6542967036099713
1	1.1385873076673984	1	$8.4078845941709750 \times 10^{-1}$
2	$-8.4463980289565041 \times 10^{-1}$	2	$5.0173545372361227 \times 10^{-2}$
3	$-1.9402578205576458 \times 10^{-2}$	3	$-3.3927814855481457 \times 10^{-2}$
4	$1.8263008233763789 \times 10^{-2}$	4	$2.2189031597479230 \times 10^{-3}$
5	$-1.0135914182203637 \times 10^{-3}$	5	$2.0794220072375433 \times 10^{-4}$
6	$-1.7691558646945688 \times 10^{-4}$	6	$-2.8561248240834349 \times 10^{-5}$
7	$3.9412349720530010 \times 10^{-5}$	7	$2.1558490385109937 \times 10^{-6}$
8	$-1.3080048703348505 \times 10^{-6}$	8	$-1.4926141973758018 \times 10^{-8}$
9	$-6.2966637235829304 \times 10^{-7}$	9	$-3.5096373106126327 \times 10^{-8}$

Table 4–2, continued.

Interval 102: Central time $T_c = 3120$, covering the time span $3080 \leq T \leq 3160$

	ε (deg)		p_A (deg)
0	23.674201019473218	0	4375.1655781544392
1	$1.7021756340578522 \times 10^{-1}$	1	55.324826445492558
2	$-3.6787912530628247 \times 10^{-2}$	2	$-6.3610159073127752 \times 10^{-2}$
3	$-1.3716059817130001 \times 10^{-3}$	3	$1.0468303581352571 \times 10^{-2}$
4	$2.1212020132054867 \times 10^{-4}$	4	$1.1072441336344600 \times 10^{-4}$
5	$-3.4919089503939915 \times 10^{-6}$	5	$-1.9609536432159341 \times 10^{-5}$
6	$3.0094038919608174 \times 10^{-7}$	6	$1.4669625985124585 \times 10^{-6}$
7	$2.6325443432720053 \times 10^{-8}$	7	$-1.2451703861588844 \times 10^{-7}$
8	$3.2270741941145026 \times 10^{-9}$	8	$4.0242531390209755 \times 10^{-9}$
9	$5.4732574662209758 \times 10^{-10}$	9	$-4.1373356983463528 \times 10^{-9}$

	ψ_A (deg)		Δ (deg)
0	4376.2662624031379	0	4379.0129245678904
1	52.212452524539980	1	54.988890549527260
2	$-1.5184811740272361 \times 10^{-1}$	2	$1.8855827638270691 \times 10^{-2}$
3	$9.4864443347211419 \times 10^{-2}$	3	$9.3684088705224959 \times 10^{-4}$
4	$-3.6200882564969438 \times 10^{-4}$	4	$-1.8456402645702535 \times 10^{-3}$
5	$-2.2813833339097410 \times 10^{-4}$	5	$1.4167522660353615 \times 10^{-4}$
6	$1.9274100357983052 \times 10^{-5}$	6	$1.3836615920680854 \times 10^{-5}$
7	$-7.0267292634898683 \times 10^{-6}$	7	$-5.1392443451437190 \times 10^{-7}$
8	$9.5682621356107279 \times 10^{-8}$	8	$-8.6265044878938074 \times 10^{-8}$
9	$1.0043967989886791 \times 10^{-8}$	9	$-1.1761506711488095 \times 10^{-9}$

	ω_A (deg)		I (deg)
0	25.283599335846751	0	24.083476801719153
1	$4.9022510386170752 \times 10^{-1}$	1	$1.1336897687685094 \times 10^{-1}$
2	$-3.4275814450424847 \times 10^{-1}$	2	$-3.4300717485964085 \times 10^{-2}$
3	$-7.4837756478446616 \times 10^{-3}$	3	$5.9077572829837710 \times 10^{-3}$
4	$4.7732520540643263 \times 10^{-3}$	4	$-4.9336780031791557 \times 10^{-4}$
5	$1.0683970115793143 \times 10^{-5}$	5	$-8.8062092067218092 \times 10^{-5}$
6	$-1.4160094194617366 \times 10^{-5}$	6	$5.7238277211270868 \times 10^{-6}$
7	$2.5262367779440941 \times 10^{-7}$	7	$4.5534542047533479 \times 10^{-7}$
8	$-1.0420166045258514 \times 10^{-7}$	8	$-1.3500308257909430 \times 10^{-8}$
9	$2.5560398421186540 \times 10^{-9}$	9	$-2.4972230783182553 \times 10^{-9}$

	χ_A (deg)		L (deg)
0	1.2048595845280925	0	$-3.3463603404764334 \times 10^{-1}$
1	-3.4138620036552833	1	$3.6251527567943364 \times 10^{-1}$
2	$-1.0299316961728852 \times 10^{-1}$	2	$-8.9687372506510949 \times 10^{-2}$
3	$9.5033611877633139 \times 10^{-2}$	3	$1.0346262922606447 \times 10^{-2}$
4	$-2.1974983418464881 \times 10^{-4}$	4	$2.1596119170820128 \times 10^{-3}$
5	$-3.3412317415728142 \times 10^{-4}$	5	$-1.7745025282999242 \times 10^{-4}$
6	$1.5581979495150460 \times 10^{-5}$	6	$-1.3864348737246655 \times 10^{-5}$
7	$-6.1134057444834170 \times 10^{-6}$	7	$4.7595100133263148 \times 10^{-7}$
8	$1.0376483103678497 \times 10^{-7}$	8	$9.5875256821498235 \times 10^{-8}$
9	$1.1076693204039114 \times 10^{-8}$	9	$-3.6735244004998594 \times 10^{-9}$

Table 4–2, continued.

Interval 103: Central time $T_c = 3200$, covering the time span $3160 \leq T \leq 3240$

	ε (deg)		p_A (deg)
0	23.707615341796104	0	4485.7087184029791
1	$-1.3408962667314965 \times 10^{-1}$	1	55.323136845383038
2	$-3.3497759019950911 \times 10^{-2}$	2	$6.1405922355591694 \times 10^{-2}$
3	$1.9542613784878742 \times 10^{-3}$	3	$9.3394952716334221 \times 10^{-3}$
4	$2.1281973988712841 \times 10^{-4}$	4	$-3.3056713090275910 \times 10^{-4}$
5	$-3.4747656767981871 \times 10^{-6}$	5	$-3.8786164290304432 \times 10^{-5}$
6	$-1.2908430967231006 \times 10^{-6}$	6	$-6.2057681883009867 \times 10^{-7}$
7	$-4.8148512458674392 \times 10^{-8}$	7	$4.1304828394906249 \times 10^{-7}$
8	$2.5310200995471357 \times 10^{-8}$	8	$2.3658469444690516 \times 10^{-8}$
9	$1.4927306935246419 \times 10^{-9}$	9	$-7.9026791146488609 \times 10^{-9}$

	ψ_A (deg)		Δ (deg)
0	4482.7110603661412	0	4489.0123807088902
1	54.984186697993349	1	54.971952462942882
2	$7.2924558892572196 \times 10^{-1}$	2	$-1.9527034961760434 \times 10^{-2}$
3	$2.4283275339871649 \times 10^{-2}$	3	$2.8394557556603943 \times 10^{-3}$
4	$-1.1685873499449556 \times 10^{-2}$	4	$1.9802112480536479 \times 10^{-3}$
5	$-9.8999680566317624 \times 10^{-4}$	5	$1.0859653066424532 \times 10^{-4}$
6	$3.1088780992852387 \times 10^{-5}$	6	$-1.6995763665864836 \times 10^{-5}$
7	$1.9633214549642906 \times 10^{-5}$	7	$-5.9739690640161845 \times 10^{-7}$
8	$1.7929089562146172 \times 10^{-6}$	8	$8.5000974957968753 \times 10^{-8}$
9	$-1.0604562162700068 \times 10^{-7}$	9	$4.2614781745610069 \times 10^{-10}$

	ω_A (deg)		I (deg)
0	24.082231768717514	0	24.117299912116511
1	-1.4283975932199375	1	$-8.1760216273991534 \times 10^{-2}$
2	$-3.4488996855372414 \times 10^{-2}$	2	$-3.6646339194453529 \times 10^{-2}$
3	$4.8889772132756490 \times 10^{-2}$	3	$-5.8942404195410500 \times 10^{-3}$
4	$6.2467882273787781 \times 10^{-4}$	4	$-3.3354067354001213 \times 10^{-4}$
5	$-5.0690430154094186 \times 10^{-4}$	5	$1.0262951854596207 \times 10^{-4}$
6	$-2.8962228539491591 \times 10^{-5}$	6	$5.1447494708871476 \times 10^{-6}$
7	$1.1701377545200937 \times 10^{-6}$	7	$-5.7792661927325140 \times 10^{-7}$
8	$4.4425628328412830 \times 10^{-7}$	8	$-2.8382262384389808 \times 10^{-8}$
9	$4.1499183281526168 \times 10^{-8}$	9	$2.0410821012018509 \times 10^{-9}$

	χ_A (deg)		L (deg)
0	-3.2775760614002182	0	$2.5318454826765467 \times 10^{-1}$
1	$-3.5311247799315958 \times 10^{-1}$	1	$3.8198014578295726 \times 10^{-1}$
2	$7.3215562266733084 \times 10^{-1}$	2	$8.8695928225753935 \times 10^{-2}$
3	$1.3566940122383917 \times 10^{-2}$	3	$7.0255319344796854 \times 10^{-3}$
4	$-1.2579597668630736 \times 10^{-2}$	4	$-2.5466343841114281 \times 10^{-3}$
5	$-9.2417782935022033 \times 10^{-4}$	5	$-1.6170851702961145 \times 10^{-4}$
6	$4.1173827483042116 \times 10^{-5}$	6	$1.8337979451602658 \times 10^{-5}$
7	$1.8946513112547867 \times 10^{-5}$	7	$1.1563294112004171 \times 10^{-6}$
8	$1.7315638135318178 \times 10^{-6}$	8	$-6.6963948724610322 \times 10^{-8}$
9	$-9.5363295030925434 \times 10^{-8}$	9	$-9.9211528084624918 \times 10^{-9}$

Table 4–2, continued.

Interval 104: Central time $T_c = 3280$, covering the time span $3240 \leq T \leq 3320$

	ε (deg)		p_A (deg)
0	23.278109978800857	0	4597.1047066553853
1	$-2.6340427760046726 \times 10^{-1}$	1	56.123410559169701
2	$4.5119493874728436 \times 10^{-3}$	2	$1.2125417275806594 \times 10^{-1}$
3	$3.8685361786791938 \times 10^{-3}$	3	$-4.5544707368339156 \times 10^{-4}$
4	$2.5528471192486154 \times 10^{-5}$	4	$-7.9143249470255193 \times 10^{-4}$
5	$-7.1887109885107556 \times 10^{-6}$	5	$-1.8794133474856847 \times 10^{-5}$
6	$-3.6251064806310175 \times 10^{-7}$	6	$-2.0521832763954216 \times 10^{-6}$
7	$-2.3930657853875375 \times 10^{-7}$	7	$-8.7417614317483333 \times 10^{-9}$
8	$-8.8286445287180269 \times 10^{-9}$	8	$9.1246455257193646 \times 10^{-8}$
9	$4.4925602231638206 \times 10^{-9}$	9	$4.5696002105726253 \times 10^{-9}$

	ψ_A (deg)		Δ (deg)
0	4596.9441206823735	0	4599.3003224496933
1	58.501147023567014	1	55.502273504831351
2	$-1.2690587172427164 \times 10^{-1}$	2	$1.9990546442779684 \times 10^{-1}$
3	$-1.1441218231670509 \times 10^{-1}$	3	$2.8411750835144802 \times 10^{-2}$
4	$7.9768589572971069 \times 10^{-3}$	4	$-1.1134261123947312 \times 10^{-4}$
5	$2.1524455593429248 \times 10^{-3}$	5	$-3.0019437397767255 \times 10^{-4}$
6	$-1.8962563706728617 \times 10^{-4}$	6	$-1.4471771610828585 \times 10^{-5}$
7	$-3.3607489985144238 \times 10^{-5}$	7	$1.6023315247446965 \times 10^{-7}$
8	$4.6386051307585332 \times 10^{-6}$	8	$2.6549297176535048 \times 10^{-8}$
9	$5.3630098883189107 \times 10^{-7}$	9	$1.2089759427639064 \times 10^{-8}$

	ω_A (deg)		I (deg)
0	22.376534912394207	0	23.484120765240491
1	$-1.7874253401621451 \times 10^{-2}$	1	$-5.7976267009411356 \times 10^{-1}$
2	$2.6143729960044737 \times 10^{-1}$	2	$-6.6862041810569249 \times 10^{-2}$
3	$-1.8762723059344353 \times 10^{-2}$	3	$5.7190614030042737 \times 10^{-3}$
4	$-5.9633942001798942 \times 10^{-3}$	4	$1.4990207931585555 \times 10^{-3}$
5	$3.9857940070464416 \times 10^{-4}$	5	$1.8962415479686585 \times 10^{-5}$
6	$7.3004394239734865 \times 10^{-5}$	6	$-9.8862573460337271 \times 10^{-6}$
7	$-6.1495562116114605 \times 10^{-6}$	7	$-2.2752334267646651 \times 10^{-7}$
8	$-8.3191348795979147 \times 10^{-7}$	8	$1.1479034094287257 \times 10^{-8}$
9	$1.1793085913929687 \times 10^{-7}$	9	$-2.3114605299488565 \times 10^{-9}$

	χ_A (deg)		L (deg)
0	$-1.7566442148203033 \times 10^{-1}$	0	1.4579195338498896
1	2.5812111946972476	1	$6.7344923311339076 \times 10^{-1}$
2	$-2.7072595634291808 \times 10^{-1}$	2	$-8.6651417682707234 \times 10^{-2}$
3	$-1.2224904623795328 \times 10^{-1}$	3	$-3.1339037141238345 \times 10^{-2}$
4	$9.3702071903516569 \times 10^{-3}$	4	$-6.6205280855763207 \times 10^{-4}$
5	$2.2720362189552307 \times 10^{-3}$	5	$3.1128663525517517 \times 10^{-4}$
6	$-1.9180590165191402 \times 10^{-4}$	6	$1.2215392102561058 \times 10^{-5}$
7	$-3.4152112524189280 \times 10^{-5}$	7	$-3.5053634622400418 \times 10^{-7}$
8	$4.5463803541791661 \times 10^{-6}$	8	$7.5570518306455069 \times 10^{-8}$
9	$5.3283891000840275 \times 10^{-7}$	9	$-6.4832815189229189 \times 10^{-9}$

Table 4–2, continued.

Interval 105: Central time $T_c = 3360$, covering the time span $3320 \leq T \leq 3400$

	ε (deg)		p_A (deg)
0	22.925417127800858	0	4710.1344687503757
1	$-5.6293826768654710 \times 10^{-2}$	1	56.830261216948366
2	$4.3215768138879337 \times 10^{-2}$	2	$3.0169445837954730 \times 10^{-2}$
3	$1.4442350081285529 \times 10^{-3}$	3	$-1.4350163268975202 \times 10^{-2}$
4	$-4.0201007580957289 \times 10^{-4}$	4	$-5.3289655305377211 \times 10^{-4}$
5	$-2.0947102531111429 \times 10^{-5}$	5	$9.8219573957691292 \times 10^{-5}$
6	$2.7926815449840735 \times 10^{-6}$	6	$6.9379954212604126 \times 10^{-6}$
7	$2.6361891749750623 \times 10^{-7}$	7	$-8.8652799129093232 \times 10^{-7}$
8	$-3.6818546849841334 \times 10^{-8}$	8	$-9.0165296073644408 \times 10^{-8}$
9	$-3.4023688275035941 \times 10^{-9}$	9	$1.2263265526023873 \times 10^{-8}$

	ψ_A (deg)		Δ (deg)
0	4711.0231135298955	0	4712.6076446601043
1	55.462713675375519	1	57.895511160615375
2	$-2.7878275574368677 \times 10^{-1}$	2	$2.9480464781219519 \times 10^{-1}$
3	$7.3597741803142751 \times 10^{-2}$	3	$-2.9918346701048943 \times 10^{-2}$
4	$4.8048785655043048 \times 10^{-3}$	4	$-6.4200721211964447 \times 10^{-3}$
5	$-1.0046300417678685 \times 10^{-3}$	5	$1.6361703337380245 \times 10^{-5}$
6	$3.2872541907719013 \times 10^{-5}$	6	$7.2290688910521227 \times 10^{-5}$
7	$-5.1720130590802916 \times 10^{-7}$	7	$4.5777409985863025 \times 10^{-6}$
8	$-1.2229338463594278 \times 10^{-6}$	8	$-5.1108211766668710 \times 10^{-7}$
9	$1.3419458732911511 \times 10^{-7}$	9	$-9.7859737167005694 \times 10^{-8}$

	ω_A (deg)		I (deg)
0	23.153869510790552	0	22.255825688808498
1	$4.3290956973107862 \times 10^{-1}$	1	$-4.9587384269973665 \times 10^{-1}$
2	$-1.5111004487411746 \times 10^{-1}$	2	$1.1191125555521380 \times 10^{-1}$
3	$-2.1656649919024510 \times 10^{-2}$	3	$1.8338125968487417 \times 10^{-2}$
4	$4.5820727033340101 \times 10^{-3}$	4	$-7.7566186221695488 \times 10^{-4}$
5	$2.1501244402306955 \times 10^{-4}$	5	$-2.3054432455930206 \times 10^{-4}$
6	$-3.6313703355093167 \times 10^{-5}$	6	$-3.0432609816427721 \times 10^{-6}$
7	$7.3900443693999439 \times 10^{-7}$	7	$1.6479685105976369 \times 10^{-6}$
8	$-1.3067801465036544 \times 10^{-8}$	8	$1.3484182518516209 \times 10^{-7}$
9	$-2.1001513538062805 \times 10^{-8}$	9	$-6.1646998188730481 \times 10^{-9}$

	χ_A (deg)		L (deg)
0	$9.6468461131348915 \times 10^{-1}$	0	1.1559375726058103
1	-1.4846619704948068	1	-1.1525584258262749
2	$-3.3703421336719684 \times 10^{-1}$	2	$-2.8467042709997187 \times 10^{-1}$
3	$9.5551234938380180 \times 10^{-2}$	3	$1.6879397656316944 \times 10^{-2}$
4	$6.0038878263587313 \times 10^{-3}$	4	$6.2338094144985640 \times 10^{-3}$
5	$-1.2111785937464462 \times 10^{-3}$	5	$7.8894279945518786 \times 10^{-5}$
6	$1.9263813148646463 \times 10^{-5}$	6	$-6.7450355057743697 \times 10^{-5}$
7	$1.1251857157839354 \times 10^{-6}$	7	$-5.5305189927260917 \times 10^{-6}$
8	$-1.0856705592184822 \times 10^{-6}$	8	$4.2241900716072024 \times 10^{-7}$
9	$1.1763095685106630 \times 10^{-7}$	9	$1.1148426608532909 \times 10^{-7}$

Table 4–2, continued.

Interval 106: Central time $T_c = 3440$, covering the time span $3400 \leq T \leq 3480$

	ε (deg)		p_A (deg)
0	23.131606114172563	0	4823.4960733038635
1	$2.4339065213619525 \times 10^{-1}$	1	56.400074363026992
2	$2.0768418933721470 \times 10^{-2}$	2	$-1.2388861358607600 \times 10^{-1}$
3	$-4.6097045337268990 \times 10^{-3}$	3	$-6.8720230731032838 \times 10^{-3}$
4	$-2.0705951895281525 \times 10^{-4}$	4	$1.1715867809732527 \times 10^{-3}$
5	$2.3092468556576843 \times 10^{-5}$	5	$2.9473000375803438 \times 10^{-5}$
6	$-4.3534269863076271 \times 10^{-8}$	6	$-3.5347657511537409 \times 10^{-6}$
7	$5.2102603275245825 \times 10^{-8}$	7	$3.5158637971132960 \times 10^{-7}$
8	$2.2384413132928070 \times 10^{-8}$	8	$-3.8988385219801697 \times 10^{-8}$
9	$-2.8165710108043387 \times 10^{-9}$	9	$-7.7157898798613835 \times 10^{-9}$

	ψ_A (deg)		Δ (deg)
0	4822.5095507386754	0	4828.7891396195956
1	56.664925197709865	1	57.702313812950425
2	$4.5483642402874439 \times 10^{-1}$	2	$-3.7786979202121996 \times 10^{-1}$
3	$2.0933243298791104 \times 10^{-3}$	3	$-4.7236112120966562 \times 10^{-2}$
4	$-1.3685588667318262 \times 10^{-2}$	4	$6.0547346638226360 \times 10^{-3}$
5	$-6.9421154771953917 \times 10^{-4}$	5	$4.5062219683280314 \times 10^{-4}$
6	$1.1720827004600666 \times 10^{-4}$	6	$-8.4513633628403125 \times 10^{-5}$
7	$2.1752026904003765 \times 10^{-5}$	7	$-1.7324159656649786 \times 10^{-6}$
8	$6.5707300115807193 \times 10^{-7}$	8	$1.1846283269326555 \times 10^{-6}$
9	$-3.2959451659335744 \times 10^{-7}$	9	$-2.8278294772227367 \times 10^{-8}$

	ω_A (deg)		I (deg)
0	22.900004364034971	0	22.515382280910897
1	$-5.3288982314731559 \times 10^{-1}$	1	$7.8300449234225195 \times 10^{-1}$
2	$2.6016566115540807 \times 10^{-2}$	2	$1.4185397386420128 \times 10^{-1}$
3	$4.1340223808063440 \times 10^{-2}$	3	$-1.7306010268266800 \times 10^{-2}$
4	$6.2104934479483236 \times 10^{-4}$	4	$-2.1778658327988757 \times 10^{-3}$
5	$-6.1282176011820164 \times 10^{-4}$	5	$1.8347882117370950 \times 10^{-4}$
6	$-2.7938387360416907 \times 10^{-5}$	6	$1.5321738178418651 \times 10^{-5}$
7	$3.1368975889097756 \times 10^{-6}$	7	$-2.0926284392671983 \times 10^{-6}$
8	$5.6322498338800010 \times 10^{-7}$	8	$-6.2134428858225925 \times 10^{-8}$
9	$2.6427097491983660 \times 10^{-8}$	9	$2.5448988386596836 \times 10^{-8}$

	χ_A (deg)		L (deg)
0	-1.0719287597043820	0	-1.8972026212108611
1	$2.8864159983409351 \times 10^{-1}$	1	-1.4157349914643999
2	$6.2844186388971789 \times 10^{-1}$	2	$2.7181436059661861 \times 10^{-1}$
3	$9.2864428903322511 \times 10^{-3}$	3	$4.3993205719242317 \times 10^{-2}$
4	$-1.6075629354096241 \times 10^{-2}$	4	$-5.0612477699248475 \times 10^{-3}$
5	$-7.3158353809244061 \times 10^{-4}$	5	$-4.5762994938386700 \times 10^{-4}$
6	$1.3114315287026541 \times 10^{-4}$	6	$8.2023966242627620 \times 10^{-5}$
7	$2.1339980789149256 \times 10^{-5}$	7	$2.3048100513981412 \times 10^{-6}$
8	$6.5411592482860542 \times 10^{-7}$	8	$-1.2333383741183398 \times 10^{-6}$
9	$-3.1964882771278454 \times 10^{-7}$	9	$1.9068643876069661 \times 10^{-8}$

Table 4–2, continued.

Interval 107: Central time $T_c = 3520$, covering the time span $3480 \leq T \leq 3560$

	ε (deg)
0	23.594711172863752
1	$1.7023322502598692 \times 10^{-1}$
2	$-3.7510482892079643 \times 10^{-2}$
3	$-3.6813653441002738 \times 10^{-3}$
4	$3.5055170982325610 \times 10^{-4}$
5	$2.6377613300263473 \times 10^{-5}$
6	$-1.6494189130081153 \times 10^{-6}$
7	$-1.6936597950694642 \times 10^{-7}$
8	$1.9529504618876563 \times 10^{-8}$
9	$2.0683487079116012 \times 10^{-9}$

	p_A (deg)
0	4935.2820169458787
1	55.416359425734812
2	$-8.7728885859206166 \times 10^{-2}$
3	$1.2315676045788645 \times 10^{-2}$
4	$9.0742810675098827 \times 10^{-4}$
5	$-6.9175191685724472 \times 10^{-5}$
6	$-4.3582049154805041 \times 10^{-6}$
7	$3.8372273712102759 \times 10^{-7}$
8	$3.3695897770495171 \times 10^{-8}$
9	$-6.8234681544377893 \times 10^{-9}$

	ψ_A (deg)
0	4937.2433983102336
1	57.230124363976956
2	$-4.9370328814142788 \times 10^{-1}$
3	$-8.7429049553316816 \times 10^{-2}$
4	$1.1760260935912313 \times 10^{-2}$
5	$1.3604772813221496 \times 10^{-3}$
6	$-2.5017473189352061 \times 10^{-4}$
7	$-1.0578246807469161 \times 10^{-5}$
8	$5.2251919252585723 \times 10^{-6}$
9	$-5.2274510370552975 \times 10^{-8}$

	Δ (deg)
0	4940.6138087679590
1	54.153543665320311
2	$-3.4923652559806583 \times 10^{-1}$
3	$4.1270853453857359 \times 10^{-2}$
4	$2.6995450834596314 \times 10^{-3}$
5	$-2.9342488879930557 \times 10^{-4}$
6	$1.7691760123001911 \times 10^{-5}$
7	$2.9909845116623602 \times 10^{-7}$
8	$-2.6898738218489023 \times 10^{-7}$
9	$2.1668279167596549 \times 10^{-8}$

	ω_A (deg)
0	23.128865305856890
1	$9.2303154009453929 \times 10^{-1}$
2	$2.0549061998793091 \times 10^{-1}$
3	$-2.9046492841229417 \times 10^{-2}$
4	$-5.5269357910458239 \times 10^{-3}$
5	$4.5010081873122987 \times 10^{-4}$
6	$5.4687014628784248 \times 10^{-5}$
7	$-7.2738342134457835 \times 10^{-6}$
8	$-3.3000206130784838 \times 10^{-7}$
9	$1.2627026961029627 \times 10^{-7}$

	I (deg)
0	24.351997969027357
1	$8.1538137317302528 \times 10^{-1}$
2	$-1.3651916809928495 \times 10^{-1}$
3	$-1.9432278056738024 \times 10^{-2}$
4	$1.5321167298930347 \times 10^{-3}$
5	$1.0016606492993380 \times 10^{-4}$
6	$-7.7492323848971784 \times 10^{-6}$
7	$3.6925430486686077 \times 10^{-7}$
8	$-1.0289127100246544 \times 10^{-9}$
9	$-5.3006116699778660 \times 10^{-9}$

	χ_A (deg)
0	2.1404318496254822
1	1.9842387587997787
2	$-4.3707163538167020 \times 10^{-1}$
3	$-1.0915302230677472 \times 10^{-1}$
4	$1.1296872773065729 \times 10^{-2}$
5	$1.5535112739183461 \times 10^{-3}$
6	$-2.4796423808280653 \times 10^{-4}$
7	$-1.1645499880368238 \times 10^{-5}$
8	$5.1875739483009069 \times 10^{-6}$
9	$-4.3532628264619888 \times 10^{-8}$

	L (deg)
0	-1.9432636102771200
1	1.3780379521841276
2	$2.8977442319585268 \times 10^{-1}$
3	$-3.1416660390388735 \times 10^{-2}$
4	$-2.1944289132436526 \times 10^{-3}$
5	$2.4129081528452169 \times 10^{-4}$
6	$-1.9894029885122426 \times 10^{-5}$
7	$4.7619218364317482 \times 10^{-8}$
8	$3.0147074191021760 \times 10^{-7}$
9	$-2.9209750929821874 \times 10^{-8}$

Table 4–2, continued.

Interval 108: Central time $T_c = 3600$, covering the time span $3560 \leq T \leq 3640$

	ε (deg)		p_A (deg)
0	23.578325100230316	0	5045.9735077348034
1	$-1.8796716165076250 \times 10^{-1}$	1	55.428274197168159
2	$-3.9026138684599136 \times 10^{-2}$	2	$9.2710461139662028 \times 10^{-2}$
3	$3.5657703511009686 \times 10^{-3}$	3	$1.3440207604394190 \times 10^{-2}$
4	$4.1067581155367287 \times 10^{-4}$	4	$-7.8945550282993296 \times 10^{-4}$
5	$-1.8391863354248000 \times 10^{-5}$	5	$-8.2073450511009747 \times 10^{-5}$
6	$-1.7598626447187519 \times 10^{-6}$	6	$1.2771754504678097 \times 10^{-6}$
7	$-1.3482582915320619 \times 10^{-8}$	7	$2.5706173035809921 \times 10^{-7}$
8	$1.0046195231916671 \times 10^{-8}$	8	$3.5004877297402838 \times 10^{-8}$
9	$2.3967749562272867 \times 10^{-9}$	9	$-2.8686376617221444 \times 10^{-9}$

	ψ_A (deg)		Δ (deg)
0	5046.9514884301130	0	5047.9953159226404
1	52.625420560337828	1	53.747416220413747
2	$-3.3755055326398311 \times 10^{-1}$	2	$2.6770567609880374 \times 10^{-1}$
3	$8.8720537250689938 \times 10^{-2}$	3	$5.2991921712737107 \times 10^{-2}$
4	$4.9429163283561883 \times 10^{-3}$	4	$-1.0287210790275464 \times 10^{-3}$
5	$-4.5248323273065544 \times 10^{-4}$	5	$-2.4576082763338710 \times 10^{-4}$
6	$6.1623431206630476 \times 10^{-5}$	6	$-2.1597057810130351 \times 10^{-5}$
7	$-2.6057378800897729 \times 10^{-6}$	7	$-1.6583459399529976 \times 10^{-6}$
8	$-8.4544483277885873 \times 10^{-7}$	8	$8.5981977007343267 \times 10^{-8}$
9	$6.7874231897354922 \times 10^{-8}$	9	$1.8762176183583654 \times 10^{-8}$

	ω_A (deg)		I (deg)
0	25.010213509548433	0	24.513376500984975
1	$5.1320775316161672 \times 10^{-1}$	1	$-6.9318721218881754 \times 10^{-1}$
2	$-3.0862198539761693 \times 10^{-1}$	2	$-1.8235153266714366 \times 10^{-1}$
3	$-3.0901776735590032 \times 10^{-2}$	3	$1.3344646847725878 \times 10^{-2}$
4	$4.3144341568812687 \times 10^{-3}$	4	$2.1566903392214580 \times 10^{-3}$
5	$2.6872578550419061 \times 10^{-4}$	5	$-3.5471278228164785 \times 10^{-5}$
6	$-1.8791399039805010 \times 10^{-5}$	6	$-7.2844203466699411 \times 10^{-6}$
7	$1.0855023110291764 \times 10^{-6}$	7	$-4.1260459722665008 \times 10^{-7}$
8	$-4.3923482697835836 \times 10^{-8}$	8	$-3.0743123842688751 \times 10^{-8}$
9	$-1.2303107158293564 \times 10^{-8}$	9	$1.8592228913552474 \times 10^{-10}$

	χ_A (deg)		L (deg)
0	1.0713138579551863	0	1.6792200079487222
1	-3.0717554162398695	1	1.8356765714778648
2	$-4.7515066131564349 \times 10^{-1}$	2	$-1.9633436529657180 \times 10^{-1}$
3	$8.4241066038525455 \times 10^{-2}$	3	$-4.3626495832656233 \times 10^{-2}$
4	$6.8496070832551938 \times 10^{-3}$	4	$5.0256227895830522 \times 10^{-4}$
5	$-4.6335137011129087 \times 10^{-4}$	5	$1.9840533748678864 \times 10^{-4}$
6	$4.9834863733125869 \times 10^{-5}$	6	$2.1459985192063002 \times 10^{-5}$
7	$-2.4220869698701442 \times 10^{-6}$	7	$1.7689904846765397 \times 10^{-6}$
8	$-8.2276552589432908 \times 10^{-7}$	8	$-4.4086986329801402 \times 10^{-8}$
9	$7.0392273523033882 \times 10^{-8}$	9	$-2.1608702236557567 \times 10^{-8}$

Table 4–2, continued.

Interval 109: Central time $T_c = 3680$, covering the time span $3640 \le T \le 3720$

	ε (deg)		p_A (deg)
0	23.078318221055645	0	5157.8640947838146
1	$-2.5748976582666787 \times 10^{-1}$	1	56.500504481072624
2	$2.5186176381277884 \times 10^{-2}$	2	$1.3705196521552273 \times 10^{-1}$
3	$5.5250121673141309 \times 10^{-3}$	3	$-7.9684184071222770 \times 10^{-3}$
4	$-2.1666599729609848 \times 10^{-4}$	4	$-1.5290630954232495 \times 10^{-3}$
5	$-3.5732908323497191 \times 10^{-5}$	5	$2.5237354669640398 \times 10^{-5}$
6	$-1.5009592868193561 \times 10^{-7}$	6	$6.5820745061996115 \times 10^{-6}$
7	$-1.2968340427988937 \times 10^{-8}$	7	$3.8313012931321823 \times 10^{-7}$
8	$1.7031851081523101 \times 10^{-8}$	8	$4.7678445837565075 \times 10^{-8}$
9	$4.6195707839983470 \times 10^{-9}$	9	$-5.5983960008995742 \times 10^{-9}$

	ψ_A (deg)		Δ (deg)
0	5153.5154139588564	0	5159.0761782527180
1	55.059109121591991	1	57.581809990011078
2	$9.8915904106196506 \times 10^{-1}$	2	$5.4769321185846659 \times 10^{-1}$
3	$1.0708869831302048 \times 10^{-1}$	3	$-2.9210822676405637 \times 10^{-2}$
4	$-7.6745073806562273 \times 10^{-3}$	4	$-9.3405862983802408 \times 10^{-3}$
5	$-2.1804720090863513 \times 10^{-3}$	5	$-1.0965889480085180 \times 10^{-4}$
6	$-2.6228504232009936 \times 10^{-4}$	6	$1.0035642177142655 \times 10^{-4}$
7	$-5.3533376572735951 \times 10^{-6}$	7	$9.1182828159398479 \times 10^{-6}$
8	$4.2749464474495871 \times 10^{-6}$	8	$-6.7962699168707055 \times 10^{-7}$
9	$8.4022762093055262 \times 10^{-7}$	9	$-1.7994915147678537 \times 10^{-7}$

	ω_A (deg)		I (deg)
0	23.407513849970402	0	22.500950218703633
1	-1.9896076392958378	1	-1.0633663173235016
2	$-1.5412261968050541 \times 10^{-1}$	2	$1.1821667791351899 \times 10^{-1}$
3	$6.0312381274508226 \times 10^{-2}$	3	$2.8301699758952425 \times 10^{-2}$
4	$5.5443554728997407 \times 10^{-3}$	4	$-1.2334979052637711 \times 10^{-3}$
5	$-2.8814547694071092 \times 10^{-4}$	5	$-3.0216900264174973 \times 10^{-4}$
6	$-6.2224137156169822 \times 10^{-5}$	6	$-2.0697401392671281 \times 10^{-6}$
7	$-6.5178833205853439 \times 10^{-6}$	7	$2.4236935267671458 \times 10^{-6}$
8	$-3.2546004678867504 \times 10^{-7}$	8	$1.8881999443836470 \times 10^{-7}$
9	$5.9151274505238314 \times 10^{-8}$	9	$-1.3790180897933403 \times 10^{-8}$

	χ_A (deg)		L (deg)
0	-4.7262419092728063	0	2.5475146062669259
1	-1.5323365819001912	1	-1.1857717789811529
2	$9.3501137610061266 \times 10^{-1}$	2	$-4.4156531923544416 \times 10^{-1}$
3	$1.2041014470764737 \times 10^{-1}$	3	$2.4072058626961274 \times 10^{-2}$
4	$-7.6430093787681915 \times 10^{-3}$	4	$8.2062969763793867 \times 10^{-3}$
5	$-2.2769462446165124 \times 10^{-3}$	5	$1.0841815696673701 \times 10^{-4}$
6	$-2.5677350776860580 \times 10^{-4}$	6	$-9.6182808694667096 \times 10^{-5}$
7	$-5.3850246778433547 \times 10^{-6}$	7	$-8.5755199055483662 \times 10^{-6}$
8	$4.1560511390485192 \times 10^{-6}$	8	$7.4362384748412295 \times 10^{-7}$
9	$8.4589939332869499 \times 10^{-7}$	9	$1.7334635223804245 \times 10^{-7}$

Table 4–2, continued.

Interval 110: Central time $T_c = 3760$, covering the time span $3720 \leq T \leq 3800$

	ε (deg)		p_A (deg)
0	22.900423046018074	0	5271.4344962762087
1	$1.0114255930006871 \times 10^{-1}$	1	56.906930704562575
2	$5.0268486506721845 \times 10^{-2}$	2	$-5.3780351838970027 \times 10^{-2}$
3	$-2.3152116995812672 \times 10^{-3}$	3	$-1.7764678882869652 \times 10^{-2}$
4	$-5.5463648241249445 \times 10^{-4}$	4	$7.8748840047359148 \times 10^{-4}$
5	$2.6879202412631536 \times 10^{-5}$	5	$1.4370432716138289 \times 10^{-4}$
6	$4.1591148812590326 \times 10^{-6}$	6	$-9.0542500340931697 \times 10^{-6}$
7	$-3.2754549990866835 \times 10^{-7}$	7	$-1.2556105047025439 \times 10^{-6}$
8	$-4.7503036741976257 \times 10^{-8}$	8	$1.2486889762818907 \times 10^{-7}$
9	$4.9377254103827986 \times 10^{-9}$	9	$1.5753295918671709 \times 10^{-8}$

	ψ_A (deg)		Δ (deg)
0	5271.5882681123686	0	5276.1936259127473
1	62.222177073054300	1	58.776571105821107
2	$1.7153210370671241 \times 10^{-2}$	2	$-3.2488564640282292 \times 10^{-1}$
3	$-2.8805353465326786 \times 10^{-1}$	3	$-6.8087404018938485 \times 10^{-2}$
4	$-3.0977779800278397 \times 10^{-3}$	4	$7.6356337419105463 \times 10^{-3}$
5	$7.8979089849586705 \times 10^{-3}$	5	$6.4108839742745234 \times 10^{-4}$
6	$1.3261472342594077 \times 10^{-4}$	6	$-1.2640062415356534 \times 10^{-4}$
7	$-2.3159717490554025 \times 10^{-4}$	7	$-2.3171080011718887 \times 10^{-6}$
8	$-5.5491279246145441 \times 10^{-6}$	8	$1.9086183558984171 \times 10^{-6}$
9	$7.4429160134923124 \times 10^{-6}$	9	$-6.6857560071595414 \times 10^{-8}$

	ω_A (deg)		I (deg)
0	20.825463766263188	0	21.921593299115650
1	$5.0790728810148721 \times 10^{-2}$	1	$5.5397545485820159 \times 10^{-1}$
2	$5.7383080393444404 \times 10^{-1}$	2	$1.9921926570246614 \times 10^{-1}$
3	$5.8396748267125778 \times 10^{-3}$	3	$-1.8515987725112205 \times 10^{-2}$
4	$-1.3820262386782862 \times 10^{-2}$	4	$-2.5428605675619596 \times 10^{-3}$
5	$-1.9050536020995832 \times 10^{-4}$	5	$2.6127486351550363 \times 10^{-4}$
6	$2.5979729321934273 \times 10^{-4}$	6	$1.5210036759899544 \times 10^{-5}$
7	$5.1014048221548572 \times 10^{-6}$	7	$-3.0080580037410663 \times 10^{-6}$
8	$-6.0979700595468349 \times 10^{-6}$	8	$-4.3802645139999770 \times 10^{-9}$
9	$-1.6648274131041069 \times 10^{-7}$	9	$3.4840550750651228 \times 10^{-8}$

	χ_A (deg)		L (deg)
0	$1.6722587585761446 \times 10^{-1}$	0	-1.3061924806641213
1	5.7320449990996647	1	-2.0295631128091147
2	$7.8241247325726874 \times 10^{-2}$	2	$2.8954080300648923 \times 10^{-1}$
3	$-2.8492284757211628 \times 10^{-1}$	3	$5.3961701204558636 \times 10^{-2}$
4	$-4.1321482728996164 \times 10^{-3}$	4	$-7.1274710759813788 \times 10^{-3}$
5	$7.9089700214062203 \times 10^{-3}$	5	$-5.2236582567371872 \times 10^{-4}$
6	$1.4417135326088812 \times 10^{-4}$	6	$1.1875995424565967 \times 10^{-4}$
7	$-2.3100762948259837 \times 10^{-4}$	7	$1.0839115398770565 \times 10^{-6}$
8	$-5.6881787340832220 \times 10^{-6}$	8	$-1.7803423214634805 \times 10^{-6}$
9	$7.4267355326781075 \times 10^{-6}$	9	$8.3944764913561943 \times 10^{-8}$

Table 4–2, continued.

Interval 111: Central time $T_c = 3840$, covering the time span $3800 \leq T \leq 3880$

	ε (deg)		p_A (deg)
0	23.347718447002971	0	5384.3769698144886
1	$2.9809523267980344 \times 10^{-1}$	1	55.959474913637233
2	$-5.9627972571557683 \times 10^{-3}$	2	$-1.4457570641491313 \times 10^{-1}$
3	$-5.2145194857048506 \times 10^{-3}$	3	$3.3547181596201293 \times 10^{-3}$
4	$1.4200782037316980 \times 10^{-4}$	4	$1.1916986276802124 \times 10^{-3}$
5	$2.0969071697670287 \times 10^{-5}$	5	$-4.7202456133831490 \times 10^{-5}$
6	$-8.8218561735455794 \times 10^{-7}$	6	$-2.1876749281288247 \times 10^{-7}$
7	$1.7589012122566637 \times 10^{-7}$	7	$9.7444315914791424 \times 10^{-8}$
8	$-9.2724127902650521 \times 10^{-9}$	8	$-8.2509994279662084 \times 10^{-8}$
9	$-3.8971125979924959 \times 10^{-9}$	9	$5.0999253751974365 \times 10^{-9}$

	ψ_A (deg)		Δ (deg)
0	5389.3871392434510	0	5390.1165284245127
1	54.624902890226970	1	55.094198670718579
2	−1.1266067539655143	2	$-4.0082138845024903 \times 10^{-1}$
3	$1.0382166832071672 \times 10^{-1}$	3	$3.9047894766426378 \times 10^{-2}$
4	$9.7544062383574256 \times 10^{-3}$	4	$2.4499757544203752 \times 10^{-3}$
5	$-2.4229426356452195 \times 10^{-3}$	5	$-4.3986425439182248 \times 10^{-4}$
6	$2.7131758586481531 \times 10^{-4}$	6	$2.4365518237576101 \times 10^{-5}$
7	$2.1098149425636705 \times 10^{-6}$	7	$5.1603260020273733 \times 10^{-7}$
8	$-5.6414881594575115 \times 10^{-6}$	8	$-3.1380235632122182 \times 10^{-7}$
9	$9.4261256602639793 \times 10^{-7}$	9	$3.1954950181871840 \times 10^{-8}$

	ω_A (deg)		I (deg)
0	23.826800474594877	0	23.704643516067262
1	2.3192105933696074	1	$9.7979315778920219 \times 10^{-1}$
2	$-1.4031048565001679 \times 10^{-1}$	2	$-9.1203676881862225 \times 10^{-2}$
3	$-7.0656586690500357 \times 10^{-2}$	3	$-1.8661570060262195 \times 10^{-2}$
4	$4.7194484998340018 \times 10^{-3}$	4	$1.7797068049187711 \times 10^{-3}$
5	$4.0830105224795028 \times 10^{-4}$	5	$7.2386456436705300 \times 10^{-5}$
6	$-6.7541130568772465 \times 10^{-5}$	6	$-1.2486342766302029 \times 10^{-5}$
7	$6.6927408477857106 \times 10^{-6}$	7	$4.2112192119268558 \times 10^{-7}$
8	$-1.5049163369016859 \times 10^{-7}$	8	$1.1617974924747155 \times 10^{-8}$
9	$-9.4982452346064573 \times 10^{-8}$	9	$-2.7279725675843683 \times 10^{-9}$

	χ_A (deg)		L (deg)
0	5.4597185700413395	0	−2.3903472447605128
1	−1.4067576243133288	1	$9.2797524383110506 \times 10^{-1}$
2	−1.0802328838140048	2	$2.8239438547175801 \times 10^{-1}$
3	$1.0297583307446095 \times 10^{-1}$	3	$-3.8154783980600534 \times 10^{-2}$
4	$1.0367408677117664 \times 10^{-2}$	4	$-1.5705293485386198 \times 10^{-3}$
5	$-2.3974537603825998 \times 10^{-3}$	5	$4.1925699747109578 \times 10^{-4}$
6	$2.5710070725142084 \times 10^{-4}$	6	$-2.2490444624988409 \times 10^{-5}$
7	$2.0187514263073722 \times 10^{-6}$	7	$-5.7744334311742913 \times 10^{-7}$
8	$-5.4843544672943271 \times 10^{-6}$	8	$2.1398249141084627 \times 10^{-7}$
9	$9.3876508755589130 \times 10^{-7}$	9	$-2.5808445310431135 \times 10^{-8}$

Table 4-2, continued.

Interval 112: Central time $T_c = 3920$, covering the time span $3880 \leq T \leq 3960$

	ε (deg)		p_A (deg)
0	23.743578563035862	0	5495.4435287844267
1	$6.6193075546317474 \times 10^{-2}$	1	55.208475241096537
2	$-4.3331264907783835 \times 10^{-2}$	2	$-2.5326016304354691 \times 10^{-2}$
3	$-3.5597903563457459 \times 10^{-4}$	3	$1.3368923968464619 \times 10^{-2}$
4	$3.8387164939750376 \times 10^{-4}$	4	$-6.7972960683813239 \times 10^{-5}$
5	$-7.0075071787745839 \times 10^{-6}$	5	$-7.5990606378481040 \times 10^{-5}$
6	$-2.4321871285361397 \times 10^{-6}$	6	$1.9892888061774416 \times 10^{-6}$
7	$5.8976425599718970 \times 10^{-8}$	7	$5.6952105135273358 \times 10^{-7}$
8	$3.0280580184729526 \times 10^{-8}$	8	$-1.1172413345340377 \times 10^{-8}$
9	$-4.7596606448424895 \times 10^{-10}$	9	$-8.0105343305426436 \times 10^{-9}$

	ψ_A (deg)		Δ (deg)
0	5494.1500857023706	0	5498.7285262077044
1	51.347371987714450	1	53.943684234084342
2	$3.0977784481726010 \times 10^{-1}$	2	$1.0215935944004677 \times 10^{-1}$
3	$1.1430617622943027 \times 10^{-1}$	3	$3.4106127692675669 \times 10^{-2}$
4	$-1.4284655385862747 \times 10^{-3}$	4	$-2.0241192982325451 \times 10^{-3}$
5	$-1.6534965652254884 \times 10^{-4}$	5	$-1.1549240357704468 \times 10^{-4}$
6	$-7.5108268660785880 \times 10^{-5}$	6	$1.3690867224302425 \times 10^{-6}$
7	$-1.0147631083247173 \times 10^{-5}$	7	$-2.7049269452146563 \times 10^{-7}$
8	$-5.2832160219858824 \times 10^{-8}$	8	$9.8256053857041446 \times 10^{-8}$
9	$1.3731224533086467 \times 10^{-8}$	9	$3.5572914213997598 \times 10^{-9}$

	ω_A (deg)		I (deg)
0	25.754667498716128	0	24.585654353315690
1	$-6.3188071997390409 \times 10^{-1}$	1	$-1.3663846401927708 \times 10^{-1}$
2	$-4.2765630124709661 \times 10^{-1}$	2	$-1.3793480099634528 \times 10^{-1}$
3	$2.4694875413269156 \times 10^{-2}$	3	$9.2296209446831921 \times 10^{-3}$
4	$5.8209810112432785 \times 10^{-3}$	4	$1.1953451455525011 \times 10^{-3}$
5	$-1.0398062886042405 \times 10^{-4}$	5	$-9.0472526854710096 \times 10^{-5}$
6	$-1.2549234698299511 \times 10^{-5}$	6	$-2.6164733826787016 \times 10^{-6}$
7	$-1.3752460412029528 \times 10^{-6}$	7	$1.1415287839851177 \times 10^{-7}$
8	$-1.8635679377290619 \times 10^{-7}$	8	$-1.7035321588257344 \times 10^{-8}$
9	$-7.9879230560329907 \times 10^{-9}$	9	$1.5599111074587680 \times 10^{-9}$

	χ_A (deg)		L (deg)
0	-1.4196192137509177	0	$2.7720768848775538 \times 10^{-1}$
1	-4.2441373430781090	1	1.3820335774748730
2	$3.7593602405832521 \times 10^{-1}$	2	$-1.3980969681781019 \times 10^{-1}$
3	$1.1447437477321452 \times 10^{-1}$	3	$-2.3162999928514380 \times 10^{-2}$
4	$-2.2324678888405905 \times 10^{-3}$	4	$2.2215084657922388 \times 10^{-3}$
5	$-2.4074048908440056 \times 10^{-4}$	5	$5.3217839386816942 \times 10^{-5}$
6	$-6.9418967367810080 \times 10^{-5}$	6	$-1.2925506071458932 \times 10^{-6}$
7	$-9.8990919645887437 \times 10^{-6}$	7	$8.7374255546438096 \times 10^{-7}$
8	$-8.8017274399719438 \times 10^{-8}$	8	$-1.0297311902142719 \times 10^{-7}$
9	$1.9545368329094962 \times 10^{-8}$	9	$-1.3013049814048490 \times 10^{-8}$

Table 4–2, continued.

Interval 113: Central time $T_c = 4000$, covering the time span $3960 \leq T \leq 4040$

	ε (deg)		p_A (deg)
0	23.574214666451682	0	5606.0973400661856
1	$-2.1658996044654553 \times 10^{-1}$	1	55.546556075796880
2	$-2.1282656132842915 \times 10^{-2}$	2	$9.5123693462425451 \times 10^{-2}$
3	$3.1274207463088761 \times 10^{-3}$	3	$5.3608125154663796 \times 10^{-3}$
4	$5.3475691370231731 \times 10^{-5}$	4	$-6.3734806079982708 \times 10^{-4}$
5	$-9.9495757378822451 \times 10^{-6}$	5	$4.0453262276731428 \times 10^{-6}$
6	$8.4185124972599853 \times 10^{-7}$	6	$-2.2138609165441823 \times 10^{-7}$
7	$-1.0734107638588457 \times 10^{-7}$	7	$-2.0947089747860223 \times 10^{-7}$
8	$-6.6062601203118389 \times 10^{-9}$	8	$4.4031645643916225 \times 10^{-8}$
9	$2.4973585091056946 \times 10^{-9}$	9	$8.1782025782388308 \times 10^{-10}$

	ψ_A (deg)		Δ (deg)
0	5602.6281390521676	0	5608.2407214828560
1	57.721167630109300	1	55.698205919026172
2	$9.6242024949959648 \times 10^{-1}$	2	$2.5578812600206843 \times 10^{-1}$
3	$-7.5972530507400568 \times 10^{-2}$	3	$-1.0953419389791596 \times 10^{-2}$
4	$-2.6248838567137607 \times 10^{-2}$	4	$-2.7951761590448253 \times 10^{-3}$
5	$-3.0872639761176020 \times 10^{-4}$	5	$1.4853920281439987 \times 10^{-4}$
6	$5.0018064997686607 \times 10^{-4}$	6	$2.4333575023648612 \times 10^{-5}$
7	$5.7708152432052404 \times 10^{-5}$	7	$-1.1331493561665901 \times 10^{-8}$
8	$-5.9419077248992413 \times 10^{-6}$	8	$-2.3155675400599757 \times 10^{-7}$
9	$-1.9804503239071459 \times 10^{-6}$	9	$-1.0417465504465018 \times 10^{-8}$

	ω_A (deg)		I (deg)
0	22.887600095945328	0	23.687322646053172
1	-1.6592042547905213	1	$-6.2698245037289435 \times 10^{-1}$
2	$2.4429739216146170 \times 10^{-1}$	2	$1.9492852343594716 \times 10^{-2}$
3	$6.2002848435834657 \times 10^{-2}$	3	$1.1546170529907926 \times 10^{-2}$
4	$-4.6118361600249953 \times 10^{-3}$	4	$-9.3505006342746555 \times 10^{-4}$
5	$-1.0610881175491601 \times 10^{-3}$	5	$-9.0883627088518425 \times 10^{-5}$
6	$-7.1853727959221394 \times 10^{-6}$	6	$6.1533152572924682 \times 10^{-6}$
7	$1.4439523567993656 \times 10^{-5}$	7	$7.0412257974937056 \times 10^{-7}$
8	$1.5897286811893265 \times 10^{-6}$	8	$-3.1447659139919830 \times 10^{-9}$
9	$-1.2321400536310043 \times 10^{-7}$	9	$-6.2162079687493909 \times 10^{-9}$

	χ_A (deg)		L (deg)
0	-3.7830598636488574	0	1.5181141815942693
1	2.3907964880551771	1	$-1.7083461751854765 \times 10^{-1}$
2	$9.3256195964897641 \times 10^{-1}$	2	$-1.7468778936884397 \times 10^{-1}$
3	$-9.1537207098199484 \times 10^{-2}$	3	$1.8173475963224003 \times 10^{-2}$
4	$-2.6779413760091272 \times 10^{-2}$	4	$2.3139206992591972 \times 10^{-3}$
5	$-1.9053881796233524 \times 10^{-4}$	5	$-1.6651919943047645 \times 10^{-4}$
6	$5.0862516547425241 \times 10^{-4}$	6	$-2.5600665848564336 \times 10^{-5}$
7	$5.7287626748651866 \times 10^{-5}$	7	$-1.1360232697685832 \times 10^{-7}$
8	$-6.0102209425779575 \times 10^{-6}$	8	$2.8523951075882267 \times 10^{-7}$
9	$-1.9796745540844675 \times 10^{-6}$	9	$1.1128165530941072 \times 10^{-8}$

Table 4–2, continued.

Interval 114: Central time $T_c = 4080$, covering the time span $4040 \le T \le 4120$

	ε (deg)		p_A (deg)
0	23.091755521147441	0	5718.0350050905894
1	$-2.3448369787412167 \times 10^{-1}$	1	56.394856166340538
2	$1.6079265628707337 \times 10^{-2}$	2	$9.9729407383202089 \times 10^{-2}$
3	$2.7060782220070042 \times 10^{-3}$	3	$-4.4338396093521169 \times 10^{-3}$
4	$-1.0684445106846852 \times 10^{-4}$	4	$-5.5109185146794982 \times 10^{-4}$
5	$-7.9887458029101657 \times 10^{-6}$	5	$1.5807212000081060 \times 10^{-5}$
6	$2.1653554971549719 \times 10^{-7}$	6	$1.7104270254029251 \times 10^{-6}$
7	$4.1316877439300694 \times 10^{-8}$	7	$1.7255132957385019 \times 10^{-8}$
8	$1.6340746864582737 \times 10^{-9}$	8	$-6.6690501061048974 \times 10^{-9}$
9	$4.4023793803630004 \times 10^{-10}$	9	$-5.3728324893552710 \times 10^{-10}$

	ψ_A (deg)		Δ (deg)
0	5720.0822354892232	0	5720.9741410586546
1	58.138715427573294	1	56.831754790802250
2	$-8.1605089327116648 \times 10^{-1}$	2	$2.2696473526995455 \times 10^{-2}$
3	$-5.5626023634954730 \times 10^{-2}$	3	$-1.3849662269964872 \times 10^{-2}$
4	$2.6407162734868008 \times 10^{-2}$	4	$2.3534108591297421 \times 10^{-3}$
5	$-7.3382433592246450 \times 10^{-4}$	5	$1.1740401195638595 \times 10^{-4}$
6	$-4.3086509067629009 \times 10^{-4}$	6	$-2.9598968189069972 \times 10^{-5}$
7	$6.3446599576066466 \times 10^{-5}$	7	$-5.0425578157224286 \times 10^{-7}$
8	$3.0948507453917652 \times 10^{-6}$	8	$2.3087971151074272 \times 10^{-7}$
9	$-1.8405039947200350 \times 10^{-6}$	9	$-5.1437833982685404 \times 10^{-9}$

	ω_A (deg)		I (deg)
0	22.322463988898912	0	22.805158693097111
1	1.0410670834806746	1	$-2.3312971835385872 \times 10^{-1}$
2	$1.8573271049465590 \times 10^{-1}$	2	$4.6908342826369688 \times 10^{-2}$
3	$-6.5474789474342642 \times 10^{-2}$	3	$-5.7534196839789234 \times 10^{-3}$
4	$-2.4400275897501451 \times 10^{-3}$	4	$-4.6975360668004306 \times 10^{-4}$
5	$1.1132498308004471 \times 10^{-3}$	5	$1.2399675865252048 \times 10^{-4}$
6	$-3.2735580383469010 \times 10^{-5}$	6	$2.7924400244733930 \times 10^{-6}$
7	$-1.1722919481101505 \times 10^{-5}$	7	$-9.2699610851504245 \times 10^{-7}$
8	$1.7927214152779484 \times 10^{-6}$	8	$-4.8549251408317068 \times 10^{-9}$
9	$2.9887222417190933 \times 10^{-8}$	9	$5.2370667398586112 \times 10^{-9}$

	χ_A (deg)		L (deg)
0	2.2215689318933996	0	$6.4825540418156588 \times 10^{-1}$
1	1.8962107536155525	1	$-4.7317617579132460 \times 10^{-1}$
2	$-9.8846355038331955 \times 10^{-1}$	2	$8.4540746699479665 \times 10^{-2}$
3	$-5.6740737330147189 \times 10^{-2}$	3	$1.0071483956355001 \times 10^{-2}$
4	$2.8536630500112227 \times 10^{-2}$	4	$-3.1524615237075788 \times 10^{-3}$
5	$-6.8008622448138637 \times 10^{-4}$	5	$-1.0617867023665985 \times 10^{-4}$
6	$-4.4480264937215544 \times 10^{-4}$	6	$3.3475471530151484 \times 10^{-5}$
7	$6.2910741915038098 \times 10^{-5}$	7	$5.4927010386687263 \times 10^{-7}$
8	$3.1420018801415762 \times 10^{-6}$	8	$-2.4739151836957506 \times 10^{-7}$
9	$-1.8367159079345415 \times 10^{-6}$	9	$4.4030209580103212 \times 10^{-9}$

Table 4–2, continued.

Interval 115: Central time $T_c = 4160$, covering the time span $4120 \leq T \leq 4200$

	ε (deg)		p_A (deg)
0	22.828228000480608	0	5831.3714428541308
1	$-1.3146629262614861 \times 10^{-2}$	1	56.865447743420300
2	$3.5108890936130920 \times 10^{-2}$	2	$9.3930075676069143 \times 10^{-3}$
3	$4.0263947724718537 \times 10^{-4}$	3	$-9.3230534280776907 \times 10^{-3}$
4	$-1.3484531798075845 \times 10^{-4}$	4	$-7.6494856720870811 \times 10^{-5}$
5	$2.8576221709055415 \times 10^{-6}$	5	$8.6544786955085844 \times 10^{-6}$
6	$-8.0029989633146298 \times 10^{-7}$	6	$-2.5986184919663691 \times 10^{-6}$
7	$-2.1430566947804990 \times 10^{-7}$	7	$2.5230464863692926 \times 10^{-7}$
8	$6.4020954705941460 \times 10^{-9}$	8	$8.9346344502525646 \times 10^{-8}$
9	$4.8159222550813048 \times 10^{-9}$	9	$2.5377759537889738 \times 10^{-10}$

	ψ_A (deg)		Δ (deg)
0	5831.2323258888564	0	5834.7175303858405
1	53.758460943119411	1	56.938553813106672
2	$2.4785691439246589 \times 10^{-2}$	2	$4.8452887772613195 \times 10^{-2}$
3	$1.2475540192427790 \times 10^{-1}$	3	$9.8216981077192474 \times 10^{-3}$
4	$1.1796706351548421 \times 10^{-4}$	4	$-7.0295549813221909 \times 10^{-4}$
5	$-5.5710238112082735 \times 10^{-4}$	5	$-2.7640902122271302 \times 10^{-4}$
6	$-4.9370019494952591 \times 10^{-6}$	6	$3.7806119054298543 \times 10^{-6}$
7	$-1.5931132920602236 \times 10^{-5}$	7	$1.3790951579778468 \times 10^{-6}$
8	$-7.6967823997890628 \times 10^{-8}$	8	$-1.5396323374120127 \times 10^{-8}$
9	$4.7714834933478874 \times 10^{-8}$	9	$3.9455105385584112 \times 10^{-9}$

	ω_A (deg)		I (deg)
0	23.751801669907536	0	22.520503367436593
1	$-6.0587226099433199 \times 10^{-2}$	1	$-8.8816338802068082 \times 10^{-2}$
2	$-3.3579960897331152 \times 10^{-1}$	2	$5.2076223437509834 \times 10^{-3}$
3	$1.1343729841266319 \times 10^{-3}$	3	$2.7075853014409929 \times 10^{-3}$
4	$6.9909373645666075 \times 10^{-3}$	4	$1.0045038935384777 \times 10^{-3}$
5	$3.9306198491447593 \times 10^{-6}$	5	$-2.5293135385427243 \times 10^{-5}$
6	$-2.2032383375509617 \times 10^{-5}$	6	$-9.3706031121538303 \times 10^{-6}$
7	$1.0773041397905719 \times 10^{-8}$	7	$1.8856134395165840 \times 10^{-7}$
8	$-3.0165615610219017 \times 10^{-7}$	8	$2.3247796600966774 \times 10^{-8}$
9	$-3.7680479962424759 \times 10^{-9}$	9	$-2.4704379421542289 \times 10^{-9}$

	χ_A (deg)		L (deg)
0	$-1.5099080115815859 \times 10^{-1}$	0	$2.1223572094207237 \times 10^{-1}$
1	-3.3805711879314342	1	$-7.5366393628319685 \times 10^{-2}$
2	$1.7364397218261603 \times 10^{-2}$	2	$-4.2277959442306141 \times 10^{-2}$
3	$1.4781666930133783 \times 10^{-1}$	3	$-2.0775372772435318 \times 10^{-2}$
4	$1.6683881478151558 \times 10^{-4}$	4	$6.7982729736079648 \times 10^{-4}$
5	$-7.4340663803660221 \times 10^{-4}$	5	$3.0698938576046332 \times 10^{-4}$
6	$-1.8087303838728790 \times 10^{-6}$	6	$-7.1101327063007340 \times 10^{-6}$
7	$-1.5024037878561732 \times 10^{-5}$	7	$-1.2239753142365757 \times 10^{-6}$
8	$-1.7869921672066440 \times 10^{-7}$	8	$1.1762923710077030 \times 10^{-7}$
9	$4.1301664218609648 \times 10^{-8}$	9	$-3.3213017050587055 \times 10^{-9}$

Table 4–2, continued.

Interval 116: Central time $T_c = 4240$, covering the time span $4200 \leq T \leq 4280$

	ε (deg)
0	23.068681055064354
1	$2.4465081530179948 \times 10^{-1}$
2	$2.4047244842686750 \times 10^{-2}$
3	$-2.6345494314250383 \times 10^{-3}$
4	$-2.4833876663622710 \times 10^{-4}$
5	$6.6458908928707020 \times 10^{-6}$
6	$2.8502777049481539 \times 10^{-6}$
7	$-6.7077399039763291 \times 10^{-8}$
8	$-5.4529453029527962 \times 10^{-8}$
9	$1.6567740875324603 \times 10^{-9}$

	p_A (deg)
0	5944.8182364468515
1	56.487883220536111
2	$-1.0166587954932121 \times 10^{-1}$
3	$-7.7330942639949630 \times 10^{-3}$
4	$5.3709890569623903 \times 10^{-4}$
5	$6.6167342591666529 \times 10^{-5}$
6	$-1.9545219319245532 \times 10^{-6}$
7	$-1.0171510173391010 \times 10^{-6}$
8	$3.8193089483790093 \times 10^{-8}$
9	$1.8816485811534556 \times 10^{-8}$

	ψ_A (deg)
0	5942.7624329466350
1	58.501314260129007
2	$8.3971822810334582 \times 10^{-1}$
3	$-6.6180006466199860 \times 10^{-2}$
4	$-2.8270282304001596 \times 10^{-2}$
5	$-6.9726537665306773 \times 10^{-4}$
6	$4.9652017056598952 \times 10^{-4}$
7	$7.0463266034695227 \times 10^{-5}$
8	$-4.2748182608938466 \times 10^{-6}$
9	$-2.2063992580522747 \times 10^{-6}$

	Δ (deg)
0	5949.0037227835787
1	57.283922422108851
2	$-3.1497607351042072 \times 10^{-2}$
3	$-2.7509458999953773 \times 10^{-2}$
4	$-2.2052023734441427 \times 10^{-3}$
5	$2.5496276368730373 \times 10^{-4}$
6	$3.1548818967936057 \times 10^{-5}$
7	$-1.2997497963756482 \times 10^{-6}$
8	$-3.3367968118589873 \times 10^{-7}$
9	$-5.6937838092053503 \times 10^{-9}$

	ω_A (deg)
0	22.170327848077565
1	-1.0479772089722715
2	$2.1413876966208702 \times 10^{-1}$
3	$6.7292471576964219 \times 10^{-2}$
4	$-2.7671505823366637 \times 10^{-3}$
5	$-1.1839120472674367 \times 10^{-3}$
6	$-3.5052220613051352 \times 10^{-5}$
7	$1.3414311726500133 \times 10^{-5}$
8	$2.0501692306543864 \times 10^{-6}$
9	$-4.8532014547362818 \times 10^{-8}$

	I (deg)
0	22.612996746804587
1	$2.5152506488632431 \times 10^{-1}$
2	$8.5382562432585574 \times 10^{-2}$
3	$5.4083341373739184 \times 10^{-3}$
4	$-9.9764309283655717 \times 10^{-4}$
5	$-1.1958249806642186 \times 10^{-4}$
6	$5.4210158108839799 \times 10^{-6}$
7	$1.0003258388371062 \times 10^{-6}$
8	$7.3798720254986859 \times 10^{-9}$
9	$-8.0849123754576657 \times 10^{-9}$

	χ_A (deg)
0	-2.2297592923737772
1	2.1872026970667533
2	1.0148860762179213
3	$-6.4360829054397269 \times 10^{-2}$
4	$-3.0392641386408912 \times 10^{-2}$
5	$-6.9582811838101450 \times 10^{-4}$
6	$5.1065239232656906 \times 10^{-4}$
7	$7.1084684758965024 \times 10^{-5}$
8	$-4.3552259383761583 \times 10^{-6}$
9	$-2.2243955472118752 \times 10^{-6}$

	L (deg)
0	$-6.9235847137126576 \times 10^{-1}$
1	$-8.6273465166503430 \times 10^{-1}$
2	$-7.7535516519314630 \times 10^{-2}$
3	$2.1184679035796750 \times 10^{-2}$
4	$2.9777903411909676 \times 10^{-3}$
5	$-1.9612946197194039 \times 10^{-4}$
6	$-3.5455990999340857 \times 10^{-5}$
7	$2.1182813319658593 \times 10^{-7}$
8	$3.8197531463991832 \times 10^{-7}$
9	$2.6715954832656216 \times 10^{-8}$

Table 4–2, continued.

Interval 117: Central time $T_c = 4320$, covering the time span $4280 \leq T \leq 4360$

	ε (deg)		p_A (deg)
0	23.617908486105475	0	6056.8310832514042
1	$2.6652792181913099 \times 10^{-1}$	1	55.509960598596466
2	$-2.0940874206137661 \times 10^{-2}$	2	$-1.1931567628423554 \times 10^{-1}$
3	$-4.0116905361574709 \times 10^{-3}$	3	$5.3073410317536495 \times 10^{-3}$
4	$6.5026904282224861 \times 10^{-5}$	4	$8.1292012576448368 \times 10^{-4}$
5	$1.0928735171635940 \times 10^{-5}$	5	$-6.8485854742662607 \times 10^{-6}$
6	$3.6492596251518243 \times 10^{-7}$	6	$1.6033361166586597 \times 10^{-6}$
7	$2.3445724597053048 \times 10^{-7}$	7	$-1.3482733737745646 \times 10^{-7}$
8	$-1.0927029559641898 \times 10^{-8}$	8	$-9.2070666586399739 \times 10^{-8}$
9	$-4.9021371131894182 \times 10^{-9}$	9	$5.1364962679323749 \times 10^{-9}$

	ψ_A (deg)		Δ (deg)
0	6060.6652043020706	0	6062.1974167664888
1	57.708580927750612	1	55.638958551776114
2	-1.0465172001282516	2	$-3.4332735093799841 \times 10^{-1}$
3	$-7.0770567401773534 \times 10^{-2}$	3	$-7.7810520859501065 \times 10^{-3}$
4	$2.7893065801373219 \times 10^{-2}$	4	$3.8917451756840696 \times 10^{-3}$
5	$-6.8087572711139315 \times 10^{-4}$	5	$5.3008543896186333 \times 10^{-5}$
6	$-5.0912620930432680 \times 10^{-4}$	6	$-3.1705139477527115 \times 10^{-5}$
7	$7.0169928347045459 \times 10^{-5}$	7	$1.4240982716157949 \times 10^{-6}$
8	$4.7657352488044751 \times 10^{-6}$	8	$2.5711279345060484 \times 10^{-7}$
9	$-2.2302200453789164 \times 10^{-6}$	9	$-2.9055036198724881 \times 10^{-8}$

	ω_A (deg)		I (deg)
0	22.921075092914538	0	23.744716922640932
1	1.8301010616859358	1	$8.2350153887456293 \times 10^{-1}$
2	$2.4815307654407036 \times 10^{-1}$	2	$1.7767778115291343 \times 10^{-2}$
3	$-6.6206772525162081 \times 10^{-2}$	3	$-1.6145559617848635 \times 10^{-2}$
4	$-4.4032911731722365 \times 10^{-3}$	4	$-8.0033779952179905 \times 10^{-4}$
5	$1.1172158100540360 \times 10^{-3}$	5	$1.3123094767244929 \times 10^{-4}$
6	$-1.8612489696565993 \times 10^{-5}$	6	$3.7475416245684127 \times 10^{-6}$
7	$-1.4607940684729859 \times 10^{-5}$	7	$-9.8339917320060514 \times 10^{-7}$
8	$1.9046787603445313 \times 10^{-6}$	8	$2.7790275249394715 \times 10^{-8}$
9	$9.2001286477467001 \times 10^{-8}$	9	$8.1768108331785821 \times 10^{-9}$

	χ_A (deg)		L (deg)
0	4.1824289877231814	0	-1.9840153882527866
1	2.4233538001801404	1	$-1.4964585957038733 \times 10^{-1}$
2	$-9.9645936892223838 \times 10^{-1}$	2	$2.4381557944595940 \times 10^{-1}$
3	$-8.6585078557380193 \times 10^{-2}$	3	$1.4946372878686099 \times 10^{-2}$
4	$2.8288371094919603 \times 10^{-2}$	4	$-3.3152973024946989 \times 10^{-3}$
5	$-5.4791082021802271 \times 10^{-4}$	5	$-8.2354426986036956 \times 10^{-5}$
6	$-5.1931508210561439 \times 10^{-4}$	6	$3.5070070360758382 \times 10^{-5}$
7	$6.9646817127084788 \times 10^{-5}$	7	$-1.4615483135914542 \times 10^{-6}$
8	$4.8885564426192499 \times 10^{-6}$	8	$-3.6735475985510423 \times 10^{-7}$
9	$-2.2340567796810422 \times 10^{-6}$	9	$3.4385475481591938 \times 10^{-8}$

Table 4–2, continued.

Interval 118: Central time $T_c = 4400$, covering the time span $4360 \leq T \leq 4440$

	ε (deg)		p_A (deg)
0	23.859240159121256	0	6167.2447747644154
1	$-5.1645961331955072 \times 10^{-2}$	1	55.016116834752225
2	$-5.1896244741409454 \times 10^{-2}$	2	$1.4492198532121235 \times 10^{-2}$
3	$-1.4832984163366140 \times 10^{-4}$	3	$1.5582103196937054 \times 10^{-2}$
4	$4.3606814781917303 \times 10^{-4}$	4	$1.9871991106158109 \times 10^{-4}$
5	$1.1700714283228856 \times 10^{-5}$	5	$-7.9684560415098609 \times 10^{-5}$
6	$-2.3134440412094971 \times 10^{-6}$	6	$-3.0285118094089652 \times 10^{-6}$
7	$-9.9647569434664454 \times 10^{-8}$	7	$4.5122934357236724 \times 10^{-7}$
8	$2.3340904972501575 \times 10^{-8}$	8	$1.2580497196087491 \times 10^{-8}$
9	$1.5559815908293307 \times 10^{-10}$	9	$-5.5538325992446935 \times 10^{-9}$

	ψ_A (deg)		Δ (deg)
0	6168.6968736149089	0	6171.1215182497305
1	50.971903048871794	1	53.483429883732289
2	$-3.2725245488725322 \times 10^{-1}$	2	$-1.1031138222939210 \times 10^{-1}$
3	$1.1559651950935194 \times 10^{-1}$	3	$4.0414486753238998 \times 10^{-2}$
4	$9.6663441408481722 \times 10^{-4}$	4	$1.6977151665582367 \times 10^{-3}$
5	$-9.7815862730311640 \times 10^{-5}$	5	$-1.1711255173427680 \times 10^{-4}$
6	$7.6750507911691775 \times 10^{-5}$	6	$3.7868054500495575 \times 10^{-6}$
7	$-1.0241608954582180 \times 10^{-5}$	7	$-6.1295477124568743 \times 10^{-7}$
8	$1.9826939480826719 \times 10^{-7}$	8	$-1.0349045499615143 \times 10^{-7}$
9	$7.0386885801365212 \times 10^{-10}$	9	$5.0266414756880152 \times 10^{-9}$

	ω_A (deg)		I (deg)
0	26.053460092325346	0	24.897750254234045
1	$7.0538296755247273 \times 10^{-1}$	1	$1.7106733197601652 \times 10^{-1}$
2	$-4.4951799244323946 \times 10^{-1}$	2	$-1.6797911684768627 \times 10^{-1}$
3	$-2.5496489299280783 \times 10^{-2}$	3	$-9.0929109491442877 \times 10^{-3}$
4	$5.8510724455926719 \times 10^{-3}$	4	$1.4539516863750402 \times 10^{-3}$
5	$8.8980437687235664 \times 10^{-5}$	5	$7.3190724439327010 \times 10^{-5}$
6	$-1.0978671439110717 \times 10^{-5}$	6	$-3.0295116170474654 \times 10^{-6}$
7	$1.3822291664640075 \times 10^{-6}$	7	$5.5981365555505328 \times 10^{-8}$
8	$-1.8272694248133815 \times 10^{-7}$	8	$-2.0351784308317567 \times 10^{-8}$
9	$1.0503342463981111 \times 10^{-8}$	9	$-1.7939709103927180 \times 10^{-9}$

	χ_A (deg)		L (deg)
0	1.5958952171748398	0	$-3.6448882930544592 \times 10^{-1}$
1	-4.4516429384709072	1	1.6772280772406715
2	$-3.8541271914307422 \times 10^{-1}$	2	$1.3735093438835516 \times 10^{-1}$
3	$1.1402059474213232 \times 10^{-1}$	3	$-2.7909476194198315 \times 10^{-2}$
4	$1.6720179161754022 \times 10^{-3}$	4	$-1.7440339278854823 \times 10^{-3}$
5	$-1.7280441151849517 \times 10^{-4}$	5	$5.6925861959466697 \times 10^{-5}$
6	$7.2043091433112866 \times 10^{-5}$	6	$-5.2808359698558283 \times 10^{-6}$
7	$-9.8522297372522789 \times 10^{-6}$	7	$1.0592335957020216 \times 10^{-6}$
8	$2.3122235339213132 \times 10^{-7}$	8	$1.1253738634818917 \times 10^{-7}$
9	$3.6986689726863530 \times 10^{-9}$	9	$-1.1610341750800448 \times 10^{-8}$

Table 4–2, continued.

Interval 119: Central time $T_c = 4480$, covering the time span $4440 \leq T \leq 4520$

	ε (deg)		p_A (deg)
0	23.418282066912001	0	6277.9387625125577
1	$-3.5700155112547809 \times 10^{-1}$	1	55.806085175485006
2	$-1.3654523437869499 \times 10^{-2}$	2	$1.6530583612779690 \times 10^{-1}$
3	$5.8425886222028893 \times 10^{-3}$	3	$5.6167078407120965 \times 10^{-3}$
4	$1.7994460369298898 \times 10^{-4}$	4	$-1.3398460866051692 \times 10^{-3}$
5	$-2.9454451315115530 \times 10^{-5}$	5	$-5.2810268950885181 \times 10^{-5}$
6	$-8.9566979954193387 \times 10^{-7}$	6	$3.0909398687510448 \times 10^{-6}$
7	$3.5609343092309074 \times 10^{-9}$	7	$1.8280266851878738 \times 10^{-7}$
8	$-1.7712228458574121 \times 10^{-9}$	8	$2.9311181482688399 \times 10^{-8}$
9	$1.5572056216588265 \times 10^{-9}$	9	$9.9881780487293527 \times 10^{-10}$

	ψ_A (deg)		Δ (deg)
0	6272.6157415101526	0	6278.9401293584365
1	54.178806001327621	1	54.806538377806057
2	1.1408573830850785	2	$4.4807595153093749 \times 10^{-1}$
3	$1.1392863853092967 \times 10^{-1}$	3	$4.3003083982932829 \times 10^{-2}$
4	$-7.8213524256250627 \times 10^{-3}$	4	$-2.4526476906662539 \times 10^{-3}$
5	$-2.3035825603194343 \times 10^{-3}$	5	$-4.5513828497593149 \times 10^{-4}$
6	$-3.0039082850811438 \times 10^{-4}$	6	$-3.1630912995821351 \times 10^{-5}$
7	$-6.4940836135664554 \times 10^{-6}$	7	$1.1642420820091466 \times 10^{-7}$
8	$4.6335745154391512 \times 10^{-6}$	8	$3.6669916105298178 \times 10^{-7}$
9	$9.6551712019616588 \times 10^{-7}$	9	$4.2146289791252983 \times 10^{-8}$

	ω_A (deg)		I (deg)
0	24.068338831188623	0	23.884533833876595
1	-2.4570737450538365	1	-1.1321372194407737
2	$-1.7019409623264868 \times 10^{-1}$	2	$-1.0448577242597772 \times 10^{-1}$
3	$7.0807384398105865 \times 10^{-2}$	3	$2.0818834418705734 \times 10^{-2}$
4	$5.0594784907353104 \times 10^{-3}$	4	$1.8424447573456620 \times 10^{-3}$
5	$-3.4451111046632074 \times 10^{-4}$	5	$-7.2852699851031758 \times 10^{-5}$
6	$-6.3037553406373629 \times 10^{-5}$	6	$-1.2693188031015295 \times 10^{-5}$
7	$-7.2221138533585448 \times 10^{-6}$	7	$-6.5784875701814439 \times 10^{-7}$
8	$-3.3755764633309561 \times 10^{-7}$	8	$1.4215477767185419 \times 10^{-9}$
9	$6.9863001308991253 \times 10^{-8}$	9	$4.8641168882482687 \times 10^{-9}$

	χ_A (deg)		L (deg)
0	-5.8057435226238999	0	2.7627781656796891
1	-1.7203151971870392	1	1.0705321238367554
2	1.0774439706255846	2	$-3.1264421797971362 \times 10^{-1}$
3	$1.1126670510763003 \times 10^{-1}$	3	$-3.9816941373512932 \times 10^{-2}$
4	$-8.3255605669219420 \times 10^{-3}$	4	$1.4662821848937905 \times 10^{-3}$
5	$-2.2774037193986516 \times 10^{-3}$	5	$4.2815952006591165 \times 10^{-4}$
6	$-2.8920197260361094 \times 10^{-4}$	6	$3.2788488606257218 \times 10^{-5}$
7	$-6.6410067759240107 \times 10^{-6}$	7	$-8.8878620804006371 \times 10^{-8}$
8	$4.5372315502890592 \times 10^{-6}$	8	$-3.2827584738228774 \times 10^{-7}$
9	$9.6590892750261010 \times 10^{-7}$	9	$-4.0395186272042162 \times 10^{-8}$

Table 4–2, continued.

Interval 120: Central time $T_c = 4560$, covering the time span $4520 \le T \le 4600$

	ε (deg)		p_A (deg)
0	22.818220166266299	0	6390.8016091206194
1	$-1.8767461828447701 \times 10^{-1}$	1	56.993543430543750
2	$5.1444766521366111 \times 10^{-2}$	2	$9.0009909094389991 \times 10^{-2}$
3	$3.2490713013973296 \times 10^{-3}$	3	$-1.7394241035648096 \times 10^{-2}$
4	$-5.0017374711925964 \times 10^{-4}$	4	$-9.8509091761111059 \times 10^{-4}$
5	$-2.5908699741938057 \times 10^{-5}$	5	$1.1175987789907281 \times 10^{-4}$
6	$2.1155034057885249 \times 10^{-6}$	6	$7.5474414078763532 \times 10^{-6}$
7	$1.4627283290708208 \times 10^{-7}$	7	$-3.8505569458584107 \times 10^{-7}$
8	$-4.9943781527866048 \times 10^{-9}$	8	$-4.0565869943057170 \times 10^{-8}$
9	$1.5920993923578004 \times 10^{-10}$	9	$1.6334628173978773 \times 10^{-9}$

	ψ_A (deg)		Δ (deg)
0	6390.1630445793134	0	6392.7668068261470
1	62.530692527114650	1	58.980912634158678
2	$1.1246802553794203 \times 10^{-1}$	2	$3.8533151714136441 \times 10^{-1}$
3	$-3.0438139814430312 \times 10^{-1}$	3	$-7.2624992071480623 \times 10^{-2}$
4	$-1.9973323325272055 \times 10^{-3}$	4	$-8.4760647940971424 \times 10^{-3}$
5	$8.6316020819875534 \times 10^{-3}$	5	$7.1450099791408448 \times 10^{-4}$
6	$8.4035713932318712 \times 10^{-5}$	6	$1.4698777712876726 \times 10^{-4}$
7	$-2.6310583048940732 \times 10^{-4}$	7	$-3.0693110457279300 \times 10^{-6}$
8	$-3.5799821339139902 \times 10^{-6}$	8	$-2.3844496414797261 \times 10^{-6}$
9	$8.7907389958258246 \times 10^{-6}$	9	$-7.2089690160604115 \times 10^{-8}$

	ω_A (deg)		I (deg)
0	20.664369082532526	0	21.777309756221039
1	$-2.7011627147396951 \times 10^{-1}$	1	$-7.0359718121475592 \times 10^{-1}$
2	$5.9644228018866194 \times 10^{-1}$	2	$2.1188174707250384 \times 10^{-1}$
3	$2.8122923395621063 \times 10^{-5}$	3	$1.9783423957759878 \times 10^{-2}$
4	$-1.4644259252147234 \times 10^{-2}$	4	$-2.8139383815866585 \times 10^{-3}$
5	$1.0520093577665273 \times 10^{-5}$	5	$-2.7895760662453246 \times 10^{-4}$
6	$2.8613801147422752 \times 10^{-4}$	6	$1.9207363982708288 \times 10^{-5}$
7	$1.1014349643972947 \times 10^{-6}$	7	$3.5577562456720600 \times 10^{-6}$
8	$-7.0120506308994858 \times 10^{-6}$	8	$-5.2738955431209054 \times 10^{-8}$
9	$-5.7866691542601349 \times 10^{-8}$	9	$-4.7090561534534415 \times 10^{-8}$

	χ_A (deg)		L (deg)
0	$-6.9201520489184416 \times 10^{-1}$	0	1.6902444215669736
1	5.9673684330450083	1	-2.1574586715398598
2	$1.8096758843222451 \times 10^{-2}$	2	$-3.1386757230011977 \times 10^{-1}$
3	$-3.0220491120885910 \times 10^{-1}$	3	$5.9304273545903463 \times 10^{-2}$
4	$-8.0041458847456546 \times 10^{-4}$	4	$7.7501077616471382 \times 10^{-3}$
5	$8.6847333739300705 \times 10^{-3}$	5	$-6.3608180141468221 \times 10^{-4}$
6	$7.4574071248968930 \times 10^{-5}$	6	$-1.4097127143609757 \times 10^{-4}$
7	$-2.6352053887395862 \times 10^{-4}$	7	$2.8391807100733278 \times 10^{-6}$
8	$-3.5367366572873428 \times 10^{-6}$	8	$2.3497207352295443 \times 10^{-6}$
9	$8.7902292368287011 \times 10^{-6}$	9	$7.3285331587524934 \times 10^{-8}$

Table 4–2, continued.

Interval 121: Central time $T_c = 4640$, covering the time span $4600 \le T \le 4680$

	ε (deg)		p_A (deg)
0	22.870620480049876	0	6504.7942451578820
1	$2.2791816255296261 \times 10^{-1}$	1	56.817030330694155
2	$3.7244400945450831 \times 10^{-2}$	2	$-1.2314871378064067 \times 10^{-1}$
3	$-5.1091686541036199 \times 10^{-3}$	3	$-1.1331461518127734 \times 10^{-2}$
4	$-2.9593676407132430 \times 10^{-4}$	4	$1.5726519042642678 \times 10^{-3}$
5	$4.6193350946427925 \times 10^{-5}$	5	$5.5540728124233716 \times 10^{-5}$
6	$1.4406174077272214 \times 10^{-6}$	6	$-1.3064995598407956 \times 10^{-5}$
7	$-3.8738625514452778 \times 10^{-7}$	7	$-2.7537185112206496 \times 10^{-7}$
8	$-2.0829374946257016 \times 10^{-8}$	8	$1.2155951905086232 \times 10^{-7}$
9	$3.8569436217422027 \times 10^{-9}$	9	$6.4015381817847615 \times 10^{-9}$

	ψ_A (deg)		Δ (deg)
0	6509.1471631246822	0	6510.5609551227964
1	55.507505777820969	1	58.002078853044574
2	-1.0327736846704422	2	$-5.4637210220513903 \times 10^{-1}$
3	$1.1036533476430211 \times 10^{-1}$	3	$-3.0320757972564758 \times 10^{-2}$
4	$8.9853043757939656 \times 10^{-3}$	4	$9.9230587741662710 \times 10^{-3}$
5	$-2.4886136042839376 \times 10^{-3}$	5	$-1.7279402993279778 \times 10^{-4}$
6	$2.9484909548594091 \times 10^{-4}$	6	$-1.0467538194968437 \times 10^{-4}$
7	$-3.6229439015849879 \times 10^{-6}$	7	$1.0955615700087452 \times 10^{-5}$
8	$-5.5338588508458172 \times 10^{-6}$	8	$5.2827643284135250 \times 10^{-7}$
9	$1.0393755129003936 \times 10^{-6}$	9	$-2.0790411301031391 \times 10^{-7}$

	ω_A (deg)		I (deg)
0	23.032176229291409	0	22.161656829204233
1	1.9844592890765541	1	1.0138989621400148
2	$-1.4026238060678385 \times 10^{-1}$	2	$1.2705484811140373 \times 10^{-1}$
3	$-6.3477076901200032 \times 10^{-2}$	3	$-2.8884904014784997 \times 10^{-2}$
4	$5.8380332823370059 \times 10^{-3}$	4	$-1.0693841229555242 \times 10^{-3}$
5	$3.2097674965257684 \times 10^{-4}$	5	$3.1760869807037439 \times 10^{-4}$
6	$-7.1058285233095835 \times 10^{-5}$	6	$-5.6041236543969868 \times 10^{-6}$
7	$7.7409313663791653 \times 10^{-6}$	7	$-2.2495835216828109 \times 10^{-6}$
8	$-3.3327520681653910 \times 10^{-7}$	8	$2.4847469221635072 \times 10^{-7}$
9	$-8.3991011179054235 \times 10^{-8}$	9	$4.9648929396631146 \times 10^{-9}$

	χ_A (deg)		L (deg)
0	4.7213228977205540	0	-2.4320454258499401
1	-1.3870756609145638	1	-1.2932772355570672
2	$-9.9423077682171315 \times 10^{-1}$	2	$4.5427681289685305 \times 10^{-1}$
3	$1.2710021813621771 \times 10^{-1}$	3	$2.1441239732573063 \times 10^{-2}$
4	$8.9673593847067723 \times 10^{-3}$	4	$-8.7618454347916958 \times 10^{-3}$
5	$-2.6186988072400038 \times 10^{-3}$	5	$2.0892814038044258 \times 10^{-4}$
6	$2.9580011434185680 \times 10^{-4}$	6	$9.3599041524863332 \times 10^{-5}$
7	$-2.9091256202624436 \times 10^{-6}$	7	$-1.1180592099046110 \times 10^{-5}$
8	$-5.5983375540377763 \times 10^{-6}$	8	$-4.0557643806901730 \times 10^{-7}$
9	$1.0311736042771058 \times 10^{-6}$	9	$2.1520495010693458 \times 10^{-7}$

Table 4–2, continued.

Interval 122: Central time $T_c = 4720$, covering the time span $4680 \leq T \leq 4760$

	ε (deg)		p_A (deg)
0	23.417012609487166	0	6617.3101383369083
1	$2.6563461926427893 \times 10^{-1}$	1	55.705901727107451
2	$-2.4954057814116526 \times 10^{-2}$	2	$-1.1676464156375753 \times 10^{-1}$
3	$-3.6910548421681352 \times 10^{-3}$	3	$9.8285270053440129 \times 10^{-3}$
4	$3.1980443119960441 \times 10^{-4}$	4	$7.0078995116482736 \times 10^{-4}$
5	$8.9900969438707564 \times 10^{-6}$	5	$-6.3969940471380123 \times 10^{-5}$
6	$-4.3411141489498007 \times 10^{-7}$	6	$1.8185078555548739 \times 10^{-6}$
7	$1.6923774890076039 \times 10^{-7}$	7	$-3.5547673961441034 \times 10^{-7}$
8	$-3.4025768815206320 \times 10^{-8}$	8	$-6.8135843633601710 \times 10^{-8}$
9	$-3.5295200934763997 \times 10^{-9}$	9	$1.3468868848025104 \times 10^{-8}$

	ψ_A (deg)		Δ (deg)
0	6616.4634907221050	0	6622.5020352352986
1	52.984767730050762	1	54.192219337777729
2	$3.5734559035763070 \times 10^{-1}$	2	$-2.6244127679176741 \times 10^{-1}$
3	$9.1800553959911326 \times 10^{-2}$	3	$5.2282535269716769 \times 10^{-2}$
4	$-5.7222016956555317 \times 10^{-3}$	4	$5.9095618525045183 \times 10^{-4}$
5	$-5.6862149955849512 \times 10^{-4}$	5	$-2.9300281280345491 \times 10^{-4}$
6	$-7.3905856665553394 \times 10^{-5}$	6	$2.0450725508554583 \times 10^{-5}$
7	$-2.4080646887959999 \times 10^{-6}$	7	$-1.5534769223158002 \times 10^{-6}$
8	$1.1190383179982680 \times 10^{-6}$	8	$-5.1733265831982772 \times 10^{-8}$
9	$9.4465389789461756 \times 10^{-8}$	9	$1.5913032576789509 \times 10^{-8}$

	ω_A (deg)		I (deg)
0	24.682525452593882	0	24.159156242511506
1	$-4.6363555673043683 \times 10^{-1}$	1	$7.3657050778284630 \times 10^{-1}$
2	$-2.9860581613247890 \times 10^{-1}$	2	$-1.6304310929405526 \times 10^{-1}$
3	$3.3472254451538559 \times 10^{-2}$	3	$-1.1193150439506847 \times 10^{-2}$
4	$4.6441121005088712 \times 10^{-3}$	4	$2.2598348506955016 \times 10^{-3}$
5	$-2.8587992326799528 \times 10^{-4}$	5	$1.9653270064357291 \times 10^{-5}$
6	$-2.3111722060701312 \times 10^{-5}$	6	$-9.2883302505398087 \times 10^{-6}$
7	$-1.5679871608104094 \times 10^{-6}$	7	$2.9870369935267747 \times 10^{-7}$
8	$-3.3081712204791461 \times 10^{-8}$	8	$-9.4132250045437672 \times 10^{-9}$
9	$1.7675582336322058 \times 10^{-8}$	9	$1.2591770560928709 \times 10^{-9}$

	χ_A (deg)		L (deg)
0	$-9.2629072606231345 \times 10^{-1}$	0	-1.8182193666589200
1	-2.9772069665041534	1	1.6497935415218814
2	$5.2117616497832259 \times 10^{-1}$	2	$1.6376088099510798 \times 10^{-1}$
3	$9.1302101406656382 \times 10^{-2}$	3	$-4.6561169897291722 \times 10^{-2}$
4	$-7.5808212737259024 \times 10^{-3}$	4	$-9.7355789202213887 \times 10^{-5}$
5	$-6.0686058072277132 \times 10^{-4}$	5	$2.6892580452571924 \times 10^{-4}$
6	$-6.5211423000814056 \times 10^{-5}$	6	$-1.7191446013941773 \times 10^{-5}$
7	$-1.4754220114852021 \times 10^{-6}$	7	$9.3802149520138264 \times 10^{-7}$
8	$1.1333395181587105 \times 10^{-6}$	8	$-2.9712699493172238 \times 10^{-8}$
9	$7.8136298607501770 \times 10^{-8}$	9	$-6.8283794732720907 \times 10^{-11}$

Table 4–2, continued.

Interval 123: Central time $T_c = 4800$, covering the time span $4760 \le T \le 4840$

	ε (deg)		p_A (deg)
0	23.676197121742111	0	6728.2314007285902
1	$-1.4804740663988391 \times 10^{-2}$	1	55.336088457332494
2	$-3.5012338176533385 \times 10^{-2}$	2	$2.5441927216585689 \times 10^{-2}$
3	$1.9540200181885548 \times 10^{-3}$	3	$1.0427453852137029 \times 10^{-2}$
4	$2.7290262057674951 \times 10^{-4}$	4	$-6.4159922191379777 \times 10^{-4}$
5	$-2.3917261479578684 \times 10^{-5}$	5	$-5.1970508439091943 \times 10^{-5}$
6	$-1.4902571631703269 \times 10^{-6}$	6	$5.4747310481028650 \times 10^{-6}$
7	$2.4445306335169318 \times 10^{-7}$	7	$3.4591038053723337 \times 10^{-7}$
8	$1.4656411370085822 \times 10^{-8}$	8	$-7.0239986863070250 \times 10^{-8}$
9	$-4.1547309277731218 \times 10^{-9}$	9	$-3.3427240621567208 \times 10^{-9}$

	ψ_A (deg)		Δ (deg)
0	6726.8955691106763	0	6730.6584169299066
1	57.505810750172698	1	54.418512766044916
2	$4.0682834502897683 \times 10^{-1}$	2	$2.7528949618961883 \times 10^{-1}$
3	$-1.0766421542204278 \times 10^{-1}$	3	$2.7453600572329952 \times 10^{-2}$
4	$-1.1628945913501421 \times 10^{-2}$	4	$-3.5405746183456620 \times 10^{-3}$
5	$1.8708670581012828 \times 10^{-3}$	5	$-2.1446482143863037 \times 10^{-4}$
6	$2.7175334675694155 \times 10^{-4}$	6	$-1.6045762680264105 \times 10^{-8}$
7	$-2.2403716067659184 \times 10^{-5}$	7	$1.5121278024776822 \times 10^{-6}$
8	$-6.4050995596104518 \times 10^{-6}$	8	$2.2610759121127454 \times 10^{-7}$
9	$1.8708597407211949 \times 10^{-7}$	9	$1.2611566416505906 \times 10^{-9}$

	ω_A (deg)		I (deg)
0	23.095316794419139	0	24.313984671860471
1	$-6.5203986096214962 \times 10^{-1}$	1	$-5.2718787895638251 \times 10^{-1}$
2	$2.4439443590724775 \times 10^{-1}$	2	$-1.0018619868713096 \times 10^{-1}$
3	$2.7735977107239516 \times 10^{-2}$	3	$1.8292393115165253 \times 10^{-2}$
4	$-6.4586649320915284 \times 10^{-3}$	4	$9.3487569101610993 \times 10^{-4}$
5	$-4.7118139358381419 \times 10^{-4}$	5	$-1.3773967059975809 \times 10^{-4}$
6	$7.0855108940731668 \times 10^{-5}$	6	$-5.5910521756791439 \times 10^{-6}$
7	$8.1468416557034741 \times 10^{-6}$	7	$2.0942092939119113 \times 10^{-8}$
8	$-6.2583538098864899 \times 10^{-7}$	8	$2.4265443791598880 \times 10^{-8}$
9	$-1.5816970159585388 \times 10^{-7}$	9	$5.7463979651575498 \times 10^{-9}$

	χ_A (deg)		L (deg)
0	-1.4581632452141967	0	1.2078730757738886
1	2.3679019068145385	1	1.0054685566878236
2	$4.1164330519382634 \times 10^{-1}$	2	$-2.7515880679744588 \times 10^{-1}$
3	$-1.2840324657638019 \times 10^{-1}$	3	$-1.8538539252131489 \times 10^{-2}$
4	$-1.1484709935741357 \times 10^{-2}$	4	$3.3105292764246852 \times 10^{-3}$
5	$2.0550121169577272 \times 10^{-3}$	5	$1.6963348907002336 \times 10^{-4}$
6	$2.6878482408808031 \times 10^{-4}$	6	$3.1767340132763360 \times 10^{-6}$
7	$-2.3473217194313621 \times 10^{-5}$	7	$-1.1411768903299093 \times 10^{-6}$
8	$-6.3270930971887606 \times 10^{-6}$	8	$-2.9769531607485224 \times 10^{-7}$
9	$1.9204208953134242 \times 10^{-7}$	9	$-5.2269063725136037 \times 10^{-9}$

Table 4–2, continued.

Interval 124: Central time $T_c = 4880$, covering the time span $4840 \leq T \leq 4920$

	ε (deg)		p_A (deg)
0	23.467721492726402	0	6839.3577358931758
1	$-1.6527940918375010 \times 10^{-1}$	1	55.833445541559495
2	$-1.7665361152276604 \times 10^{-3}$	2	$7.7589921060287007 \times 10^{-2}$
3	$2.5243542181494244 \times 10^{-3}$	3	$-1.3719843093283198 \times 10^{-3}$
4	$-1.3781139614334395 \times 10^{-4}$	4	$-5.4629499148159592 \times 10^{-4}$
5	$-8.6452329019879882 \times 10^{-6}$	5	$3.6165694074749879 \times 10^{-5}$
6	$9.4179168399329002 \times 10^{-7}$	6	$8.8860242652344438 \times 10^{-7}$
7	$-1.8903844195250650 \times 10^{-8}$	7	$-5.1984536238194375 \times 10^{-8}$
8	$6.8711313932321633 \times 10^{-9}$	8	$3.8249044669732895 \times 10^{-9}$
9	$3.7380808618350893 \times 10^{-12}$	9	$-3.6076559467028432 \times 10^{-9}$

	ψ_A (deg)		Δ (deg)
0	6841.0782605878385	0	6841.9150826404655
1	55.799590548440034	1	56.762784562271115
2	$-5.8912997052256182 \times 10^{-1}$	2	$1.8616200067920070 \times 10^{-1}$
3	$1.3268904359089988 \times 10^{-2}$	3	$-4.0303795863546128 \times 10^{-2}$
4	$1.3897273624526529 \times 10^{-2}$	4	$-2.3600059273158092 \times 10^{-3}$
5	$-9.6496890251137706 \times 10^{-4}$	5	$5.2044865501247204 \times 10^{-4}$
6	$-7.8179683210018643 \times 10^{-5}$	6	$3.1971755601192933 \times 10^{-5}$
7	$2.4797217330568351 \times 10^{-5}$	7	$-4.5889405136051830 \times 10^{-6}$
8	$-1.6436203152044634 \times 10^{-6}$	8	$-4.6066822028954418 \times 10^{-7}$
9	$-2.5669470514404556 \times 10^{-7}$	9	$4.3457961369407859 \times 10^{-8}$

	ω_A (deg)		I (deg)
0	23.537015392172075	0	23.173373374658202
1	$8.7122285286289514 \times 10^{-1}$	1	$-4.3875570221823691 \times 10^{-1}$
2	$-5.6007712892337440 \times 10^{-3}$	2	$1.0312881154972785 \times 10^{-1}$
3	$-4.8144538945002723 \times 10^{-2}$	3	$7.5421570766006486 \times 10^{-3}$
4	$8.6521429559976106 \times 10^{-4}$	4	$-2.1061351237401471 \times 10^{-3}$
5	$6.1300771819802297 \times 10^{-4}$	5	$-6.8615386738549276 \times 10^{-5}$
6	$-3.5557460055419359 \times 10^{-5}$	6	$1.7287613573775773 \times 10^{-5}$
7	$-2.0982019804486842 \times 10^{-6}$	7	$8.0722357845541741 \times 10^{-7}$
8	$6.1682566690513504 \times 10^{-7}$	8	$-1.1851431149467077 \times 10^{-7}$
9	$-4.6981372537466530 \times 10^{-8}$	9	$-9.7660500051390622 \times 10^{-9}$

	χ_A (deg)		L (deg)
0	1.8760797180582457	0	1.0678200106335453
1	$-3.2810748860552903 \times 10^{-2}$	1	-1.0123000342453304
2	$-7.2708569977923652 \times 10^{-1}$	2	$-1.1699179607224553 \times 10^{-1}$
3	$1.4688902468087160 \times 10^{-2}$	3	$4.2364466425932615 \times 10^{-2}$
4	$1.5784839248001172 \times 10^{-2}$	4	$1.8787678352120043 \times 10^{-3}$
5	$-1.0036697769819112 \times 10^{-3}$	5	$-5.1928587283456078 \times 10^{-4}$
6	$-9.0041765462660084 \times 10^{-5}$	6	$-3.1460727280594530 \times 10^{-5}$
7	$2.4783191096963583 \times 10^{-5}$	7	$4.7088902328202858 \times 10^{-6}$
8	$-1.6009326791763078 \times 10^{-6}$	8	$4.6738684227823443 \times 10^{-7}$
9	$-2.5114767129151402 \times 10^{-7}$	9	$-4.7954457121605631 \times 10^{-8}$

Table 4–2, continued.

Interval 125: Central time $T_c = 4960$, covering the time span $4920 \le T \le 5000$

	ε (deg)		p_A (deg)
0	23.188823631241068	0	6951.5266870927663
1	$-1.0094910439540065 \times 10^{-1}$	1	56.297869550021695
2	$1.3528840290474921 \times 10^{-2}$	2	$3.3811386485587906 \times 10^{-2}$
3	$1.4138374375241296 \times 10^{-4}$	3	$-4.1443952305701973 \times 10^{-3}$
4	$-9.6049097551946857 \times 10^{-5}$	4	$1.3529085957587571 \times 10^{-4}$
5	$9.9589212874607648 \times 10^{-6}$	5	$1.6056952145206749 \times 10^{-5}$
6	$1.1881058130036544 \times 10^{-7}$	6	$-2.4800687003058616 \times 10^{-6}$
7	$-6.8523048469618859 \times 10^{-8}$	7	$1.6209914045028481 \times 10^{-8}$
8	$3.7839093075517994 \times 10^{-9}$	8	$1.5180358882542919 \times 10^{-8}$
9	$5.0265336993451665 \times 10^{-10}$	9	$-1.7539661704469478 \times 10^{-9}$

	ψ_A (deg)		Δ (deg)
0	6950.1629570160811	0	6955.4829611824177
1	54.035946952141830	1	56.484901739103247
2	$2.8191997034758717 \times 10^{-1}$	2	$-1.8317402951858377 \times 10^{-1}$
3	$9.2676822694587038 \times 10^{-2}$	3	$-2.2776486731418545 \times 10^{-3}$
4	$-2.0417328730575110 \times 10^{-3}$	4	$4.6698972022874455 \times 10^{-3}$
5	$-6.9833016604944183 \times 10^{-4}$	5	$-1.1913085638109604 \times 10^{-4}$
6	$-5.0291934364281871 \times 10^{-5}$	6	$-3.6470751055451883 \times 10^{-5}$
7	$-7.1889280279858131 \times 10^{-6}$	7	$2.9183599583072817 \times 10^{-6}$
8	$4.9085675183925939 \times 10^{-7}$	8	$7.6349296079709697 \times 10^{-8}$
9	$1.0467953513518096 \times 10^{-7}$	9	$-3.8118545915881334 \times 10^{-8}$

	ω_A (deg)		I (deg)
0	23.991574800043881	0	23.028719083565249
1	$-6.2437905670712945 \times 10^{-1}$	1	$2.2025869488809900 \times 10^{-1}$
2	$-2.4711001079768207 \times 10^{-1}$	2	$1.9282813497032315 \times 10^{-2}$
3	$1.8106793321239638 \times 10^{-2}$	3	$-1.5458948049034050 \times 10^{-2}$
4	$5.1085142180836745 \times 10^{-3}$	4	$1.2192141155936512 \times 10^{-4}$
5	$-1.1351271352006359 \times 10^{-4}$	5	$1.8840978186274336 \times 10^{-4}$
6	$-2.5820743168058098 \times 10^{-5}$	6	$-4.8320211994775649 \times 10^{-6}$
7	$-9.6463807211233002 \times 10^{-7}$	7	$-9.5672319461216363 \times 10^{-7}$
8	$-1.4740096996653601 \times 10^{-7}$	8	$7.1298929341885261 \times 10^{-8}$
9	$5.0608363292600033 \times 10^{-9}$	9	$7.4123118705798273 \times 10^{-10}$

	χ_A (deg)		L (deg)
0	-1.4847477189215144	0	$-4.5375941190135616 \times 10^{-1}$
1	-2.4633258398745768	1	$-2.0346710133428027 \times 10^{-1}$
2	$2.7518657045609766 \times 10^{-1}$	2	$2.3557145102737750 \times 10^{-1}$
3	$1.0621742018678135 \times 10^{-1}$	3	$-1.9557041163693521 \times 10^{-3}$
4	$-2.7601972525029799 \times 10^{-3}$	4	$-4.9098558144029168 \times 10^{-3}$
5	$-8.3468026238127847 \times 10^{-4}$	5	$1.3868585439737487 \times 10^{-4}$
6	$-4.2316611245645123 \times 10^{-5}$	6	$3.6689879576609133 \times 10^{-5}$
7	$-6.4546417547320033 \times 10^{-6}$	7	$-2.9109232618498862 \times 10^{-6}$
8	$4.4177459309447159 \times 10^{-7}$	8	$-7.1304556615981344 \times 10^{-8}$
9	$1.0333451255245648 \times 10^{-7}$	9	$3.6106535745533817 \times 10^{-8}$

CHAPTER 5
CONCLUSIONS

The previous three chapters have discussed the orientation of the invariable plane of the Solar System and have presented new formulations for precession using the invariable plane as a reference plane. This final chapter will compare the behavior of the short-term and long-term theories near $T = 0$, contrast the traditional approach to precession with the methods developed here, and finally give a summary of the dissertation as a whole.

5.1. The Short-Term and Long-Term Theories Compared

In both the short-term and long-term theories, which were presented in Chapters 3 and 4 above, the precession matrix $\mathbf{P}$ was expressed as a product of five elementary rotations:

$$\mathbf{P} = \mathbf{R}_3(-L)\,\mathbf{R}_1(-I)\,\mathbf{R}_3(-\Delta)\,\mathbf{R}_1(I_0)\,\mathbf{R}_3(L_0). \tag{3–4}$$

The only real difference between the two theories is in the method by which L, I, and Δ are calculated as functions of T. In the short-term theory, these angles are developed as fourth-degree polynomials in T, and the numerical values of the coefficients are derived from the coefficients of the classical precession angles ζ_A, θ_A, and z_A as given by Lieske *et al.* (1977). (Since Lieske *et al.* do not carry their expansion to T^4, the coefficients of the T^4 terms in the new short-term theory could not be calculated and were consequently omitted.) In the long-term theory, Chebyshev polynomials to degree nine were fit over 8000-year intervals to the angles obtained from a million-year numerical integration based

on the work of Laskar (1990) for the motion of the ecliptic and Kinoshita (1977) for the motion of the equator.

Coefficients for the sixty-third interval of the long-term theory, covering the time span $-40 \leq T \leq +40$ (*i.e.*, within 4000 years of J2000), are presented on page 179 as part of Table 4–2. For this special case, since the central time is zero, the dimensionless time τ that is the argument of the Chebyshev polynomials is simply $T/40$. The Chebyshev polynomials through degree nine are given, *e.g.*, in Table 22.3 of Abramowitz and Stegun (1964):

$$T_0(\tau) = 1; \tag{5–1}$$

$$T_1(\tau) = \tau; \tag{5–2}$$

$$T_2(\tau) = 2\tau^2 - 1; \tag{5–3}$$

$$T_3(\tau) = 4\tau^3 - 3\tau; \tag{5–4}$$

$$T_4(\tau) = 8\tau^4 - 8\tau^2 + 1; \tag{5–5}$$

$$T_5(\tau) = 16\tau^5 - 20\tau^3 + 5\tau; \tag{5–6}$$

$$T_6(\tau) = 32\tau^6 - 48\tau^4 + 18\tau^2 - 1; \tag{5–7}$$

$$T_7(\tau) = 64\tau^7 - 112\tau^5 + 56\tau^3 - 7\tau; \tag{5–8}$$

$$T_8(\tau) = 128\tau^8 - 256\tau^6 + 160\tau^4 - 32\tau^2 + 1; \tag{5–9}$$

$$T_9(\tau) = 256\tau^9 - 576\tau^7 + 432\tau^5 - 120\tau^3 + 9\tau. \tag{5–10}$$

Let the Chebyshev coefficients of some angle α be denoted by $\alpha_k^{(C)}$. Similarly, let the corresponding polynomial coefficients be denoted by $\alpha_k^{(p)}$. For any time T, the relationship between the two sets of coefficients is just

$$\alpha(T) = \sum_{k=0}^{9} \alpha_k^{(C)} T_k(\tau) = \sum_{k=0}^{9} \alpha_k^{(p)} T^k, \tag{5–11}$$

where $T_k(\tau)$ again is the Chebyshev polynomial of the first kind of degree k, as given in equations (5–1) through (5–10) above, and T^k is just the kth power of the time T. When

the literal expansions of the Chebyshev polynomials are inserted into equation (5–11), and τ replaced by $T/40$, the short-term coefficients follow by equating like powers of T:

$$\alpha_0^{(p)} = \alpha_0^{(C)} - \alpha_2^{(C)} + \alpha_4^{(C)} - \alpha_6^{(C)} + \alpha_8^{(C)}; \tag{5–12}$$

$$\alpha_1^{(p)} = (\alpha_1^{(C)} - 3\alpha_3^{(C)} + 5\alpha_5^{(C)} - 7\alpha_7^{(C)} + 9\alpha_9^{(C)})/40; \tag{5–13}$$

$$\alpha_2^{(p)} = (2\alpha_2^{(C)} - 8\alpha_4^{(C)} + 18\alpha_6^{(C)} - 32\alpha_8^{(C)})/40^2; \tag{5–14}$$

$$\alpha_3^{(p)} = (4\alpha_3^{(C)} - 20\alpha_5^{(C)} + 56\alpha_7^{(C)} - 120\alpha_9^{(C)})/40^3; \tag{5–15}$$

$$\alpha_4^{(p)} = (8\alpha_4^{(C)} - 48\alpha_6^{(C)} + 160\alpha_8^{(C)})/40^4; \tag{5–16}$$

$$\alpha_5^{(p)} = (16\alpha_5^{(C)} - 112\alpha_7^{(C)} + 432\alpha_9^{(C)})/40^5; \tag{5–17}$$

$$\alpha_6^{(p)} = (32\alpha_6^{(C)} - 256\alpha_8^{(C)})/40^6; \tag{5–18}$$

$$\alpha_7^{(p)} = (64\alpha_7^{(C)} - 576\alpha_9^{(C)})/40^7; \tag{5–19}$$

$$\alpha_8^{(p)} = 128\alpha_8^{(C)}/40^8; \tag{5–20}$$

$$\alpha_9^{(p)} = 256\alpha_9^{(C)}/40^9. \tag{5–21}$$

Table 5–1 presents the polynomial coefficients derived in this way from the long-term theory. The layout of this table is the same as for Table 4–2, except that the coefficients now multiply powers of T instead of the Chebyshev polynomials and that angles are now expressed in units of arcseconds instead of degrees. This table was also generated by computer and is reproduced without any changes. Round-off error has produced very small values for the leading terms in ψ_A, χ_A, p_A, and Δ; these should all be exactly zero at $T = 0$. The T^1 terms in ω_A and p_A also do not have quite their correct values. This can be attributed to errors in the Chebyshev fitting process itself, as the differences are below the μarcsec level typical of the reliability of the fit.

The accumulated angles ζ_A, θ_A, and z_A, which could not in general be fit without using either extremely small intervals or prohibitively high degree, can indeed be fit reliably over the center interval, as there is no near discontinuity there. Table 5–2 presents the Chebyshev

Table 5–1. Polynomial Coefficients from the Long-Term Theory Near $T = 0$

	ε ($''$)		p_A ($''$)
0	84381.447999999997	0	$3.2096354937947308 \times 10^{-14}$
1	−46.806497655369478	1	5029.0966001167056
2	$-1.7482661370733616 \times 10^{-4}$	2	1.1118779225027495
3	$1.9989756933499876 \times 10^{-3}$	3	$7.7773431374850854 \times 10^{-5}$
4	$-5.1185667204494724 \times 10^{-7}$	4	$-2.3525767443551284 \times 10^{-5}$
5	$-2.4947518619731177 \times 10^{-8}$	5	$-1.8114977819674100 \times 10^{-8}$
6	$-3.8916044464855812 \times 10^{-11}$	6	$1.7485819599146446 \times 10^{-10}$
7	$7.0507297924325245 \times 10^{-14}$	7	$1.3097223478486466 \times 10^{-12}$
8	$2.4416281017405718 \times 10^{-15}$	8	$1.5299648134210817 \times 10^{-15}$
9	$4.8019407223117058 \times 10^{-18}$	9	$-4.3216673180582778 \times 10^{-17}$

	ψ_A ($''$)		Δ ($''$)
0	$-2.7475132059457638 \times 10^{-12}$	0	$-1.7997745475347533 \times 10^{-14}$
1	5038.7863892515397	1	5116.1890147545821
2	−1.0725071925904761	2	2.9247603504428157
3	$-1.1431963045530742 \times 10^{-3}$	3	$-5.6326196820531548 \times 10^{-3}$
4	$1.3280912459688519 \times 10^{-4}$	4	$-9.222694874598149 7 \times 10^{-5}$
5	$-9.3981039224371332 \times 10^{-8}$	5	$-2.4515708859606546 \times 10^{-8}$
6	$-3.5052371754289082 \times 10^{-9}$	6	$2.3106548406850795 \times 10^{-9}$
7	$1.7020050932359007 \times 10^{-11}$	7	$8.8345579649427775 \times 10^{-12}$
8	$4.0719020380178632 \times 10^{-14}$	8	$-4.3285606007994351 \times 10^{-14}$
9	$-8.3135839486680791 \times 10^{-16}$	9	$-3.7244008535888019 \times 10^{-16}$

	ω_A ($''$)		I ($''$)
0	84381.448000000000	0	82831.996999999998
1	$2.4352645618446073 \times 10^{-7}$	1	−134.66870936181825
2	$5.1313374324774216 \times 10^{-2}$	2	$4.9758299780304608 \times 10^{-1}$
3	$-7.7279620767219688 \times 10^{-3}$	3	$6.1705421597441326 \times 10^{-3}$
4	$-4.8928730531731757 \times 10^{-7}$	4	$-1.8571794327493753 \times 10^{-5}$
5	$3.3277076443854503 \times 10^{-7}$	5	$-1.2930540672120392 \times 10^{-7}$
6	$-3.0808680901398760 \times 10^{-10}$	6	$1.2596797631849006 \times 10^{-10}$
7	$-5.4999131521020564 \times 10^{-12}$	7	$2.7747885594664064 \times 10^{-12}$
8	$2.2094006043264014 \times 10^{-14}$	8	$5.2707950955197274 \times 10^{-15}$
9	$3.2864671555067150 \times 10^{-17}$	9	$-4.8760926987798460 \times 10^{-17}$

	χ_A ($''$)		L ($''$)
0	$-7.6977189576168340 \times 10^{-14}$	0	13869.262000000001
1	10.561284314705913	1	−96.731815116024752
2	−2.3813672261570309	2	−1.9475397995104050
3	$-1.2130689169912052 \times 10^{-3}$	3	$6.6275370677766483 \times 10^{-3}$
4	$1.7026320623123781 \times 10^{-4}$	4	$7.1172391138639654 \times 10^{-5}$
5	$-7.6757734483341848 \times 10^{-8}$	5	$-1.7199898887319890 \times 10^{-8}$
6	$-3.9900133606309957 \times 10^{-9}$	6	$-2.1533502066675072 \times 10^{-9}$
7	$1.5528808073164337 \times 10^{-11}$	7	$-7.1770963974344956 \times 10^{-12}$
8	$4.2446737113117769 \times 10^{-14}$	8	$4.5035949742291784 \times 10^{-14}$
9	$-7.7819785068516923 \times 10^{-16}$	9	$3.2314957519653324 \times 10^{-16}$

coefficients for these three angles, in units of degrees as in Table 4–2, and Table 5–3 presents the derived polynomial coefficients in units of arcseconds as in Table 5–1.

For the sake of comparison, the polynomial coefficients from T^1 to T^3 from Tables 5–1 and 5–3 are repeated, along with their short-term counterparts, in Table 5–4. Except for the coefficients for L, I, and Δ, which are copied from Table 3–2, the short-term values are those of Lieske *et al.* (1977).

The obliquity at epoch (ε_0) and the speed of general precession in longitude at epoch (p_1) are identical for both theories. One might expect that the remaining coefficients would also be identical, but this is clearly not the case. There are two major reasons for the disagreement.

First, the motion of Laskar's ecliptic is slightly different from that of Lieske *et al.* (1977). Laskar's table gave a value for s_1 (the rate of $\sin \pi_A \sin \Pi_A$ at $T = 0$) of $4\overset{''}{.}2010$ per century (see equation (4–37) above), whereas Lieske *et al.* found $s_1 = 4\overset{''}{.}1976$/century. The difference is not in the adopted planetary masses, as both sources use the same values, nor is it likely to be in Laskar's neglect of Pluto, as Table 3 of Lieske *et al.* (1977) give Pluto's effect on s as $-0\overset{''}{.}0004$/century. The difference is most likely due to differences in the way long-period perturbations on the ecliptic are treated by Newcomb and by Laskar.

The difference in s_1 values immediately affects χ_1, the rate of planetary precession at $T = 0$, since χ_1 is just $s_1 \csc \varepsilon_0$. And since p_1 is held constant, a change in χ_1 will force a compensating change in ψ_1, the rate of luni-solar precession at $T = 0$, because these quantities are related by

$$\psi_1 = p_1 + \chi_1 \cos \varepsilon_0. \tag{4–13}$$

Changes to ζ_1, θ_1, and z_1 follow.

The second major numerical difference between the short-term and long-term results can be traced to the terms in Kinoshita's force model which contain M_1 and M_3. These

Table 5–2. Chebyshev Coefficients for ζ_A, θ_A, and z_A from the Long-Term Theory Near $T = 0$

	ζ_A (deg)
0	$6.5535995169492508 \times 10^{-2}$
1	25.859714192258054
2	$6.4990131263406525 \times 10^{-2}$
3	$7.7476413454556624 \times 10^{-2}$
4	$-5.5106804482914616 \times 10^{-4}$
5	$-5.3496658055591389 \times 10^{-4}$
6	$-5.0971853824247144 \times 10^{-6}$
7	$-3.9268416348947041 \times 10^{-6}$
8	$1.0695336061196845 \times 10^{-7}$
9	$4.5569025496622459 \times 10^{-8}$

	θ_A (deg)
0	$-9.6623117053158758 \times 10^{-2}$
1	21.710358783993135
2	$-9.7207740860621353 \times 10^{-2}$
3	$-1.8692053808322820 \times 10^{-1}$
4	$-5.7157566394378652 \times 10^{-4}$
5	$-1.8429987072279923 \times 10^{-4}$
6	$1.3064377738139140 \times 10^{-5}$
7	$6.0182402260548316 \times 10^{-6}$
8	$1.6234219264927075 \times 10^{-8}$
9	$1.0514313353239362 \times 10^{-8}$

	z_A (deg)
0	$2.3583842394992432 \times 10^{-1}$
1	25.862770541533251
2	$2.3332288095854970 \times 10^{-1}$
3	$7.8454570986236268 \times 10^{-2}$
4	$-2.5168150045011350 \times 10^{-3}$
5	$-5.5812815704333173 \times 10^{-4}$
6	$-1.1500791889797691 \times 10^{-6}$
7	$-3.0757413486748074 \times 10^{-6}$
8	$1.2193393735137808 \times 10^{-7}$
9	$3.3372158296169025 \times 10^{-8}$

Table 5–3. Polynomial Coefficients for ζ_A, θ_A, and z_A from the Long-Term Theory Near $T = 0$

	ζ_A ($''$)
0	$-4.5574789574403493 \times 10^{-13}$
1	2306.2174215303851
2	$3.0216067884230173 \times 10^{-1}$
3	$1.8021352810925796 \times 10^{-2}$
4	$-5.8313907338479862 \times 10^{-6}$
5	$-2.8476467079208087 \times 10^{-7}$
6	$-1.6742284214244399 \times 10^{-10}$
7	$-6.0988541204700905 \times 10^{-12}$
8	$7.5201584340234576 \times 10^{-15}$
9	$1.6020361092810187 \times 10^{-16}$

	θ_A ($''$)
0	$-2.3864172467750383 \times 10^{-13}$
1	2004.3141179252800
2	$-4.2661853348720036 \times 10^{-1}$
3	$-4.1830896120925914 \times 10^{-2}$
4	$-7.3084187027460287 \times 10^{-6}$
5	$-1.2720580656168292 \times 10^{-7}$
6	$3.6378291830159077 \times 10^{-10}$
7	$8.3300786657955114 \times 10^{-12}$
8	$1.1414685824399036 \times 10^{-15}$
9	$3.6964384189943970 \times 10^{-17}$

	z_A ($''$)
0	$-6.4814414125903593 \times 10^{-13}$
1	2306.2174216495372
2	1.0952002769438511
3	$1.8270258317263512 \times 10^{-2}$
4	$-2.8209102105137765 \times 10^{-5}$
5	$-3.0132950415077149 \times 10^{-7}$
6	$-5.9781112067212404 \times 10^{-11}$
7	$-4.7476277219832168 \times 10^{-12}$
8	$8.5734802732700903 \times 10^{-15}$
9	$1.1732399815981924 \times 10^{-16}$

Table 5–4. Polynomial Coefficients from the Long-Term and Short-Term Theories Near $T = 0$

	Long-Term (Chapter 4)	Short-Term (Chapter 3)
ε_1	$-46''.8065$	$-46''.8150$
ε_2	$-0''.00017$	$-0''.00059$
ε_3	$0''.001999$	$0''.001813$
p_1	$5029''.0966$	$5029''.0966$
p_2	$1''.11188$	$1''.11113$
p_3	$0''.000077$	$-0''.000006$
ζ_1	$2306''.2174$	$2306''.2181$
ζ_2	$0''.30216$	$0''.30188$
ζ_3	$0''.018021$	$0''.017998$
θ_1	$2004''.3141$	$2004''.3109$
θ_2	$-0''.42662$	$-0''.42665$
θ_3	$-0''.041831$	$-0''.041833$
z_1	$2306''.2174$	$2306''.2181$
z_2	$1''.09520$	$1''.09468$
z_3	$0''.018270$	$0''.018203$
ψ_1	$5038''.7864$	$5038''.7784$
ψ_2	$-1''.07251$	$-1''.07259$
ψ_3	$-0''.001143$	$-0''.001147$
χ_1	$10''.5613$	$10''.5526$
χ_2	$-2''.38137$	$-2''.38064$
χ_3	$-0''.001213$	$-0''.001147$
ω_1	$0''.0000$	$0''.0000$
ω_2	$0''.05131$	$0''.05127$
ω_3	$-0''.007728$	$-0''.007726$
L_1	$-96''.7318$	$-96''.7230$
L_2	$-1''.94754$	$-1''.94824$
L_3	$0''.006628$	$0''.006539$
I_1	$-134''.6687$	$-134''.6685$
I_2	$0''.49758$	$0''.49754$
I_3	$0''.006171$	$0''.006173$
Δ_1	$5116''.1890$	$5116''.1809$
Δ_2	$2''.92476$	$2''.92466$
Δ_3	$-0''.005633$	$-0''.005636$

terms were not known to Newcomb; if they are omitted, Kinoshita's rate of luni-solar precession (an instantaneous rate measured along the moving ecliptic of date) reduces to

$$R = (M_0 k_{☾} + S_0 k_{\odot}) \cos\varepsilon. \tag{5–22}$$

(The terms in M_2 and S_2, also unknown to Newcomb, have been eliminated from this equation as well. This however will not change the gist of the argument.) The expression in parentheses becomes Newcomb's Precessional Constant, P; its rate at $T = 0$, P_1, arises through the dependence of S_0 on the Earth's eccentricity:

$$P_1 = k_{\odot} \left(\frac{dS_0}{de_{\oplus}} \frac{de_{\oplus}}{dT} \right)_{T=0}. \tag{5–23}$$

Lieske *et al.* (1977) adopted $P_1 = -0''.00369$/century.

When Kinoshita's terms with M_1 and M_3 are added, the additional dependence on ε gives rise to new terms in P_1. The revised equation is

$$\begin{aligned} P_1 &= k_{\odot} \left(\frac{dS_0}{de_{\oplus}} \frac{de_{\oplus}}{dT} \right)_{T=0} \\ &\quad + k_{☾} \left[M_1 \frac{d}{dT} \left(\frac{2\cos 2\varepsilon}{\sin 2\varepsilon} \right) - M_3 \frac{m_{☾}}{m_{\oplus} + m_{☾}} \frac{n_{☾}^2}{\Omega n_{\Omega}} H \frac{d}{dT} \left(\frac{6\cos^2\varepsilon - 1}{\cos\varepsilon} \right) \right]_{T=0}. \end{aligned} \tag{5–24}$$

The change in P_1 shows up in the T^2 terms in ψ_A and p_A, and extends to the angles ζ_A, θ_A, and z_A through them. The differences in the T^2 terms of these last three angles can be analyzed, using equation (22) of Lieske *et al.* (1977), to infer a change of $-0''.00024$/century in P_1; therefore Kinoshita's model implies

$$P_1 = -0''.00393/\text{century}. \tag{5–25}$$

Should the currently-accepted value of P_1 be changed to the value given above? There is little sense in making such a change until a more precise constant of precession is adopted

by the IAU. Similarly, the value of s_1 above derives ultimately from DE200. Numerically determined values for all the c and s coefficients, using a post-Neptune successor to the DE200 ephemeris, might be adopted as part of the same package of changes.

5.2. Comparing the Classical and Invariable Plane Precession Formulations

We have now three ways of computing the precession matrix for transforming equatorial coordinates from mean of J2000 to mean of date:

$$\mathbf{P} = \mathbf{R}_3(-z_A)\,\mathbf{R}_2(\theta_A)\,\mathbf{R}_3(-\zeta_A) \tag{2–1}$$

$$= \mathbf{R}_3(\chi_A)\,\mathbf{R}_1(-\omega_A)\,\mathbf{R}_3(-\psi_A)\,\mathbf{R}_1(\varepsilon_0) \tag{4–55}$$

$$= \mathbf{R}_3(-L)\,\mathbf{R}_1(-I)\,\mathbf{R}_3(-\Delta)\,\mathbf{R}_1(I_0)\,\mathbf{R}_3(L_0). \tag{3–4}$$

The first method uses three Euler angles and is the most direct way of transforming equatorial coordinates. An additional advantage is that the first and last rotations are about the z-axis; the arguments ζ_A and z_A combine trivially with the initial and final right ascension, respectively, in effect reducing the problem to one rotation by θ_A. However, the three angles undergo rapid changes whenever the celestial pole passes near its position at $T = 0$; consequently this method is not well suited for long-term work.

The second method uses a sequence of four rotations, but only three of the four rotation angles are functions of time. The formulation is a bit more complicated than the first method, partly because of the extra rotation and partly because the first rotation matrix is not an $\mathbf{R}_3$ matrix. This deficiency is more than overcome by the better long-term behavior of the rotation angles. Over the long term, ψ_A is close to a linear function of time, and χ_A and ω_A oscillate slowly in a complicated fashion; nevertheless, these angles are all well behaved and amenable to Chebyshev economization. Consequently the second formulation is better for long-term use.

The third formula for **P** is at first glance even more involved than the first two, because it contains five rotations. Again, only three of the angles are functions of time, as I_0 and L_0 are constant. However, if the intersection of the mean equator and the invariable plane were to be adopted as the origin for right ascensions, the first and fifth rotations would vanish, leaving once again three rotations, but only two of them (I and Δ) functions of time. This change of origin would therefore make the third method the simplest of all.

A comparison of equations (4–55) and (3–4) for **P** shows that Δ has the character of the accumulated luni-solar precession ψ_A, as both are the arguments of the central rotation in a 1–3–1 sequence. Similarly, I is comparable to ω_A, and L corresponds to the accumulated planetary precession χ_A. Comparisons of the plots of these three pairs of angles (Figure 4–2) shows that the angles involving the invariable plane are better behaved than their classical counterparts. The range of χ_A is some 20°, from $-10^\circ\!.2$ to $+9^\circ\!.8$, but L is restricted to $\pm 7^\circ\!.5$ over the same million-year interval. Furthermore, the variations in L seem smoother than those in χ_A. Similar behavior holds for I and ω_A: the range of I is $7^\circ\!.7$, while that of ω_A is $9^\circ\!.8$. These results merely indicate that the ecliptic, on average, is closer to the invariable plane than it is to the J2000 ecliptic. This is shown also by Figure 5–1, in which the right ascension and declination of the wandering ecliptic pole are plotted together with the pole of the adopted invariable plane.

Even over the short term, the angles L, I, and Δ are just as well behaved as ζ_A, θ_A, and z_A. The polynomial coefficients L_k, I_k, and Δ_k given in Table 5–1 are all of similar magnitude to the corresponding coefficients ζ_k, θ_k, and z_k in Table 5–3. This indicates that polynomials in either set of angles will yield results of roughly the same accuracy.

As shown in Section 3.6, the formulation in terms of the invariable plane is much simpler for precession between two arbitrary epochs. In this case only the change in Δ from T_1 to T_2 needs to be developed as a two-argument polynomial in the short-term

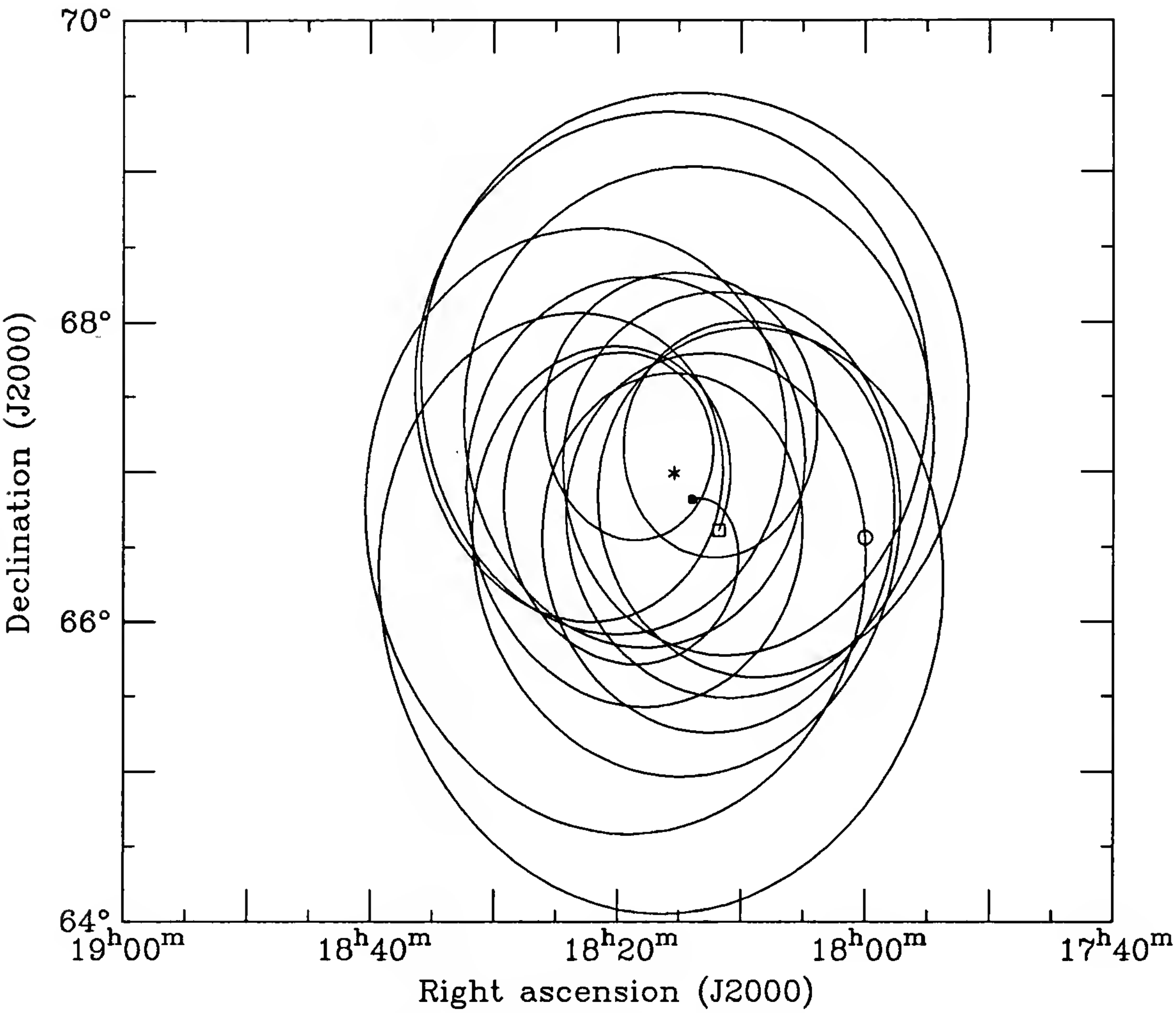

Figure 5-1. J2000 Equatorial Coordinates of the Ecliptic Pole and of the Pole of the Invariable Plane. The positions of the ecliptic pole at $T = -5000$, $T = 0$, and $T = +5000$ are marked with an open square, open circle, and filled circle, respectively. The position of the pole of the invariable plane is marked with an asterisk.

theory; the polynomials for I and L are evaluated once at T_1 and again at T_2. By contrast, the classical formulation expresses all its angles as functions of both T_1 and $(T_2 - T_1)$.

Another comparison that can be made involves nutation, the periodic displacement of the "true" celestial pole relative to the "mean" celestial pole defined by precession only. Standard practice defines two angles, the nutation in obliquity $\Delta\varepsilon$ and the nutation in longitude $\Delta\psi$; the current expressions for these angles are given by Wahr (1979). The nutation matrix $\mathbf{N}$, which rotates from mean-of-date coordinates into true-of-date, is

$$\mathbf{N} = \mathbf{R}_1[-(\varepsilon + \Delta\varepsilon)]\,\mathbf{R}_3(-\Delta\psi)\,\mathbf{R}_1(\varepsilon). \tag{5–26}$$

This matrix, however, does not interact nicely with either the first or second formulation for $\mathbf{P}$. It should be possible to derive new nutation angles, ΔI and $\delta\Delta$, such that the combined precession-nutation matrix would be given by

$$\mathbf{N}'\mathbf{P}' = \mathbf{R}_1[-(I + \Delta I)]\,\mathbf{R}_3(\Delta + \delta\Delta)\,\mathbf{R}_1(I_0). \tag{5–27}$$

(The primes denote, as in Section 3.7, that right ascensions are reckoned from the invariable plane.) Such a formulation would make nutation much easier to apply; a full development of equation (5–27) lies outside the scope of this work.

Finally, an important consequence of switching to the invariable plane is that precession theory itself is simplified: the equatorial coordinate system would be defined by one moving plane (the equator) and one fixed plane (the invariable plane), rather than by two moving planes (the ecliptic and equator). The standard concepts of luni-solar precession, planetary precession, and general precession have historically been rather difficult ones for generations of astronomy students; their interaction can be subtle. With the use of the invariable plane to define the origin for right ascensions, the need for a fourth rotation (planetary precession) is abandoned; this is so because (unlike the ecliptic) the invariable plane is fixed in space. Thus there would be no need to distinguish between luni-solar and general precession. The ecliptic is still necessary, as its orientation determines the speed and direction of the motion of the mean celestial pole (Section 4.3); but since the ecliptic appears explicitly only in the differential equations (not in the rotation matrices), its motion is in effect now subordinated to that of the equator. The pedagogical advantages of the new scheme should be apparent.

Precession theory referred to the invariable plane, when compared to the current formulation in terms of the ecliptic, is thus seen to be simpler in practice (fewer polynomials

to evaluate, with fewer arguments) and far simpler conceptually. The price to be paid in order to put the new theory into practice is steep and perhaps prohibitive: the origin of the right ascension system would be changed. Not only would all right ascensions change, but so would their annual variations. The formulation for sidereal time would change to reflect the displacement and motion of the new origin relative to the mean equinox. Future rereductions of published observations would have to include the transformation into the new system (a subtraction of $L(T)$ from their right ascensions). And students would still be required to learn both new and old equatorial coordinates. Such a change would consequently be even more far-reaching than the 1960 redefinition of galactic coordinates or the 1976 adoption of the J2000 system; it could only be adopted, after careful consideration, by the entire astronomical community.

5.3. Summary

This dissertation has given an alternative method for computing the precession matrix **P**, a rotation matrix that transforms from the Earth mean equator and vernal equinox of the standard epoch J2000.0 to the Earth mean equator and vernal equinox of date. The new method employs the invariable plane of the Solar System, whose inclination I_0 and ascending node L_0 (both reckoned in the mean equatorial system of epoch J2000.0) were found in Chapter 2 to be

$$I_0 = 23^\circ 00' 31\overset{''}{.}997 \pm 0\overset{''}{.}013, \qquad (2\text{–}46)$$

$$L_0 = \ \ 3^\circ 51' 09\overset{''}{.}262 \pm 0\overset{''}{.}038. \qquad (2\text{–}45)$$

The precession matrix is written:

$$\mathbf{P} = \mathbf{R}_3(-L)\,\mathbf{R}_1(-I)\,\mathbf{R}_3(-\Delta)\,\mathbf{R}_1(I_0)\,\mathbf{R}_3(L_0), \qquad (3\text{–}4)$$

where the angles L, I, and Δ are functions of time.

For dates not more than a few centuries from J2000.0, the time-varying angles may be expressed as cubic polynomials in T, which is the time measured in Julian centuries from J2000.0:

$$L = 3°51'09''.262 - 96''.7230T - 1''.94824T^2 + 0''.006539T^3; \tag{5–28}$$

$$I = 23°00'31''.997 - 134''.6685T + 0''.49754T^2 + 0''.006173T^3; \tag{5–29}$$

$$\Delta = 5116''.1809T + 2''.92466T^2 - 0''.005636T^3. \tag{5–30}$$

The coefficients in these three equations were found in Chapter 3; they are based on the work of Lieske *et al.* (1977). The matrix **P** found this way agrees with theirs to the limits of their theory.

For dates within 500,000 years of J2000.0, the angles are expressed in sums of Chebyshev polynomials; the polynomial coefficients for each of 125 different 8000-year time intervals are given in Table 4–2. Here the angles are found by:

$$L = \sum_{k=0}^{9} L_k T_k(\tau); \tag{5–31}$$

$$I = \sum_{k=0}^{9} I_k T_k(\tau); \tag{5–32}$$

$$\Delta = \sum_{k=0}^{9} \Delta_k T_k(\tau). \tag{5–33}$$

In these equations, the T_k are the Chebyshev polynomials of the first kind of degree k, and τ is a dimensionless time given in terms of T by

$$i = \lfloor (T + 5080)/80 \rfloor; \tag{4–67}$$

$$T_c = 80i - 5040; \tag{4–68}$$

$$\tau = (T - T_c)/40. \tag{4–69}$$

The coefficients L_k, I_k, and Δ_k are found on the ith page of Table 4–2. These coefficients, along with similar ones for the classical precession angles, were obtained from a numerical integration of the motion of the mean celestial pole, using Kinoshita's (1977) formulation for its rate and Laskar's (1985, 1986, 1988, 1990) solution for the orientation of the ecliptic.

Precession between two arbitrary times T_1 and T_2 can in theory be obtained by matrix multiplication; with the starting and ending times now explicit in the notation,

$$\mathbf{P}(T_1 \rightarrow T_2) = \mathbf{P}(0 \rightarrow T_2)\,\mathbf{P}(0 \rightarrow T_1)^{\mathrm{T}}. \tag{5–34}$$

When the right-hand side is expanded, two pairs of elementary rotations cancel, leaving

$$\mathbf{P}(T_1 \rightarrow T_2) = \mathbf{R}_3[-L(T_2)]\,\mathbf{R}_1[-I(T_2)]\,\mathbf{R}_3[\Delta(T_1) - \Delta(T_2)]\,\mathbf{R}_1[I(T_1)]\,\mathbf{R}_3[L(T_1)]. \tag{5–35}$$

For short-term work, the angles L and I are found by equations (5–28) and (5–29) as before, and the difference between the two values for Δ is best evaluated by defining $t \equiv T_2 - T_1$ and then evaluating

$$\begin{aligned}\Delta(T_2) - \Delta(T_1) = {} & (5116''.1809 + 5''.89432T_1 - 0''.016908T_1^2)t \\ & + (2''.92466 - 0''.016908T_1)t^2 - 0''.005636t^3.\end{aligned} \tag{5–36}$$

For long-term work, $\Delta(T_2)$ and $\Delta(T_1)$ would probably be found independently and subtracted as written.

The matrix $\mathbf{P}$ would be simplified considerably if astronomers were to shift the origin of right ascensions from the vernal equinox to the nearby intersection of the equator and the invariable plane. Arguments in favor of this radical redefinition of right ascension were presented in Sections 3.7 and 5.2, primarily that precession theory would thereby be made simpler both in concept and in practice. The only substantial argument against this proposal is that it might be more difficult to reduce old observations referred to the vernal

equinox. Whether the precession theory proposed here is to come into general use depends therefore on a decision that can only be made by the entire astronomical community.

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BIOGRAPHICAL SKETCH

William Mann Owen, Jr., a native of Baltimore, Maryland, received his B.S. degree with honors from the California Institute of Technology in 1976 and his M.S. degree from the University of Florida in 1988. He has been employed since 1979 at the Jet Propulsion Laboratory in Pasadena, California, where as a member of Voyager's navigation team he estimated the orbits of the small satellites of Uranus and Neptune. He has received a National Merit Scholarship, a University of Florida Graduate Council Fellowship, a National Science Foundation Graduate Fellowship, and a National Aeronautics and Space Administration Exceptional Service Medal. He was elected to the Phi Kappa Phi honor society in 1988. He is a member of the American Astronomical Society.

Owen lives in Altadena, California, with his wife, Elizabeth, and three children. His interests include piano, organ, singing in mixed choirs, coin collecting, and amateur astronomy.

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

Heinrich K. Eichhorn, Chair
Professor of Astronomy

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

Kwan-Yu Chen
Professor of Astronomy

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

John P. Oliver
Associate Professor of Astronomy

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

Haywood C. Smith
Associate Professor of Astronomy

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

Jay H Lieske

Jay H. Lieske
Senior Research Scientist,
Jet Propulsion Laboratory,
California Institute of Technology

This dissertation was submitted to the Graduate Faculty of the Department of Astronomy in the College of Liberal Arts and Sciences and to the Graduate School and was accepted as partial fulfillment of the requirements for the degree of Doctor of Philosophy.

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Dean, Graduate School